Internal Rate of Return: A suggested Alternative Formula and its Macro-economics Implications

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Abstract: The study focusing in formulating an accurate mathematical formula that calculates precisely the internal rate of return. Internal rate of return uses as crucial criteria in assessment and evaluating investment projects. The study discussed different properties of the suggested formula with respect to the existing ones, particularly the unique solution which the formula could able to provide. Furthermore the shortcomings of the suggested formula were introduced with their recommended remedies. Another concern to the study is the macroeconomics implications of the formula, here the study demonstrated a positive and direct relationship between investment and market rate of interest. The paradox of interest and investment is argued, in addition to the impacts which convey when tackling the issue of macro- policies and consequently stability fallout as well as repercussions.

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1. Introduction:

As far as micro-considerations focuses upon accuracy in calculation, macro considerations emphasis on the flow of investment on national income as well as stability matters. Consequently the role of internal rate of return which play in investment and then macro word is stemmed out. Investment partly determines the national income, while mainly responsible from determination and shaping business cycle.

As mentioned earlier our target is a mathematical derivation of an accurate formula to the internal rate of return, beside the identification of its advantages as well as its shortcomings respecting the existence formulas in a narrow micro sense. While broadly the applications of the formula in macro word gives proper insight about investment as a core factor affecting national income as a flow of real sector and at the same time affecting the financial sector via interaction of interest rate with money market when the identity equation of aggregate expenditure is solved.

2. Aim of the Study:

The aim of this study is a suggested attention mathematical formula to the internal rate of return. Then comparing the suggested formula with the existing ones in term of accuracy and property. Mean while the existing formulas concentrate on evaluation of the feasibility of a given project or projects the suggested formula jumps to the macro implication of investment outcome.

2.1 The Problem:

This study focusing on the concept of internal rate of return (IRR) by finding unique solution to its value for the following reasons:-

- Such unique value facilitates the assessment of investment feasibility.
- The suggested formula explains the relationship between investment and internal rate of return in algebraic form.
- The detected role of interest rate from the formula justifies certain role to the exchange rate and then the adaptation path to the national income in the open economy.
- The suggested formula may be contributed in the stability debates.

2.2 Study Objectives:

- Highlighting the ambiguous role interest rate in macroeconomics.
- Defining the limitations of economic policy or the extent of the policy effectiveness when it uses interest rate and exchange rate.
- Contributing in school debate concerning the macro stability of the economic.
- Elaborating an easier formula of internal rate of return has properties of simplicity, accuracy, in addition to a few efforts in calculation the project feasibility.

3. Theoretical Framework

Investment is a hot issue in macroeconomics because the stability of the economic depends on. Here the main theories of business cycle and then growth considering investment and its explanatory variables are crucial factors which explain the phenomena of upswing and downswing of the national income.

According to the above mentioned the determination of the direction in addition to the extent of the impacts of the explanatory variables is important keys in understanding the macro world of economic activity (Rosolind Levacic,1982).

Primary; investment is defined as capital formation. Investment plays vital role in macroeconomic theory like income determination and income growth moreover business cycles theories.

Many theories explain investment from its determination like accelerator theory (income), internal stock theory (profit) as well as classical modern theory (income & interest rate)

In order to incorporate investment in income determination theory we convey inverse relationship between investment and interest rate. If we convey direct relationship it is impossible to use demand side policies (fiscal and monitory policy).

Also all theories which seek interpretation to the business cycles they hold investment as crucial variable like damping inventory theory and accelerator multiplier theory.

Importantly the theories which formulate the growth models emphasis on the interpretation of investment like Harrod growth theory, classical modern theory of growth, Rosto theory of growth beside Caldore theory of growth

The decision of investment is either undertook by net present value or internal rate of return, theoretically no conflict between the two approaches. The interest result is that the suggested derived formula in this study about internal rate of return reaches to a contradiction between the two approaches. There fore the formula needs more discussion (Michael, 1999).

4. Definition and Uses of IRR

The internal rate of return on an investment or project is the "annualized effective compounded return rate" or "rate of return" that makes tenet present value (NPV as NET*1/(1+IRR)^year) of all cash flows (both positive and negative) from a particular investment equal to zero. It can also be defined as the discount rate at which the present value of all future cash flow is equal to the initial investment or in other words the rate at which an investment breaks even.

In more specific terms, the IRR of an investment is the discount rate at which the net present value of costs (negative cash flows) of the investment equals the net present value of the benefits (positive cash flows) of the investment.

IRR calculations are commonly used to evaluate the desirability of investments or projects. The higher a project's IRR, the more desirable it is to undertake the project. Assuming all projects require the same amount of up-front investment, the project with the highest IRR would be considered the best and undertaken first.

A firm (or individual) should, in theory, undertake all projects or investments available with IRRs that exceed the cost of capital. Investment may be limited by availability of funds to the firm and/or by the firm's capacity or ability to manage numerous projects (Bruce, 2003).

Because the internal rate of return is a rate quantity, it is an indicator of the efficiency, quality, or yield of an investment. This is in contrast with the net present value, which is an indicator of the value or magnitude of an investment.

An investment is considered acceptable if its internal rate of return is greater than an established minimum acceptable rate of return or cost of capital. In a scenario where an investment is considered by a firm that has shareholders, this minimum rate is the cost of capital of the investment (which may be determined by the risk-adjusted cost of capital of alternative investments). This ensures that the investment is supported by equity holders since, in general, an investment whose IRR exceeds its cost of capital adds value for the company (i.e., it is economically profitable).

One of the uses of IRR is by corporations that wish to compare capital projects. For example, a corporation will evaluate an investment in a new plant versus an extension of an existing plant based on the IRR of each project. In such a case, each new capital project must produce an IRR that is higher than the company's cost of capital. Once this hurdle is surpassed, the project with the highest IRR would be the wiser investment, all other things being equal (including risk).

IRR is also useful for corporations in evaluating stock buyback programs. Clearly, if a company allocates a substantial amount to a stock buyback, the analysis must show that the company's own stock is a better investment (has a higher IRR) than any other use of the funds for other capital projects, or than any acquisition candidate at current market prices (Pogue, 2004).

5. Calculation of IRR (Moten, J. and Thron, C, 2013)

Given a collection of pairs (time, cash flow) involved in a project, the internal rate of return follows from the net present value as a function of the rate of return. A rate of return for which this function is zero is an internal rate of return.

Given the (period, cash flow) pairs (n, C_n) where n is a positive integer, the total number of periods N, and the net present value NPV, the internal rate of return is given by $r$in:

$$\mathrm{NPV} = \sum_{n=0}^{N} \frac{C_n}{(1+r)^n} = 0$$

Where;

NPV stands for net present value

C_n stands for cost in year n

r stands for market interest rate

The period is usually given in years, but the calculation may be made simpler if r is calculated using the period in which the majority of the problem is defined (e.g., using months if most of the cash flows occur at monthly intervals) and converted to a yearly period thereafter.

Any fixed time can be used in place of the present (e.g., the end of one interval of an annuity); the value obtained is zero if and only if the NPV is zero.

In the case that the cash flows are random variables, such as in the case of a life annuity, the expected values are put into the above formula.

Often, the value of r cannot be found analytically. In this case, numerical methods or graphical methods must be used.

Numerical solution

Since the above is a manifestation of the general problem of finding the roots of the equation $\mathrm{NPV}(r)$, there are many numerical methods that can be used to estimate r. For example, using the secant method, r is given by

$$r_{n+1} = r_n - \mathrm{NPV}_n \left(\frac{r_n - r_{n-1}}{\mathrm{NPV}_n - \mathrm{NPV}_{n-1}} \right).$$

Where;

NPV_n stands for net present value in year n

r_{n+1} stands for market interest rate in future year

NPV_{n-1} stands for present value in past year

r_{n-1} stands for market interest rate in past year

where r_n is considered the nth approximation of the IRR.

This r can be found to an arbitrary degree of accuracy. An accuracy of 0.00001% is provided by Microsoft Excel.

The convergence behaviour of the function by the following:

- If the function $\mathrm{NPV}(i)$ has a single real root r, then the sequence converges reproducibly towards r.
- If the function $\mathrm{NPV}(i)$ has n real roots $r_1, r_2, \ldots, r_n$, then the sequence converges to one of the roots, and changing the values of the initial pairs may change the root to which it converges.
- If function $\mathrm{NPV}(i)$ has no real roots, then the sequence tends towards +∞.

Having $r_1 > r_0$ when $\mathrm{NPV}_0 > 0$ or $r_1 < r_0$ when $\mathrm{NPV}_0 < 0$ may speed up convergence of r_n to r.

Of particular interest is the case where the stream of payments consists of a single outflow, followed by multiple inflows occurring at equal periods. In the above notation, this corresponds to:

$$C_0 < 0, \quad C_n \geq 0 \text{ for } n \geq 1.$$

In this case the NPV of the payment stream is a convex, strictly decreasing function of interest rate. There is always a single unique solution for IRR.

Given two estimates r_1 and r_2 for IRR, the secant method equation (see above) with $n = 2$ always produces an improved estimate r_3. This is sometimes referred to as the Hit and Trial (or Trial and Error) method. More accurate interpolation formulas can also be obtained: for instance the secant formula with correction

$$r_{n+1} = r_n - \mathrm{NPV}_n \left(\frac{r_n - r_{n-1}}{\mathrm{NPV}_n - \mathrm{NPV}_{n-1}} \right) \left(1 - 1.4 \frac{\mathrm{NPV}_{n-1}}{\mathrm{NPV}_{n-1} - 3\mathrm{NPV}_n + 2C_0} \right),$$

(which is most accurate when $0 > \mathrm{NPV}_n > \mathrm{NPV}_{n-1}$) has been shown to be almost 10 times more accurate than the secant formula for a wide range of interest rates and initial guesses. If applied iteratively, either the secant method or the improved formula always converges to the correct solution.

Both the secant method and the improved formula rely on initial guesses for IRR. The following initial guesses may be used:

$$r_1 = (A/|C_0|)^{2/(N+1)} - 1$$
$$r_2 = (1 + r_1)^p - 1$$

where

$$A = \text{ sum of inflows } = C_1 + \cdots + C_N$$
$$p = \frac{\log(A/|C_0|)}{\log(A/\mathrm{NPV}_{1,in})}.$$

where, $\mathrm{NPV}_{1,in}$ refers to the NPV of the inflows only (that is, set $C_0 = 0$ and compute NPV).

5.1 Limitation with IRR

As an investment decision tool, the calculated IRR should not be used to rate mutually exclusive projects, but only to decide whether a single project is worth investing in. NPV vs discount rate comparison for two mutually exclusive projects. Project 'A' has a higher NPV (for certain discount rates), even though its IRR (= x-axis intercept) is lower than for project 'B'

In cases where one project has a higher initial investment than a second mutually exclusive project, the first project may have a lower IRR (expected return), but a higher NPV (increase in shareholders' wealth) and should thus be accepted over the second project (assuming no capital constraints).

Since IRR does not consider cost of capital, it should not be used to compare projects of different duration. Modified Internal Rate of Return (MIRR) does consider cost of capital and provides a better indication of a project's efficiency in contributing to the firm's discounted cash flow.

When a project has multiple IRRs it may be more convenient to compute the IRR of the project with the benefits reinvested. Accordingly, MIRR is used, which has an assumed reinvestment rate, usually equal to the project's cost of capital.

It has been shown that with multiple internal rates of return, the IRR approach can still be interpreted in a way that is consistent with the present value approach provided that the underlying investment stream is correctly identified as net investment or net borrowing (Hazen, 2003).

Despite a strong academic preference for NPV, surveys indicate that executives prefer IRR over NPV. Apparently, managers find it easier to compare investments of different sizes in terms of percentage rates of return than by dollars of NPV. However, NPV remains the "more accurate" reflection of value to the business. IRR, as a measure of investment efficiency may give better insights in capital constrained situations. However, when comparing mutually exclusive projects, NPV is the appropriate measure (Gitman & Joehnk, 2008).

6. Derivation of the suggested formula:

Internal rate of return is a rate of interest rate which equates the total cost of the project and the sum of successive present values of the project. Or when the net present value equal zero. The following equation represents the matter:-

$$C=\left(\frac{Y}{1+r}\right)+\left(\frac{Y}{(1+r)^2}\right)+\left(\frac{Y}{(1+r)^3}\right)+\dots\dots+\left(\frac{Y}{(1+r)^N}\right) \_\_\_\_\_\_\_ (1)$$

Where;

C stands for the total cost of the project

Y stands for the initial return of the project

N stands for the duration of the project

r stands for the market interest rate

Let

$$(1+r)=d \_\_\_\_\_\_\_\_\_\_\_\_\_ (2)$$

Then equation (1) becomes;

$$C=\left(\frac{Y}{d}\right)+\left(\frac{Y}{(d)^2}\right)+\left(\frac{Y}{(d)^3}\right)+\dots\dots+\left(\frac{Y}{(d)^N}\right) \_\_\_\_\_ (3)$$

Let

$$\left(\frac{1}{d}\right)=f \_\_\_\_\_\_\_\_\_ (4)$$

Then equation (3) becomes;

$$C=Y\left[f+f^2+f^3+\dots\dots+f^N\right] \_\_\_\_\_\_\_\_\_\_\_\_ (5)$$

The terms between the brackets in equation (5) are regressive geometric series because (f) is fraction. The sum of the series is given by:-

$$a\left(\frac{1-r^N}{1-r}\right)$$

Where;

A is the first term which from equation (5) is (f)

R is the base which from equation (5) is also (f)

N is the number of terms

Accordingly equation (5) will become;

$$C=Yf\left[\frac{1-f^N}{1-f}\right] \_\_\_\_\_\_\_\_ (6)$$

Then from equation (6) we reach;

$$C-f\left(Y+C\right)+Yf^{(N+1)}=o \_\_\_\_\_\_\_\_\_\_ (7)$$

In equation (7) we must find the value of (f) which solves the equation

Then from equation (2) and (4) we can find the internal rate of return by

$$IRR=\left(\frac{1-f}{f}\right) \_\_\_\_\_\_\_\_\_\_\_\_ (8)$$

As far as (f) is fraction the logic value of (f) must lie between (0.5 & 1) in order to keep (IRR) neither negative nor greater than one.

Alternatively the formula also could be derived by following steps

Let C = [f + f^2 + ... + f^N]

f * C = [F^2 + ...+ f^N + f^(N+1)]

Then C - f *C = f - f^(N+1)

Then C = f * (1 - f^N)/(1-f)

from equation (5)

C = Y * [f * (1 - f^N)/(1-f)]

6.1 Advantages of the formula:

- It gives unique (ratio) solution the internal rate of return.
- It needs only three variables namely:-

A- The expected initial return of the project.

B- The total cost of the project.

C- The duration of the project.

- It can be used manually, within a limit of a few minutes the accurate solution can be reached.
- The factorization of the equation not needs more than fifteen substitutions.
- Computer programming of the equation solution can be done easily for any required decimal points.
- No need to any supplementary tables to find the required value.

6.2 Limitation of the formula:

The formula supposes only one value to the initial project return, while practically, in some times; there exist more than one value for each year in the life span of the project. Such critique can also be said to the cost factor. In order to solve this problem the formula can resort to the statistical measures specifically the weighted mean. Here the weights would be given proportionally according to the denominator of the present value and the sequence of the years.(Also we can use geometric & harmonic means)

7. Beyond the formula:

There exists direct relationship between investment and interest rate. This relation can be traced as follow from the equations:-

- Any increase in r equation (2) will reduce f equation (4) then increase IRR in equation (9). Hence increase in IRR will stimulate the investment.
- Based on the above statement exchange rate should has direct relation with export if and only if interest rate effects negatively exchange rate. This because to the direct effect of interest rate to investment as proved.
- The repercussions of the above two statements are the following:-

a- IS & LM analysis will break down as they established from the assumption of negative relationship between investment and interest rate.

b- The crowding out mechanism of money supply will be reversed to crowding in mechanism in the multipliers of government expenditure and taxation. This result can be reached straight forward from the identity of aggregate expenditure if a direct relation between interest rate and investment is hold.

c- Money supply has negative relation with income because the money supply is engaging in the aggregate expenditure identity through investment equation.

d- If the above statement (c) is correct, the quantity theory of exchange and then the monetarism are critically les influential.

7.1 Views on the paradox of interest and exchange rates:

- The ambiguous impact of interest rate and then exchange rate which was traced historically postulates a very limit role of them as explanatory variables.
- If interest and exchange rates are not explanatory variables, the only out let from the paradox is considering them as appetizers.
- Such appetence justifies the usage of the expression of (to some extent) in defining positive or negative role of interest and exchange rates in the process of orienting the economic activity.
- The exactness in manipulating any economic policy could be hardly reached had the mathematical logic contradict the intuitive wisdom as the formula showed.
- In order to reach tranquil and trustable principle the probability likelihood concept must involve in interpretation and predicting any phenomena.
- The above concept can be implemented with view to mathematical logic and intuitive wisdom holding in mind the historical experiences of phenomena behavior.
- To keep and sustain stability inclusive look to all schools of thought is necessary.
- The paradox which was defined justifies a situation of self independence to any phenomena in responding to any exogenous factors.
- It is high time to test the sign of the parameter is it correct or wrong without reference to the theoretical back ground or intuition.

Finally all we stated is wrong if we look to investment through net present value (equation 1), clearly there exists an inverse relationship between interest rate and investment. What we want to inquire about is which key is leading in investment decisions (Net present value or internal rate of return).

8. Conclusion and Macro- Implications of the formula

As a matter of deduction if we looked to investment from net present value prospect there exist negative effect of market interest rate on investment, while if we introduced the other side of the story which is evaluating of investment through the internal rate of return the formula demonstrates positive effect of market interest rate on investment. The contradiction of the two approaches out set the necessity of different interpretations to the role of investment which played in macro-level.

Economic solution to the paradox of interest rate respecting is urgently needed. Here saving definition as counter part of investment can add much arguments in solving the paradox. Another point is if interest rate has conflicted effect on investment, it is feasible to investigate about response variable (investment) behavior in isolation from stimulate variable (interest rate) when policy makers strike it as instrument.

From the above paragraph we can state that the issue of economic stability can never be managed if and only if inner behavior of investment outweighs the strikes of market interest rate on investment decisions, consequently, economics mechanism in down rooting the phenomena is significantly suspected.

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