

## Deflection of Free End Lapped Connected Z Cold Formed Purlins

H.M. Maaly

Structural Engineering Dept., Faculty of Engineering, Zagazig University, Egypt  
[dr\\_h\\_maaly@yahoo.com](mailto:dr_h_maaly@yahoo.com)

**Abstract:** Cold-formed steel Z sections are widely used in modern roof systems as purlins. To maintain their continuity, these sections might be overlapped over their interior supports. The overlapping operation introduces cross-sectional changes which would affect the structural behavior of the purlins upon loading. In this study, many parameters that affect the structural behavior of Z purlins are introduced. The finite element technique was used to simulate the structural behavior of lapped connections over the internal supports in multi-span cold-formed stiffened steel Z purlin systems. The considered beams had span lengths of 500,600,700 and 800 cm with nominal web depths of 20.0 cm. and thickness 2.0 mm. The work program includes modeling of a single Z section beam with lap lengths 50,100,125,150 and 175 cm. The considered lap models for free ends lap joint without bolts on the web at the lap ends plus self-drilling screws at the top flange. The case of simply supported Z purlins is also considered in the work for comparison. Based on this analysis using the (ABAQUS 6.8) the deflections of lapped beams of stiffened Z sections are studied and presented. Empirical equations were obtained to predict the deflection of free end lapped purlin. The results based on these equations were compared with the experimental results and good agreement is achieved.

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### 1. Introduction

Cold-formed steel purlins are widely used in modern roof systems due to their high structural efficiency and build ability. The most common shapes of cold-formed steel purlins are C and Z sections, and the section depth ranges from 100 to 350 mm while the thickness ranges from 1.2 to 3.0 mm. Common yield strengths are 280 and 350 N/mm<sup>2</sup>, but recently, sections with yield strength up to 450 N/mm<sup>2</sup> may be found in some purlin systems giving improved load carrying capacities. **Ghossn** and **Sinno**[3,4] performed a series of tests on unrestrained lapped cold-formed steel Z- sections. A total of twenty-eight stiffened Z-section beams were tested by applying lateral load at the mid-span until failed. The test series covered various section sizes (section depth to thickness ratio ranges from 79 to 131) and lap lengths (lap length to span ratio ranges from 0.25 to 1.00). It was noted that the lapped sections enhanced the load carrying capacities and the stiffnesses, significantly with the lap length to span ratio up to 0.5. Little enhancement was discovered when the lap length to span ratio increased any further. For the lap length to span ratio less than 0.5, the specimens were failed in the single section at the ends of lap while they failed at the lapped region near mid-span for the lap length to span ratio larger than 0.5. **Ghossn** and **Sinno** concluded that these specimens were failed by bending after

stress analyses. A reduction factor,  $R_s$ , was

introduced to estimate the moment capacities of the specimens failed in the lapped region. The ratio between the predicted and the measured moment resistances was found to range from 0.85 to 1.23. This reflected that the predicted moment resistances were overestimated in some cases which gave unsafe design. They also proposed two empirical formulae to estimate both the load resistance ratio and the stiffness. These formulae were only applicable for limited section sizes, material properties and specific fastener sizes. **Ho H.C., K.F. Chung**[6] presented an analytical method to predict the deformation characteristics of lapped connections of cold-formed Z sections. **Cheng Yu, and Benjamin W. Schafer** [7] presented a nonlinear finite element model to Simulate two series of flexural tests for cold-formed steel C and Z section beams in local and distortional buckling with applications to the direct strength method. **Lei Zhang, Gengshu Tong** [8] reported their research work on the structural behavior of multiple span lapped purlin systems restrained by roof sheeting using cold-formed Z sections.

**Ghossn** [5] tested stiffened Z-section beam purlins to evaluate the deflection behavior of lap joints under combined bending and shear. The results were obtained at failure load and the mode of failure for each test was illustrated. Twenty eight sets of

specimens were tested where their dimensions and material properties are shown in Tables (1) and (2) and Fig.(1) show the details of test setup

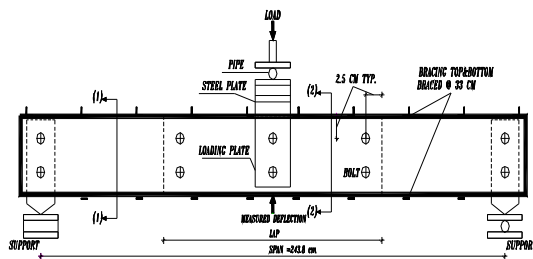


Fig.(1) Show the Details of Test Setup

**2. Verification by Finite Element Model**

Table (1): Test Parameters for Tested Beams [4]

Section <sup>a</sup>	Web depth to thickness, h/t	Lap length (cm)	Number of testes
Z 8X060	131	0 <sup>b</sup> , 61.0, 121.9, 243.8 <sup>c</sup>	4
Z 9.5X075	125	0, 61.0, 91.4, 121.9, 152.4, 243.8	12
Z 8X075	105	0, 61.0, 121.9, 243.8	4
Z 9.5X101	92	0, 61.0, 121.9, 243.8	4
Z 8X099	79	0, 61.0, 121.9, 243.8	4

Where:

<sup>a</sup> Section is identified as: Z (h)\*(t); where "h" is the web depth in inches (1 in. =25.4 mm) and "t" is the thickness in thousandths of an inch.

<sup>b</sup> Single continuous Z section ( no lap).

<sup>c</sup> Full lap: double Z section over entire span.

Table (2): Cross-sectional and Mechanical Properties of Tested Beams [4].

Section	h/t	I <sub>g</sub> (cm <sup>4</sup> ) <sup>a</sup>	I <sub>e</sub> (cm <sup>4</sup> ) <sup>b</sup>	S <sub>e</sub> (cm <sup>3</sup> ) <sup>c</sup>	F <sub>y</sub> (Mpa) <sup>d</sup>	F <sub>u</sub> (Mpa) <sup>e</sup>
Z 8X060	131	344.2	314.3	29.17	418	529
Z 9.5X075	125	638.0	591.9	47.05	342	521
Z 8X075	105	425.0	392.5	36.87	342	521
Z 9.5X101	92	854.9	854.9	70.79	421	531
Z 8X099	79	557.3	557.3	54.88	421	531

Where:

<sup>a</sup> I<sub>g</sub> =Gross moment of inertia.

<sup>b</sup> I<sub>e</sub> =Effective moment of inertia.

<sup>c</sup> S<sub>e</sub> = Effective section modulus.

<sup>d</sup> F<sub>y</sub> =Yield strength.

<sup>e</sup> F<sub>u</sub> =Ultimate strength.

**3. Comparison Study**

Four specimens of section Z8X060 (Single Z, Lap 61 cm, Lap 121.9 cm, Lap 243.8 cm) with material properties mentioned before are modeled by Using F.E.M (ABAQUS V 6.8) [2] results are plotted together with the experimental data in Figs. (3), (4), (5) and (6). The graphs indicate that the FEM models are approximately agree with the experimental work for each specimen with acceptable accuracy. These figures show the relation between deflection and load, the deflection increase with load up to the failure of the beam. It is being noted that the deflections decrease with the increase of lap length. Fig.(2) shows the deflection for lapped purlin with lap length=61cm

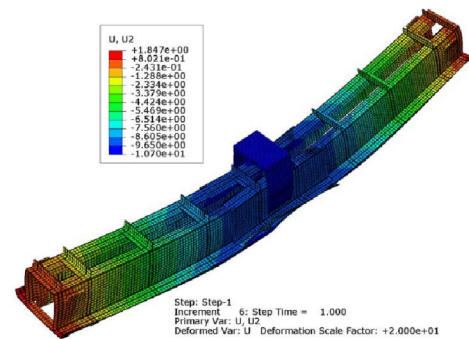


Fig.(2) Deflection for Lap Length=61cm

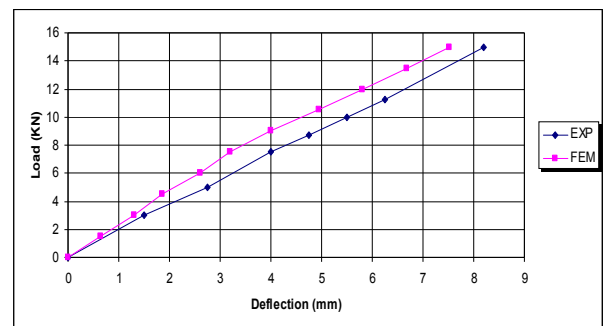


Fig.(3) Load- Deflection Curve at Mid-span For (Single Beam)

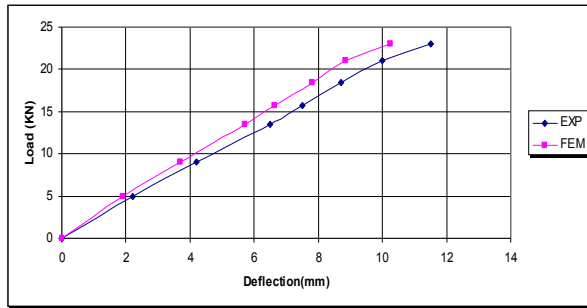


Fig.(4) Load- Deflection Curve at Mid-span For (Lap Length=61cm)

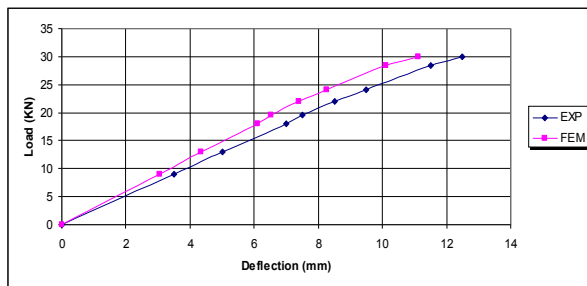


Fig.(5) Load- Deflection Curve at Mid-span For (Lap Length=121.9cm)

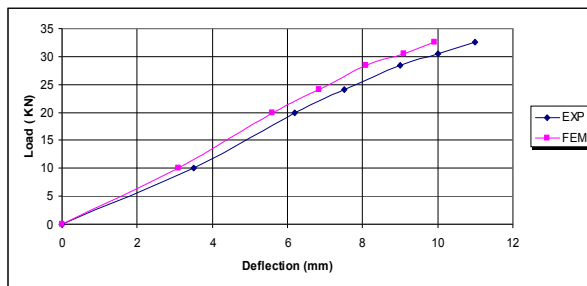


Fig.(6) Load- Deflection Curve at Mid-span For (Full Lap=243.8cm)

Table (3): Values of Maximum Deflection VS. Test Results at Mid-Span.

Test No.	FEM deflection (mm)	Lab. Test deflection (mm)	FEM/Test deflection Ratio
1	-7.52	-8.2	0.917
2	-10.25	-11.5	0.891
3	-11.12	-12.5	0.889
4	9.85	-11	0.895

A comparison of the FEM results with test results listed in table (3) indicates that the F.E method is an effective means to predict the deflection of Z purlin. Generally, there is a shift between experimental test and FEM deflection curve due to

fixed simulation of the bolts in FEM where the bolts are simulate with tie constrain. Maximum deflection values are approximately equal.

#### 4. Parametric Study

The corrugated sheets with thickness of 0.50 mm and width 1.0 m as shown in Fig.(7), give:  $A=6.09 \text{ cm}^2$ ,  $Wt=4.78 \text{ kg/ m}^2$

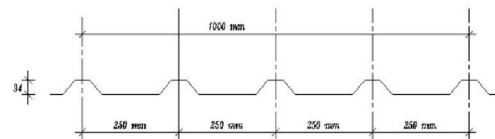


Fig.(7)The Considered Section of corrugated steel sheets

The considered section in this study is shown in Fig.(8) where, The purlin section is 200Z20 with properties:  $I_x = 409.1 \text{ cm}^4$ ,  $I_y = 57.30 \text{ cm}^4$ .  $A=6.90 \text{ cm}^2$ ,  $A_{eff}=6.74 \text{ cm}^2$ .  $Wt=5.42 \text{ kg/m}$ .

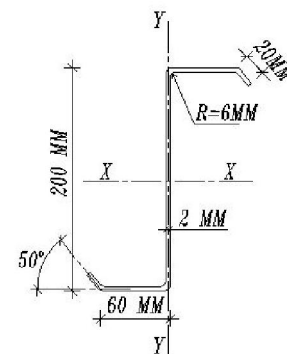


Fig.(8) The Considered Section of Z purlin

The seat angle dimensions 160\*80\*8 mm and length =110 mm is used. The spacing between the purlins equals 150 cm. The load considered in this work is uniform vertical load equals  $105 \text{ kg/m}^2$ , which represent own weight of purlin, own weight of corrugated sheets and live load equals  $53.50 \text{ kg/m}^2$ . The properties of steel that used in the study are,  $E = 200000 \text{ N/mm}^2$ ,  $\gamma= 7800 \text{ kg /m}^3$ ,  $f_y = 360 \text{ N/mm}^2$ ,  $f_u = 520 \text{ N/ mm}^2$   $\nu=0.30$  where:

$E =$  Young's modulus,  $\text{N/ mm}^2$ ,  $\nu =$  Poisson's ratio

$\gamma =$  unit weight,  $\text{kg / m}^3$

$f_y =$  steel yield stress,  $\text{N/ mm}^2$

$f_u$  = steel ultimate stress, N/ mm<sup>2</sup>

### 4.1 Deflection Analysis of Free End Lapped Sections.

The considered model consists of two Z sections with spacing equals 150 cm connected by corrugated steel sheets. One seat angle is used for each connection; purlins used are with different lengths (L) 500,600,700 and 800 cm with different lap length ( $L_p$ ) 50,100,125,150 and 175 cm for each length as shown in Fig. (9). 4 bolts of diameter 12 mm are used in the lap joint as shown in Fig. (9).The deflection is studied at centerline of the web of the purlin along the whole length. Fig. (10) shows the deflection for ( $L_p = 50$  cm, L=500 cm).

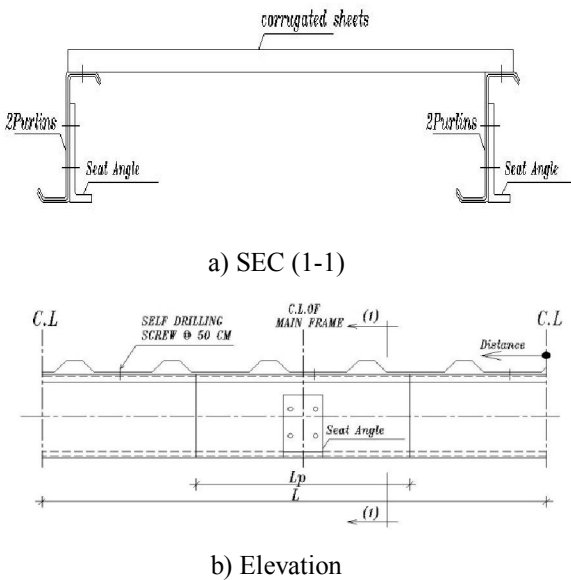


Fig.(9) Free End Lapped Connection

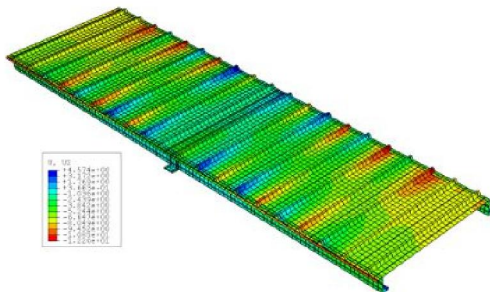


Fig.(10) Deflection for ( $L_p = 50$  cm, L=500 cm)

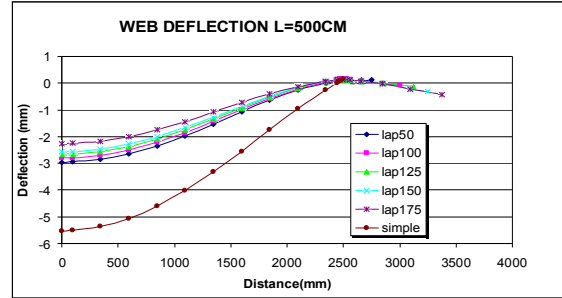


Fig.(11) Deflection along Centerline of Web for Span 500cm

Fig (11) shows the relation between deflections versus the distance along web center line with different lengths of lap joints for span length equals 500 cm. For simple purlin the deflection increases from zero at the seat angle to the maximum value (5.528 mm) at mid-span of purlin. The deflection decreases with the increase of lap length, where the maximum deflection is ranging from 2.95 mm to 2.257 mm for lap length ranging from 50 to 175cm respectively. The ratio between the maximum deflection from lapped span with lap length 175 cm to the deflection of simple span equals to 0.408. Table (4) shows the ratio between the deflections for different lap lengths at maximum value to the maximum deflection value for the simple span.

Table (4): Deflection at Mid-Span for Span 500cm

Lap length (cm)	Deflection at mid span	
	Deflection(mm)	Ratio
$L_p=50$	-2.95	0.533
$L_p=100$	-2.790	0.505
$L_p=125$	-2.660	0.481
$L_p=150$	-2.556	0.462
$L_p=175$	-2.257	0.408
Simple	-5.528	1

The ratios of maximum deflection of free end lapped purlins to simple one( $\Delta/\Delta_s$ ) are plotted versus the ratios of lap length to span length ( $L_p / L$ ) in

Fig.(12).

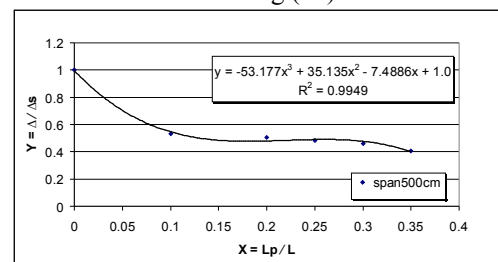


Fig.(12) Relation between  $\Delta/\Delta_s$  and  $L_p / L$  for Span 500 cm

From Fig.(12) the following equation can be obtained.

$$\Delta/\Delta_s = -53.177 (L_p/L)^3 + 35.135 (L_p/L)^2 - 7.4886 (L_p/L) + 1.0 \dots\dots\dots (1)$$

The work is repeated for spans 600,700 and 800 cm and the results are shown in Figs. (13,14,15,16,17&18) and Tables(5,6&7).

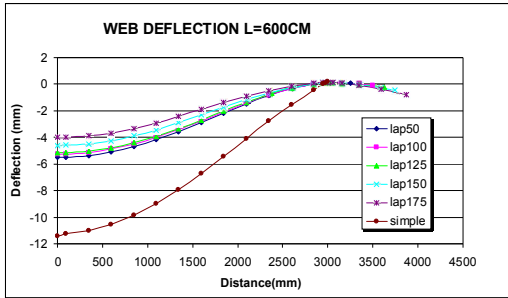


Fig.(13) Deflection along Centerline of Web for Span 600cm

Table (5): Deflection at Mid-Span for Span 600cm

Lap length (cm)	Deflection at mid span	
	Deflection(mm)	Ratio
L <sub>p</sub> = 50	-5.506	0.482
L <sub>p</sub> =100	-5.248	0.460
L <sub>p</sub> =125	-5.157	0.452
L <sub>p</sub> =150	-4.60	0.403
L <sub>p</sub> =175	-3.998	0.350
Simple	-11.408	1

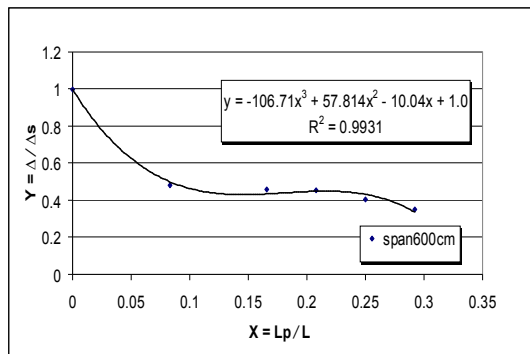


Fig.(14) Relation between  $\Delta/\Delta_s$  and  $L_p/L$  for Span 600 cm

From Fig.(14) the following equation can be obtained.

$$\Delta/\Delta_s = -106.71 (L_p/L)^3 + 57.814 (L_p/L)^2 - 10.04 (L_p/L) + 1.0 \dots\dots\dots (2)$$

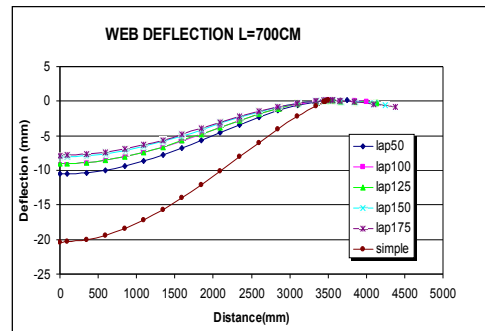


Fig.(15) Deflection along Centerline of Web for Span 700cm

Table (6): Deflection at Mid-Span for Span 700cm

Lap length (cm)	Deflection at mid span	
	Deflection(mm)	Ratio
L <sub>p</sub> = 50	-10.544	0.517
L <sub>p</sub> =100	-9.05	0.444
L <sub>p</sub> =125	-9.03	0.443
L <sub>p</sub> =150	-8.07	0.396
L <sub>p</sub> =175	-7.83	0.384
Simple	-20.36	1

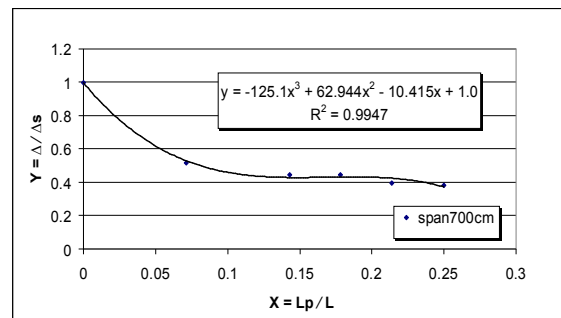


Fig.(16) Relation between  $\Delta/\Delta_s$  and  $L_p/L$  for span 700 cm

From Fig.(16) the following equation can be obtained.

$$\Delta/\Delta_s = -125.1 (L_p/L)^3 + 62.944 (L_p/L)^2 - 10.415 (L_p/L) + 1.0 \dots\dots\dots (3)$$

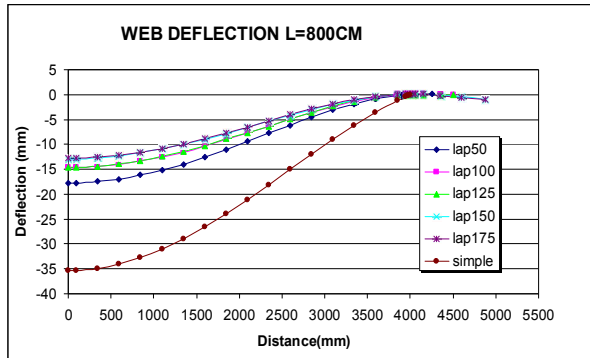


Fig.(17) Deflection along Centerline of Web for Span 800cm

Table (7): Deflection at Mid-Span for Span 800cm

Lap length (cm)	Deflection at mid span	
	Deflection(mm)	Ratio
L <sub>p</sub> = 50	-17.76	0.50
L <sub>p</sub> =100	-14.68	0.415
L <sub>p</sub> =125	-14.55	0.411
L <sub>p</sub> =150	-12.87	0.364
L <sub>p</sub> =175	-12.77	0.361
Simple	-35.39	1

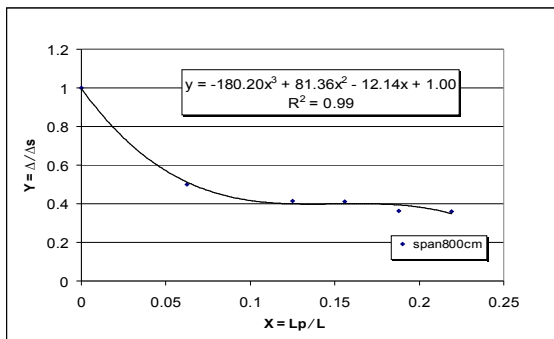


Fig.(18) Relation between  $\Delta/\Delta_s$  and  $L_p/L$  for span 800 cm

From Fig.(18) the following equation can be obtained.

$$\Delta/\Delta_s = -180.20 (L_p/L)^3 + 81.36 (L_p/L)^2 - 12.14 (L_p/L) + 1.0 \dots\dots\dots (4)$$

From the previous equations (1), (2), (3) and (4) we can get a general empirical equation for deflection of free lapped purlin with respect to simple beam deflection by using curve fitting as follow:

$$\Delta/\Delta_s = A (L_p/L)^3 + B (L_p/L)^2 + C (L_p/L) + 1.0 \dots\dots\dots (5)$$

The constants A, B and C are functions of span L as follows:

$$A = -11.97 (L)^3 + 233.1 (L)^2 - 1528.03 (L) + 3256.15$$

$$B = 5.14 (L)^3 - 101.29 (L)^2 + 669.15 (L) - 1420.85$$

$$C = -0.5871 (L)^3 + 11.65 (L)^2 - 77.29 (L) + 161.12$$

Where:

$\Delta$  = max-deflection for lapped purlin, mm

$\Delta_s$  = max-deflection for simple purlin, mm

$L_p$  = lap length, m

L = span of purlin, m

Table (8) show the deflection values that obtained from the study and that values from general empirical equation. The ratio between the deflection values from empirical equation to that from F.E.M results are given also in Table(8). From the table we find that the ratio is ranging from 0.946 %to 1.082 % so that the general empirical equation can be used to predict the deflection value to this case with good results.

### 5. Summary and Conclusions

A Finite Element model for studying the deflection of bolted lapped purlin is introduced to check the results of the model with experimental ones. A good agreement is achieved.

A new study for free end lapped purlins is introduced. In this study the deflection results for different spans and lap lengths are plotted in curves. Finally a general empirical equation is obtained which can predict the ratio of the maximum deflection of free end lapped purlins to simple one with high accuracy.

Table (8): Deflection values of F.E.M and Empirical General Equation

SECTION	SPAN (cm)	LAP LENGTH (cm)	FREE END		GENERAL		RATIO
			F.E.M		EQUATION		
			DEFL	$\Delta/\Delta_s$	$\Delta/\Delta_s$		
200Z20	L=500	Lp=50	-2.95	0.533	0.546	1.024	
		Lp=100	-2.790	0.505	0.479	0.948	
		Lp=125	-2.660	0.481	0.489	1.017	
		Lp=150	-2.556	0.462	0.476	1.031	
		Lp=175	-2.257	0.408	0.399	0.979	
		Simple	-5.528	1	1	1	
	L=600	Lp=50	-5.506	0.482	0.499	1.037	
		Lp=100	-5.248	0.460	0.435	0.946	
		Lp=125	-5.157	0.452	0.449	0.99452	
		Lp=150	-4.60	0.403	0.432	1.074	
		Lp=175	-3.998	0.350	0.338	0.968	
		Simple	-11.408	1	1	1	
	L=700	Lp=50	-10.544	0.517	0.528	1.021	
		Lp=100	-9.05	0.444	0.428	0.965	
		Lp=125	-9.03	0.443	0.431	0.974	
		Lp=150	-8.07	0.396	0.424	1.071	
		Lp=175	-7.83	0.384	0.372	0.969	
		Simple	-20.36	1	1	1	
	L=800	Lp=50	-17.76	0.50	0.512	1.024	
		Lp=100	-14.68	0.415	0.399	0.961	
		Lp=125	-14.55	0.411	0.399	0.972	
		Lp=150	-12.87	0.364	0.393	1.082	
		Lp=175	-12.77	0.361	0.349	0.967	
		Simple	-35.39	1	1	1	

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