Impact of Order Batching on Compound Bullwhip Effect

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Abstract: Order batching in supply chains provides economic benefit in aggregating demand to save in production and transportation costs. However, rounding of orders to achieve a batch size is recognized as a source of the bullwhip effect problem within supply chains. Conditions are established under which two or more causes may attenuate or dampen the net BWE. The proposed supply chain consists of a supplier feeding two retailers with stochastic demand, described by a first order autoregressive AR(1) time series process. Supplier feeds retailers in batches for a number of future time units based on the MMSE demand forecasting method. Two BWE measures are studied, one for each demand stream individually and one for the aggregated demand. These two measures are related to demand parameters of the retailers and the number of forecasting time units. Supplier should select the optimum batch size based on demand forecasting, such that the aggregate BWE of the two retailers is less than the sum of the separate BWE.

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1.Introduction:

Supplier feeding more than one retailer could gain economic benefit of batching by supplying orders to retailers in batches to cover demand for n future time units. However, order batching increases the BWE depending on the batch size. If demand interaction exists between retailers, aggregate BWE is introduced resulting from the synergistic effect between retailers. Depending on its value, aggregate BWE of the retailers could be less than the sum of separate BWE, producing higher system efficiency and lower costs to the supplier.

2. Literature review:

Forrester (1961) was first to explain bullwhip phenomenon as lack of information exchange between the components of the supply chain and thus it is also known as the *Forrester Effect*. Sterman (1989) through the Beer Game interpreted the phenomenon as a consequence of players' irrational behaviors or misperception of feedback.

Lee *et al.* (1997b) defined bullwhip effect as the increase of order variability as one move up the supply chain from retailers to wholesalers, manufacturers, and suppliers and stated four causes for the BWE including: Demand forecast updating, price variations, order batching, rationing and shortage gaming. The reasons for order batching include the economic order quantity (EOQ), periodic inventory review, and transportation economies.

Lee *et al.* (1997a) suggested breaking order batches, spreading customers' orders or replenishments evenly over time, and the use of third-party logistics companies to reduce the negative effect of batching.

Cachon (2000) measured the value of information sharing on supply chains by reducing lead times and increasing delivery frequency by reducing shipments batch sizes.

Riddalls (2001) studied the impact of batch production cost on the bullwhip effect. He found that the relationship between batch size and demand amplification is non-linear and depends on the remainder of the quotient of average demand and batch size.

Holland (2004) aimed to quantify the level of bullwhip induced in a two-echelon supply chain as a result of batching in the ordering rules. He found that the level of bullwhip across one echelon was proportional to the square of the batch size.

Matloub (2011) showed that when the quotient of the average demand and batch size is integer, demand amplification does not grow with increase in batch size.

Zhang (2004a) studied the bullwhip effect based on three different forecasting methods for a simple inventory system with a first order auto-regressive process AR(1). Results showed that forecasting methods play an important role in determining the impact of lead time and demand auto correlation on the bullwhip effect. He concluded that MMSE forecasting method is clearly the winner among forecasting methods if only inventory costs are considered, because it leads to the lowest inventory cost in a first order autoregressive AR(1) demand process.

Cachon *et al.* (2007) noted that "various factors influencing production can occur simultaneously: e.g., it is possible that a firm faces increasing marginal costs and fixed ordering costs and positively correlated demand. Hence, whether or not a firm smoothes production relative to sales depends on the relative importance of the factors.

Zhang (2010) investigated compound causes of the bullwhip effect in supply chains by considering an inventory system with multiple price-sensitive demand streams. Joint price and demand dynamics are captured by a vector time-series process that incorporates the stochastic co-movements in price and demand. Two BWE measures are introduced, one for each demand stream individually and one for the aggregated demand resulting from the interaction of two or more BWE causes. Conditions are established under which a pair of simultaneous compound causes may attenuate or dampen the net BWE.

3. Proposed research

This paper continues on the mathematical model developed by Zhang (2010). Order batching is introduced to the model by supposing the supplier is feeding the two retailers in batches to cover demand for *n* future time units based on MMSE demand forecast. Separate and aggregate BWE are measured for the supply chain.

4. Mathematical model:

The proposed model calculates separate and aggregate BWE by measuring order variance to a supplier for a certain period and comparing it to demand variance to retailers for the same period.

4.1 Order model: The model consists of a supplier feeding two identical retailers with stochastic demand and price described by a first order auto-regressive AR(1) time series process. Orders are delivered in batches to cover demand for *n* future time units. Batch size is based on MMSE demand forecasting.

From Zhang (2010), conservation of flow implies that:

$$q_{i,t} = y_{i,t} - y_{i,t-1} + d_{i,t-1} \quad (1)$$

Where, $q_{i,t}$ is the order to supplier for period *t*, $y_{i,t}$ is the order-up-to level for period *t*, $y_{i,t-1}$ is the order-up-to level for period *t-1*, $d_{i,t-1}$ is the demand for period *t-1*.

Let $D_i[t, t+n-1]$ be the aggregated demand forecast for *n* time units which is defined as the sum of demand for periods $[t, t+1, ..., t+n-1]$.

Therefore: $D_i[t, t+n-1] = \sum_{\tau=1}^{n} d_{i,t+\tau-1}$ (2)

Let $\hat{D}_i[t, t+n-1]$ be the MMSE (minimum mean square error) aggregated demand forecast.

As shown in Zipkin (2000), for normally distributed demand, an approximately order-up-to level can be calculated as the sum of forecasted demand for n time units and safety stock.

$$y_{i,t} = \hat{D}_i[t, t+n-1] + \zeta\hat{\sigma}_{i,t} \quad (3)$$

Where, $\hat{\sigma}_{i,t}$ is the standard deviation of demand forecast error, which is considered equal to zero as the demand forecast error is constant over time.

By sub. eq. (3) in (1), therefore:

$$q_{i,t} = \hat{D}_i[t, t+n-1] - \hat{D}_i[t-1, t+n-2] + d_{i,t-1} \quad (4)$$

4.2 BWE measure: Zhang (2000) identified two types of BWE measures; one is based on the demand and order stream of an individual retailer and the other is constructed to measure the BWE of pooled retail demands and orders.

Separate BWE: It exists if $V(q_{i,t}) \rangle V(d_{i,t})$ and the size of the effect is measured by their difference, $M_i = V(q_{i,t}) - V(d_{i,t})$.

Aggregate BWE: It exists if $V(q_{1,t} + q_{2,t}) \rangle V(d_{1,t} + d_{2,t})$ and $\overline{M} = V(q_{1,t} + q_{2,t}) - V(d_{1,t} + d_{2,t})$ measures the size of the BWE.

Expanding the variance of the sum, the aggregate BWE measure can be expressed as:

$$\overline{M} = M_1 + M_2 + 2[\text{cov}(q_{1,t}, q_{2,t}) - \text{cov}(d_{1,t}, d_{2,t})] \quad (5)$$

The sum of the two separate BWE measures plus an adjustment to account for the covariance between the order streams and the demand streams. In subsequent discussions, this adjustment is referred to as the pooling factor. When the pooling factor is negative, $\overline{M}$ is less than the sum of the separate M_i .

4.3 Demand model: As shown in Zhang (2000), the demand streams facing two interacting retailers are jointly determined by a linear combination of demand inertia, interaction between retailers, and demand forecasting error.

$$d_{1,t} = a_1 + \rho_{11}d_{1,t-1} + \rho_{12}d_{2,t-1} + \varepsilon_{1,t} \quad (6)$$

$$d_{2,t} = a_2 + \rho_{21}d_{1,t-1} + \rho_{22}d_{2,t-1} + \varepsilon_{2,t} \quad (7)$$

Where, a_1 and a_2 are constant intercept terms that determine the means of $d_{1,t}$ and $d_{2,t}$, coefficients ρ_{11} and ρ_{22} determine autocorrelation due to demand inertia, coefficients ρ_{12} and ρ_{21} describe the interaction between the two demand streams, $\varepsilon_{1,t}$

and $\varepsilon_{2,t}$ are the error due to demand forecasting update.

For analytical convenience, demand process could be expressed in a mean-centered form by subtracting the unconditional (stationary) demand from its respective demand.

Let, μ_1 and μ_2 be the unconditional mean demand for retailers 1 and 2, and let $z_{i,t}$ be the non-stationary demand. Therefore,

$$z_{i,t} = d_{i.,t} - \mu \quad (8)$$

By sub. eq. (8) in (6), (7); the joint demand process can be written as:

$$z_{1,t} = \rho_{11} z_{1,t-1} + \rho_{12} z_{2,t-1} + \varepsilon_{1,t} \quad (9)$$

$$z_{2,t} = \rho_{21} z_{1,t-1} + \rho_{22} z_{2,t-1} + \varepsilon_{2,t} \quad (10)$$

In matrix notation, let:

$$d_t = [d_{1,t}, d_{2,t}]^T, a = [a_1, a_2]^T, z_t = d_t - \mu = [z_{1,t}, z_{2,t}]^T,$$

$$\varepsilon_t = [\varepsilon_{1,t}, \varepsilon_{2,t}]^T, \rho = \begin{vmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{vmatrix}, V(\varepsilon_t) = \Psi = \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix}$$

Then, eq. (6), (7) can be expressed as:

$$d_t = a + \rho d_{t-1} + \varepsilon_t \quad (11)$$

Eq. (9), (10) can be expressed as:

$$z_t = \rho z_{t-1} + \varepsilon_t \quad (12)$$

4.4 Bullwhip effect calculation: Joint demand equations will be substituted in the order model, then the covariance matrix for separate and aggregate BWE will be calculated.

The covariance matrix of joint demand $V(q_t)$ can be easily obtained from Yule-Walker equations for a vector autoregressive first order VAR(1) time series process.

Re-arranging eq. (11):

$$\rho d_{t-1} = d_t - \varepsilon_t - a \quad (13)$$

By taking variance for both sides of eq. (13):

$$\rho V(d_t) \rho^T = V(d_t) - \Psi \quad (14)$$

Demand for the number of future time units' $k = (1,2,....)$ could be considered as demand for period $(t-1)$ multiplied by the demand correlation k times and the sum of the associated errors for period k.

By sub. in eq. (12), therefore:

$$Z_{t+k-1} = \rho^k Z_{t-1} + \sum_{j=0}^{k-1} \rho^j \varepsilon_{t+k-j-1} \quad (15)$$

The above expression shows future demand given observed demand history and can be decomposed into MMSE joint demand forecasts and their associated errors:

$$\hat{Z}_{t+k-1} = \rho^k Z_{t-1} \quad (16)$$

$$\hat{e}_{t+k-1} = Z_{t+k-1} - \hat{Z}_{t+k-1} = \sum_{j=0}^{k-1} \rho^j \varepsilon_{t+k-j-1} \quad (17)$$

Eq. (4) could be written as:

$$q_{i,t} = \hat{D}_t - \hat{D}_{t-1} + d_{i,t-1} \quad (18)$$

Where, $\hat{D}_t = \sum_{\tau=1}^{n} d_{i,t+\tau-1}$ (19)

By sub. eq. (8) in (19), therefore:

$$\hat{D}_t = \sum_{k=1}^{n} \hat{Z}_{t+k-1} + n\mu \quad (20)$$

By sub. eq. (16) in (20), therefore:

$$\hat{D}_t - n\mu = \sum_{k=1}^{n} \rho^k Z_{t-1} \quad (21)$$

Assume:

$$\phi_n = \sum_{k=1}^{n} \rho^k = \rho + \rho^2 + \ldots + \rho^n \quad (22)$$

Which is a geometric series with common ratio ρ, and its sum is: $\rho\left(\frac{1-\rho^n}{1-\rho}\right)$

By sub. eq. (22) in (21), therefore:

$$\hat{D}_t - n\mu = \phi_n Z_{t-1} \quad (23)$$

By sub. eq. (23) in (18), therefore:

$$q_t = \phi_n Z_{t-1} - \phi_n Z_{t-2} + Z_{t-1} + \mu \quad (24)$$

Taking Z_{t-1} as a common factor:

$$q_t = (\phi_n + I) Z_{t-1} - \phi_n Z_{t-2} + \mu \quad (25)$$

By sub. eq. (12) in (25) for Z_{t-1}, therefore:

$$q_t = (\phi_n + I)(\rho Z_{t-2} + \varepsilon_{t-1}) - \phi_n Z_{t-2} + \mu \quad (26)$$

From eq. (22):

$$(\phi_n + I)\rho = \rho + \rho^2 + ... + \rho^{n+1} = \phi_{n+1} \quad (27)$$

By sub. eq. (27) in (26), therefore:

$$q_t = (\phi_n + I)\varepsilon_{t-1} + (\phi_{n+1} - \phi_n) Z_{t-2} + \mu \quad (28)$$

$$q_t = (\phi_n + I)\varepsilon_{t-1} + \rho^{n+1} Z_{t-2} + \mu \quad (29)$$

By taking variance for both sides of eq. (29):

$$V(q_t) = (\phi_n + I)\Psi(\phi_n + I)^T + (\rho^{n+1}) V(d_t) (\rho^{n+1})^T \quad (30)$$

By sub. eq. (14) in (30) repeatedly:

$$V(q_t) = V(d_t) + (\phi_n + I)\Psi(\phi_n + I)^T - \sum_{i=o}^{n} \rho^i \Psi (\rho^i)^T \quad (31)$$

By summing eq. (31):

$$V(q_t) = V(d_t) + \phi_n \Psi + \Psi \phi_n^T \quad (32)$$

By applying definitions for separate and aggregate on eq. (32):

$$BWE = V(q_t) - V(d_t) = \phi_n \Psi + \Psi \phi_n^T \quad (33)$$

By sub. the matrix of ϕ_n and Ψ in eq. (33):

$$BWE = \begin{vmatrix} \rho_{11}\left(\frac{1-\rho_{11}^{n}}{1-\rho_{11}}\right) & \rho_{12}\left(\frac{1-\rho_{12}^{n}}{1-\rho_{12}}\right) \\ \rho_{21}\left(\frac{1-\rho_{21}^{n}}{1-\rho_{21}}\right) & \rho_{22}\left(\frac{1-\rho_{22}^{n}}{1-\rho_{22}}\right) \end{vmatrix} \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix} + \quad (34)$$

$$\begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix} \begin{vmatrix} \rho_{11}\left(\frac{1-\rho_{11}^{n}}{1-\rho_{11}}\right) & \rho_{21}\left(\frac{1-\rho_{21}^{n}}{1-\rho_{21}}\right) \\ \rho_{12}\left(\frac{1-\rho_{12}^{n}}{1-\rho_{12}}\right) & \rho_{22}\left(\frac{1-\rho_{22}^{n}}{1-\rho_{22}}\right) \end{vmatrix}$$

By solving the matrix in equation (34), the diagonal elements represent separate BWE for retailers 1 and 2 respectively, while the sum of the off-diagonal elements represent the pooling factor resulting from the synergistic effect between retailers.

Separate BWE for retailer 1:

$$M_1 = 2\left[\rho_{11}\left(\frac{1-\rho_{11}^{n}}{1-\rho_{11}}\right)\sigma_{11} + \rho_{12}\left(\frac{1-\rho_{12}^{n}}{1-\rho_{12}}\right)\sigma_{12}\right] \quad (35)$$

Separate BWE for retailer 2:

$$M_2 = 2\left[\rho_{22}\left(\frac{1-\rho_{22}^{n}}{1-\rho_{22}}\right)\sigma_{22} + \rho_{21}\left(\frac{1-\rho_{21}^{n}}{1-\rho_{21}}\right)\sigma_{12}\right] \quad (36)$$

Aggregate BWE for the two retailers:

$\overline{M} = M_1 + M_2 + \text{pooling factor}$

$$\overline{M} = M_1 + M_2 + 2\left[\rho_{11}\left(\frac{1-\rho_{11}^{n}}{1-\rho_{11}}\right) + \rho_{22}\left(\frac{1-\rho_{22}^{n}}{1-\rho_{22}}\right)\right]\sigma_{12}$$

$$+ 2\left[\rho_{12}\left(\frac{1-\rho_{12}^{n}}{1-\rho_{12}}\right)\sigma_{22} + \rho_{21}\left(\frac{1-\rho_{21}^{n}}{1-\rho_{21}}\right)\sigma_{11}\right]$$

(37)

5. Implementation:

Matlab simulation is run for the equations calculating separate and aggregate for a supply chain consisting of a supplier feeding two retailers in batches. Random demand variables are assumed for each retailer (demand autocorrelation, and demand cross-correlation). In each case, demand forecasting is done to determine future demand after *n* time units. Minimum demand interaction between retailers is concluded, such that, aggregate BWE of the two retailers is less than the sum of separate BWE for each retailer. Output graphs show the effect on demand parameters and demand forecasting on separate and aggregate BWE.

7. Case study:

Using the equations for the calculation of separate and aggregate BWE, a sample case study is run by assuming demand values. In each case, Matlab simulation is run at different forecasting times to select the optimum batch size based on demand parameters (demand autocorrelation, demand cross-correlation, and demand interaction), such that aggregate BWE of the two retailers is less than the sum of separate BWE.

Assume the two retailers are symmetric, resulting in: $\rho_{12} = \rho_{21}, \sigma_{12} = \sigma_{21}$; σ_{11}, σ_{22} represents the variance (error) in each demand stream and is considered equal to one (no error).

In the following graphs, random values are assumed for demand autocorrelation and demand cross-correlation, demand interaction between retailers is concluded based on the desired batch size :

- X-axis represents the degree of demand interaction between the two streams.
- Y-axis represents BWE measure.
- The solid line represents the aggregate BWE ($\overline{M}$).
- The dotted line represents the sum of separate BWE ($M_1 + M_2$).

1) Both, demand autocorrelation and demand cross-correlation are positive.

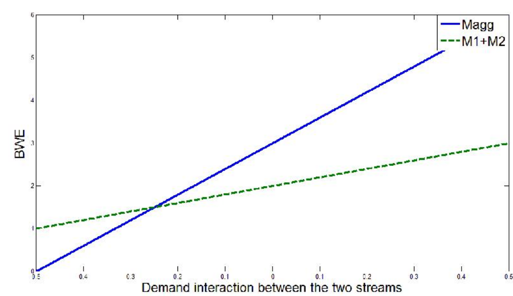

Fig. 1: BWE for ($\rho_{11} = 0.5, \sigma_{12} = 0.5, n = 1$)

In Fig. 1, $\rho_{11} = 0.5, \sigma_{12} = 0.5$, if forecasting is done for one time unit (n=1, no batching), interaction between the two streams should be less than (-0.25), for the aggregate BWE to be less than the sum of the separate BWE.

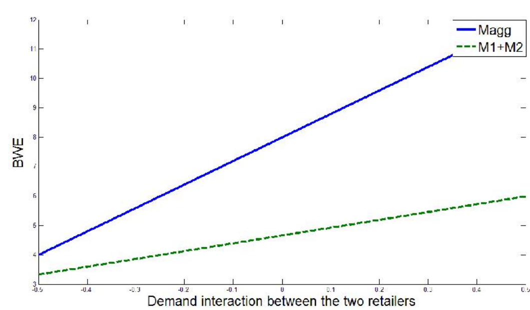

Fig. 2: BWE for ($\rho_{11} = 0.5, \sigma_{12} = 0.5, n = 10$)

In Fig. 2, if forecasting is increased to 10 time units, pooling factor is always positive, $\overline{M}$ is always greater than $M_1 + M_2$, so it is not preferred to do batching for such conditions.

From Fig.1 and Fig.2, we conclude that it is not preferred to do order batching when $\rho_{11} = 0.5, \sigma_{12} = 0.5$. It is clear that if the number of forecasting time units' n is even, aggregate BWE is always larger than the sum of the separate BWE.

2) Demand autocorrelation is positive and demand cross-correlation is negative.

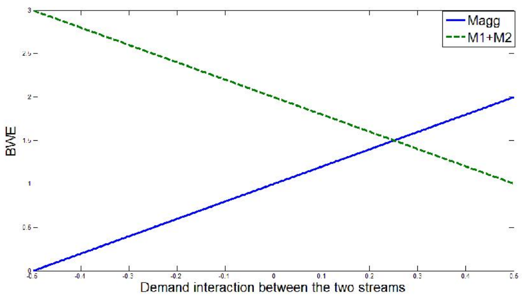

Fig. 3: BWE for ($\rho_{11} = 0.5, \sigma_{12} = -0.5, n = 1$)

In Fig. 3, $\rho_{11} = 0.5, \sigma_{12} = -0.5$, if forecasting is done for one time unit (n=1, no batching), interaction between the two streams should be less than (0.25), for the aggregate BWE to be less than the sum of the separate BWE.

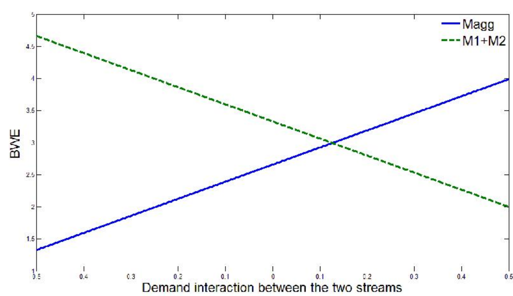

Fig. 4: BWE for ($\rho_{11} = 0.5, \sigma_{12} = -0.5, n = 10$)

In Fig. 4, when forecasting is extended for 10 time units, interaction between the two streams should be less than (0.13) for the aggregate BWE to be less than the sum of the separate BWE.

From the above we conclude that by increasing forecasting time units, demand interaction between retailers should be decreased to keep $\left(\overline{M} \langle M_1 + M_2\right)$.

3) Demand autocorrelation is negative and demand cross-correlation is positive.

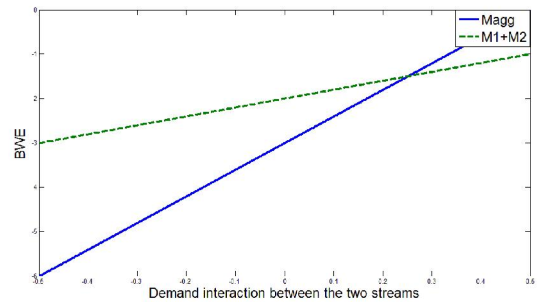

Fig. 5: BWE for ($\rho_{11} = -0.5, \sigma_{12} = 0.5, n = 1$)

In Fig. 5, $\rho_{11} = -0.5, \sigma_{12} = 0.5$, if forecasting is done for one time unit (n=1, no batching), interaction between the two streams should be less than (0.25), for the aggregate BWE to be less than the sum of the separate BWE.

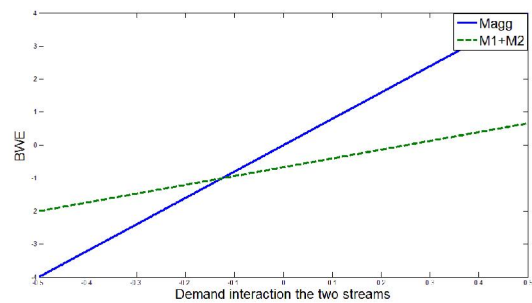

Fig. 6: BWE for ($\rho_{11} = -0.5, \sigma_{12} = 0.5, n = 10$)

In Fig. 6, if forecasting is extended for 10 time units, interaction between the two streams should be less than (-0.13) for the aggregate BWE to be less than the sum of the separate BWE.

4) Both, demand autocorrelation and demand cross-correlation are negative.

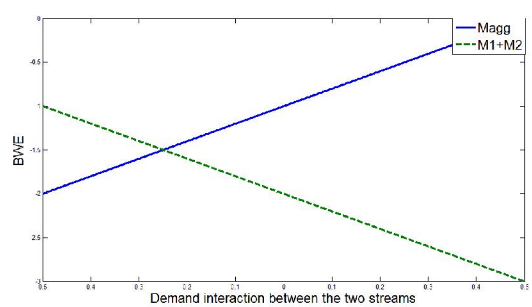

Fig. 7: BWE for ($\rho_{11} = -0.5, \sigma_{12} = -0.5, n = 1$)

In Fig. 7, $\rho_{11} = -0.5, \sigma_{12} = -0.5$. if forecasting is done for one time unit (n=1, no batching), interaction between the two streams should be less than (-0.25) for the aggregate BWE to be less than the sum of the separate BWE.

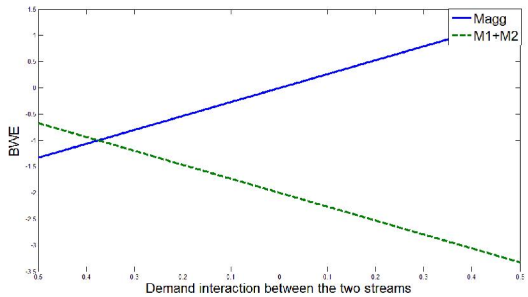

Fig. 8: BWE for ($\rho_{11} = -0.5, \sigma_{12} = -0.5, n = 10$)

In fig. 8, if forecasting is extended for 10 time units, interaction between the two streams should be less than (-0.37) for the aggregate BWE to be less than the sum of the separate BWE.

6. Conclusion:

This paper considered a supplier feeding more than one retailer simultaneously in batches to cover demand for *n* future time units based on MMSE forecasting. Synergistic effect exists due to demand interaction between retailers. Equations are reached that relate both, separate and aggregate BWE to demand parameters (demand autocorrelation, demand cross-correlation, and demand interaction) and the number of forecasting time units based on the desired batch size. Matlab simulation is run for different demand conditions, in each case, based on the desired batch size, critical demand interaction that dampens the aggregate BWE is concluded.

The following recommendations are concluded:

1) For high demand interaction between retailers, net BWE is amplified due to the increase of demand forecasting update error, so it is preferred that demand interaction is as low as possible.
2) For large batch sizes, as the number of forecasting time units n is increased, forecasting error is aggregated, so demand interaction between retailers should be low to keep $(\overline{M} \langle M_1 + M_2)$.
3) For large batch sizes (n=10,20,…), critical demand interaction between retailers to keep $(\overline{M} \langle M_1 + M_2)$ is almost constant. Forecasting is done by aggregating demand correlation for n future time units in a geometric series $(\rho + \rho^2 + ... + \rho^n)$, since correlation ranges between -1 and 1, the sum will not vary largely at high forecasting time units.

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