Predictive Controller for Pitch Controller Aircraft

Amir Torabi\(^1\) (Corresponding author), Sobhan Salehi\(^2\), Ali Karsaz\(^3\), Ebrahim Tarsayi\(^4\)

\(^1\) Faculty of Electrical Engineering, Khorasan University, Mashhad, Iran  
\(^2\) Faculty of Electrical Engineering, Islamic Azad University, Mashhad, Iran  
\(^3\) Faculty of Electrical Engineering, Khorasan University, Mashhad, Iran  
\(^4\) Faculty of Electrical Engineering, Khorasan University, Mashhad, Iran  

amirrorabi1@gmail.com

Abstract: This paper exhibits a comparative assessment based on time response specification performance between fuzzy and Model predictive control (MPC) for a pitch control system of an aircraft system. The dynamic modeling of pitch control system is considered on the design an autopilot that controls the pitch angle. It starts with a derivation of suitable mathematical model to describe the dynamics of an aircraft. For get close to actual conditions. The white noise disturbance applied to the system. The performances of pitch control systems are investigated and analyzed based on common criteria of step’s response in order to identify which control strategy delivers better performance with respect to the desired pitch angle. The design of MPC gave response less quality than that was given from Fuzzy controller but acceptable responses. Finally, it is found from simulation, predictive controller proposed gives the best performance compared to fuzzy controller.


Keywords: controller, Fuzzy, Model predictive, pitch controller

1. Introduction

Today’s aircraft designs rely heavily on automatic control system to monitor and control many of aircraft’s subsystem. The development of automatic control system has played an important role in the growth of civil and military aviation. Modern aircraft include a variety of automatic control system that aids the flight crew in navigation, flight management and augmenting the stability characteristic of the airplane.

To reduce the complexity of analysis, the aircraft is usually assumed as a rigid body and aircraft’s motion consist of a small deviation from it is equilibrium flight condition [2]. The pitch of aircraft is control by elevator which usually situated at the rear of the airplane running parallel to the wing that houses the ailerons. Pitch control is a longitudinal problem, and this work gives on design an autopilot that controls the pitch of an aircraft. Autopilot is a pilot relief mechanism that assists in maintaining an attitude, heading, altitude or flying to navigation or landing references [3].

The combination of nonlinear dynamics, modeling uncertainties and parameter variation in characterizing an aircraft and its operating environment are the one major problem of flight control system. This work is attempted to Survey the control strategies required to address the complex longitudinal dynamic characteristics of such aircraft. Many the research works has been done in [4], [5], [6], [7] and [8], to control the pitch or longitudinal dynamic of an aircraft for the purpose of flight stability. This research is still remains an open issue in the present and future efforts [9].

In this paper used predictive controller to improve performance system in the past paper, Disturbance effect haven’t apply to system but in this paper it applied. The simulation results shown that the dynamic characteristics of control systems can be improved by this method.

Problem Statement

Control of dynamic systems with present day sophistication and complexities has often been an important research area due to the difficulties in modeling, nonlinearities, and uncertainties, particularly when there is a constant change in system dynamics. It is also known that the response of a dynamic nonlinear plant cannot be tracked into a desired pattern with a linear controller. Thus, a changing dynamic controller is important to control such a plant. [10]

Pitch is defined as a rotation around the lateral or transverse axis, which is parallel to the wings, and is measured as the angle between the direction of speed in a vertical plan and the horizontal line.

Changes of pitch are caused by the deflection of the elevator, which rises or lowers the nose and tail of the aircraft. When the elevator is raised (defined as negative value), the force of the airflow will push the tail down. Hence, the nose of the aircraft will rise and the altitude of the aircraft will increase. One of the targets of a pitch control system is to control or help a pilot to control an aircraft to keep the pitch attitude constant, that is, make the aircraft return to desired attitude in a reasonable length of time after a
disturbance of the pitch angle, or make the pitch follow a given command as quickly as possible [11].

**Modeling of a Pitch Control**

This section provides a brief description on the modeling of pitch control longitudinal equation of aircraft, as a basis of a simulation environment for development and performance evaluation of the proposed controller techniques. The system of longitudinal dynamics is considered in this investigation and derived in the transfer function and states space forms. The pitch control system considered in this work is shown in Figure 1 where $X_b, Y_b$ and $Z_b$ represent the aerodynamics force components, $\theta, \Phi$ and $\delta$ represent the orientation of aircraft (pitch angle) in the earth-axis system and elevator deflection angle.

The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. Although, under certain assumptions, they can be decoupled and linearized into longitudinal and lateral equations. Aircraft pitch is governed by the longitudinal dynamics. In this example we will design an autopilot that controls the pitch of an aircraft. The basic coordinate axes and forces acting on an aircraft are shown in the figure given below.

**Figure 1.** Description of pitch control system.

Second, the change in pitch angle does not change the speed of an aircraft under any circumstance.

Referring to the Figure 1 and Figure 2, the following dynamic equations include force and moment equations are determined as shown in equation (1), (2) and (3). Referring to the Figure 1 and Figure 2, the following dynamic equations include force and moment equations are determined. The longitudinal stability derivatives parameter used are denoted in Table 1 [1].

![Figure 2: Definition of force, moments and velocity in body fixed coordinate.](http://www.jofamericanscience.org/

<table>
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<tr>
<th>Table 1. Longitudinal Derivative Stability Parameters</th>
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<th>Components</th>
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<tr>
<td>$X - mg \sin(\theta) = m(u +qv - rv)$</td>
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<tr>
<td>$Z + mgC_0 \cos(\theta) = m(\dot{u} + pv - qv)$</td>
</tr>
<tr>
<td>$M = I_y \dot{\theta} + r q (l_x - l_2) + l_{xz} (p^2 - r^2)$</td>
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It is required to completely solved the aircraft problem with considering the following assumption: (1) rolling rate $\dot{\theta} = \dot{\theta} - \delta_s$; (2) yawing rate $\dot{\psi} = \dot{\psi} - C_\psi S_\psi$; (3) pitching rate, $\dot{r} = \dot{\theta}$; (4) Pitch Angle, $\theta = q S_\psi - r S_\theta$; (5) roll Angle, $\delta = p + q S_\theta T_\theta + r C_\theta T_\theta$ and (6) Yaw Angle, $\psi = (q S_\theta + r S_\theta) \sec \theta$.
Equation (1), (2) and (3) should be linearized using small disturbance theory. The equations are replaced by a variable or reference value plus a perturbation or disturbance, as shown below.

\[
\begin{align*}
  w &= u_0 + \Delta u \\
  v &= v_0 + \Delta v \\
  w &= w_0 + \Delta w \\
  p &= p_0 + \Delta p \\
  q &= q_0 + \Delta q \\
  r &= r_0 + \Delta r \\
  X &= X_0 + \Delta X \\
  M &= M_0 + \Delta M \\
  Z &= Z_0 + \Delta Z \\
  \delta &= \delta_0 + \Delta \delta
\end{align*}
\]

For convenience, the reference flight condition is assumed to be symmetric and the propulsive forces are assumed to remain constant. This implies that \(v_u = p_u = q_u = r_u = \dot{\varphi}_u = \dot{w}_u = 0\). After linearization the (4), (5) and (6) are obtained.

\[
\begin{align*}
  \left( \frac{d}{dt} - X_0 \right) \Delta u - X_0 \Delta w + \left( g \cos \theta_0 \right) \Delta \theta &= X \Delta \delta \Delta \theta \quad (4) \\
  -Z_u \Delta u + \left( 1 - Z_0 \right) \Delta w &= \left[ u_0 - Z_0 \right] \sin \theta_0 \quad (5) \\
  -M_u \Delta u - \left( M_w + M_w \right) \Delta w &= \left[ \frac{r_0}{r_0} \right] \Delta \delta \quad (6)
\end{align*}
\]

By manipulating the (4), (5), (6) and substituting the parameters values of the longitudinal stability derivatives, the following transfer function for the change in the pitch change in the pitch rate to the change in elevator deflection angle is shown as (7) obtained.

\[
\frac{\Delta q(s)}{\Delta \delta(s)} = \frac{-M_u M_w + M_u Z_0 / u_0 \sin \theta_0 - (M_w Z_0 / u_0 - Z_0 \Delta \delta / u_0)}{s^2 \left( M_u + M_w \right) \Delta w + \left( \frac{r_0}{r_0} \right) \Delta \delta}
\]

The transfer function of the change in pitch angle to the change in elevator angle can be obtained from the change in pitch rates to the change in elevator angle in the following way.

\[
\begin{align*}
  \Delta q &= \Delta \dot{\theta} \\
  \Delta q(s) &= s \Delta \theta(s) \quad (8) \\
  \Delta \theta(s) &= \frac{1}{s \cdot \theta(s)} \quad (9) \\
  \frac{\Delta q(s)}{\Delta \theta(s)} &= \frac{1}{s \cdot \theta(s)}
\end{align*}
\]

Therefore the transfer function of the pitch control system is obtained in (11) and (12) respectively.

\[
\begin{align*}
  \Delta q(s) &= \frac{1}{s} - \left( M_u M_w + M_u Z_0 / u_0 \sin \theta_0 - (M_w Z_0 / u_0 - Z_0 \Delta \delta / u_0) \right) \\
  \Delta \theta(s) &= \frac{1}{s \cdot \theta(s)} + \left( M_u + M_w \right) \Delta w + \left( \frac{r_0}{r_0} \right) \Delta \delta
\end{align*}
\]

**Transfer function**

To find the transfer function of the above system, we need to take the Laplace transform of the above modeling equations. Recall that when finding a transfer function, zero initial conditions should be assumed. The Laplace transform of the above equations are shown below. [20]

\[
\Delta \theta(s) = \frac{1.151 s + 0.1774}{s^2 + 0.739 s^2 + 0.921 s}
\]

These values are taken from the data from one of Boeing's commercial aircraft.

**The Design of Fuzzy Controller**

Fuzzy control is based on the artificial experience. Therefore, for those control problems which can’t be resolved by traditional methods can often be resolved by the fuzzy control technology. By the fuzzy control technology, it does not know the mathematical model of the plant and easy to control uncertain systems or nonlinear control systems and can restrain the strong disturbance.

![Figure 3. The basic structure of fuzzy control system](#)

The only difference is to control the device by fuzzy controller to achieve the desired performance. Fuzzy self-tuning PID controller is a conventional PID regulator based on the fuzzy set theory, under the absolute control error and deviation change and the absolute value of the rate, on-line automatically adjusting the proportional coefficient KP, integral coefficient of KI and differential factor KD of the fuzzy controller. Fuzzy controller is a nonlinear control device, using fuzzy reasoning algorithm. The sample data of the controlled process are taken as the clear amount of input to the controller, and then after input quantization factor calculation, are transferred into fuzzy values, so they can be used for fuzzy reasoning by fuzzy language and rules.

To the other part of process, the reasoning results are firstly transferred into clear values by anti-fuzzy inference and then derive the control output with quantified factor calculation used as the control value for the controlled process. Based on the MATLAB fuzzy logic toolbox, the above control algorithm can be easily implemented [17].

**Design of nominal fuzzy controller**

In order to design the PID parameters based on fuzzy controller, at first the simplest structure of two-input single output nominal fuzzy controller is given. At any given time instance n with a sampling time Ts, the two input variables of fuzzy controller, error state variable and error change are defined as

\[
\Delta e(n) = e(n) - e(n-1)
\]

And its output variable u(n) is the control signal of process. Without loss the generality, the
The generalized predictive control approach gives an analytic solution for tracking problems of multivariable nonlinear systems in terms of a generalized predictive control performance index. A novel guidance law is developed in the following section by employing this algorithm. The generalized predictive control gives the approximation of the tracking error in the receding horizon by its Taylor-series expansion to any specified order. A closed-form optimal predictive controller is obtained by minimizing a quadratic performance index with integral action. Online optimization is not required and stability of the closed-loop system is guaranteed. For more detail, the reader is referred to Refs. [11,12]. Consider the nonlinear system

$$\begin{align*}
\dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\
y(t) &= h(x(t))
\end{align*}$$

(16)

Where $x \in \mathbb{R}^{n_a}$, $u \in \mathbb{R}^{n_u}$ and $y \in \mathbb{R}^{n_y}$ are the state, control and output vectors, respectively. It is assumed that each of the system output $y(t)$ has the same well-defined relative degree $\rho$, the output $y(t)$ and the reference trajectory $w(t)$ are
sufficiently many times continuously differentiable with respect to $t$ and the control order is chose to be $r$. The future output $y(t + \tau)$ is approximately predicted by its Taylor-series expansion up to order $\rho + r$, given by

$$y(t + \tau) \approx T(\tau)Y(t)$$

(17)

$$T(\tau) = \begin{bmatrix} I_{m \times m} & \tau & \cdots & \tau^r(\rho + r) \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0_{mx1} & 0_{mx1} & \cdots & L_g^r h(x) \end{bmatrix}$$

(19)

$$Y(t) = \begin{bmatrix} y(t) \\ y(t) \cdot \tau \\ y(t) \cdot \tau^2 \\ \vdots \\ y(t) \cdot \tau^{r-1} \\ y(t) \cdot \tau^r \\ y(t+\tau) \\ y(t+\tau) \cdot \tau \\ y(t+\tau) \cdot \tau^2 \\ \vdots \\ y(t+\tau) \cdot \tau^{r-1} \\ y(t+\tau) \cdot \tau^r \\ y(t+\tau+\tau) \\ y(t+\tau+\tau) \cdot \tau \\ y(t+\tau+\tau) \cdot \tau^2 \\ \vdots \\ y(t+\tau+\tau) \cdot \tau^{r-1} \\ y(t+\tau+\tau) \cdot \tau^r \\ y(t+\tau+\tau+\tau) \\ y(t+\tau+\tau+\tau) \cdot \tau \\ y(t+\tau+\tau+\tau) \cdot \tau^2 \\ \vdots \\ y(t+\tau+\tau+\tau) \cdot \tau^{r-1} \\ y(t+\tau+\tau+\tau) \cdot \tau^r \\ \vdots \\ y(t+\tau+\tau+\tau+\tau) \\ y(t+\tau+\tau+\tau+\tau) \cdot \tau \\ y(t+\tau+\tau+\tau+\tau) \cdot \tau^2 \\ \vdots \\ y(t+\tau+\tau+\tau+\tau) \cdot \tau^{r-1} \\ y(t+\tau+\tau+\tau+\tau) \cdot \tau^r \end{bmatrix}$$

(20)

Where $L_g^p$, $L_g^r$ represent the Lie derivative with respect to $f$ and $g$. $P_i$ is nonlinear in both $u(t),..,u^{i+1}(t)$ and $x(t)$ for $i=1,...,r$. In the moving time frame, the reference trajectory $w(t+\tau)$ is also approximated by the Taylor expansion of $w(t)$ up to $(\rho + r)$th order, given by

$$w(t+\tau) \approx T(\tau)w(t)$$

(21)

$$w(t) = \begin{bmatrix} w(t)^T \\ \vdots \\ w(t+\tau)^T \\ \vdots \\ w(t+\tau+\tau)^T \\ \vdots \\ w(t+\tau+\tau+\tau)^T \\ \vdots \\ w(t+\tau+\tau+\tau+\tau)^T \end{bmatrix}$$

(22)

The receding-horizon performance index with built-in integral action is given by

$$J = \frac{1}{2} \int_0^T (y(t+\tau) - w(t+\tau))^T (\rho + r) \int_0^T (y(t+\tau) - w(t+\tau)) d\tau$$

(23)

Where $T$ is the predictive period. The actual control input $u(t)$ given by the initial value of the optimal control input $u(t+\tau)$, $0 \leq \tau \leq T$, which minimizes the performance index by setting

$$\frac{\partial J}{\partial u} = 0$$

is described as

$$u(t) = -\left( L_g^p h(x) \right)^{-1}(KM_p + L_g^p h(x) - w_0(x))$$

(24)

Where $M_p \in \mathbb{R}^{mp \times mp}$ is given by

$$M_p = \begin{bmatrix} h(x) - w(x) \\ L_g^p h(x) - w(x) \\ \vdots \\ L_g^{p-1} h(x) - w(x) \\ L_g^p h(x) - w(x) \end{bmatrix}$$

(25)

$K \in \mathbb{R}^{m \times mp}$ is the first rows of matrix $T_{fp}^{-1}T_{pf}$, which are the submatrices of $T(\tau)$, given by

$$T(\tau) = \begin{bmatrix} T_{pf} \\ T_{fp} \end{bmatrix}$$

(26)

Figure 6: comparing between predictive controller with reference input, two missile trajectory and fuzzy controller
As we can see, the optimal predictive control law (26) is a nonlinear time invariant state feedback law. The control gain $K$ is constant, which only depends on the predictive time $T$, the control order $r$, and the relative degree $p$.

**Simulation**

The proposed control schemes have been implemented within simulation environment in Matlab and Simulink. In the previous section, the controllers were introduced which used to control the processes. As expected, controller fuzzy in compare with other controllers despite the severe disturbance on pitch system (caused by severe storms, rainy weather, etc.) had the desirable step response. Severe disturbances (means high amplitude disturbance) to proof the robustness of the fuzzy controller has been applying on pitch system. Performance of the control schemes has been evaluated in term of time domain specification.

**Conclusion**

A new control approach to pitch-rate command tracking off lighter aircraft has been proposed in this paper. Modeling is done on an aircraft pitch control and predictive controller is proposed successfully. The proposed control schemes have been implemented within simulation environment in Matlab and Simulink. Performance of the control schemes has been evaluated in term of time domain specification. The results obtained, demonstrate that the effect of the disturbances in the system can successfully be handled by predictive controller. MPC controller with constraints will be developed and able to compensate for constraints that represent physical limits of actuators in pitch angle. The design of MPC gave response less quality than that was given from Fuzzy controller but acceptable responses.

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