Aggregate Demand & aggregate Supply: Formulating Equations and their Policy Implications

Yassin El Tahir

King Khalid University, Faculty of Administrative and Financial Sciences, Abha,KSA
Email: yasn_99@hotmail.com

Abstract: Macroeconomic indicators like inflation, unemployment as well as payment balance influence by national income level and growth. To manage macro indicators ought to compromise national income growth. Hence factorization of income helps in assessing these indicators beside achieving their target levels. This study approaching national income through aggregate demand and supply, specifically the attempt to formulate equations to these concept facilitates the analysis as well as management of national income. Accordingly by solving the final equations a unique equilibrium values to price level and income will be found out. From these values exchange rate and interest rate equilibrium values were reached, consequently, sub markets equilibrium quantities will be obtained. The approach of this study concentrates on economy's potential which frame the limits within which the policy makers design their plans. Finally the choice between different policies will belong to the geometric version of the model, which also rationalizes the adoption of specific school of thought.

Keywords: Aggregate demand, Aggregate supply, AS & AS analysis, Equilibrium.

1. Introduction:

National income represents the core of macro-economic theory, because it determines the general economic indicators such as inflation and unemployment. To large extent, these indicators determine the levels of development and income distribution. Thus, the article depends on the mathematical modeling of national income determinants in order to interpret, explain and controlling such phenomena.

It is well known that national income has central or pivotal role in the economic literature as it explains all the economic phenomena that relates to the public interest. Formally the article had used the definition equation of the national income in the dynamic analysis to interpret the cycle span of the economy. This study aims to mathematize the aggregate supply and demand in order to explain and test different macroeconomic theories(Inprage, 1951).

2. The problem:

The problem of the study stems from the disequilibrium of sectors or markets of the economy that leads to weaken economic growth. Hence the idea of macro model is crucial in treating the economy because it reflects the equilibrium values of aggregate demand and supply. Thus, the article explores the equilibrium values of different sectors, in addition to formulate adequate and rational economic policies on the macro level in accordance with economic potentialities. In short, the study focuses on the following statement: " Mathematization of national income main components facilitates the tackling of economic status". However, the study's approach highlight the following query:

Is the geometrization of AD & AS better off for analyzing national income? in light of:-

- Determining the potential of the economy via equilibrium values of different variables which constitute national income sectors.

- Rolling out the conflicted policy measures according to the slope of AD & AS.

- Tolerating school debate by determining the shape of AD & AS.

3. Development of AD & AS analysis: A General Overview

Michael (1987) argued that aggregate demand and aggregate supply are economic tools developed to analysis the national income, which trace the origin of national income beside the factors that affect it. Furthermore the technique design policy measures in order to control its movement.

AD & AS approach divided the income into sectors or markets which cover real sectors and financial ones. The demand side includes the product market, money market as well as foreign exchange market. While the supply side includes labour market and production function.

The interaction of product market, money market, and foreign exchange market generate what so called IS & LM curves. IS curve represents equilibrium in the product market , while LM represents equilibrium in money market. From IS & LM aggregate demand will be produced, which in turn is inverse relationship between national income and price level.

The main factors which affect aggregate demand are government expenditure and tax rate via real sector. in addition interest rate and exchange rate

affect aggregate demand through transmission channels, the former affect real market via investment and financial market via demand of money, while the latter affect real sector in principle. Money market could affected by controlling money supply through statuary reserve, discount window as well as open market operations.

Aggregate supply produced by interaction of labour market and production function. The former affected by wages rates while the latter motivated by number of labour force , capital as well as state of technology. In effect aggregate supply is a direct relationship between national income and price level.

The interaction of aggregate demand and aggregate supply determines the equilibrium level of income at the same time price level equilibrium will be determined. In effect full employment level income is set mainly by economic capacity and partly by policy makers compromises. According to these concerns deflationary and inflationary gaps can exist.

4. School Debates:

The shape and usage of aggregate demand and supply rest significantly on conflicted school thoughts. For example flexibility of prices and wages leads to vertical aggregate supply curve, while rigidity of them produces, to some extent, horizontal or flat aggregate supply curve. Flexible price and wage assumption pertains to hidden market mechanism and full employment moreover none effective demand. The rigidity assumption of prices and wages, at least in the short run, necessitates appreciated role of demand in determining and controlling equilibrium level of income . Also within demand concern money market considerations could shaped beside directing aggregate demand(Imprage, 1986).

Rational expectations school touched national income analysis via its assumption about mark up and interest rate which incorporate into the model through each demand and supply components. New classical school debates concerning Lafer curve states more emphasis on tax leave from individuals and cooperates, then, it transmits its affects to over whole economy through production and supply(Gwartney & Stroup, 1999).

5. Economic Policy:

According to the position of national income the economic policy can be smoothly tackled. The determination of full employment state is crucial in designing certain economic policy, economy's potency as well as warranted future growth of income were important in defining full employment state. Whenever the level of full employment was pre-determined inflationary and its counterpart contraction policy can easily run to achieve the macro targets. Depression or inflation will be corrected by using either demand or supply side policies. The decision of using either policy rest on which school of thought the policy makers adopted. In an open economy exchange rate and capital mobility play substantial role in the effectiveness of certain policy(Exlender, 1951)

For a given policy to be effective the policy makers ought to pre-determine their objectives and targets along with the tools or instruments that run in order to realize the targets. Needed tools and instruments must exceed the numbers of targets to give the policy more flexibility, otherwise must at least be equal to the targets. The adequate quantity of each instrument can be calculated through likelihood , loss functions as well as Lagrange multiplier techniques(Rosolind, 1982).

6. Stability & model Dynamic(Chiang , 2005):

The stability of the model is essential in analyzing national income, it checks whether the equilibrium exist or in addition to stability of equilibrium. Stability properties of the model were tested via geometric characteristics of model relationships, namely the slopes of aggregate demand and supply curves. The slopes determine the degree of model stability, here we refer to final curves as well as sub curves from which the final curves were derived.

As far as aggregate demand curve the IS and LM curves determine the stability as well as existence of the equilibrium in product and money markets. From the steepness and flatness of each curve equilibrium stability and existence can be detected. Upward and downward slope of LM and IS respectively implies existence and stability of equilibrium, while if such position were violated any deviation from equilibrium the economy could not return back. The same deduction holds to aggregate demand and supply curves in the whole economy. The normal positions of aggregate demand and supply are affected by sub curves positions, upward and downward slopes of AS and AD respectively require sub curves slopes combat with economic theory.

The dynamic analysis of the model reveals something about its stability characteristics. Dynamic state of a model realized through inserting lag variables in the model, the lags clarify long run tendency in the model to the equilibrium. Another way for looking to long run state in the system is to establish characteristic equation then testing its roots, negative roots implies stability that either momentic or oscillated according to the root magnitude. positive roots implies instability that either momentic or oscillated according to the root magnitude.

The economic policy designs according to the features of stability, such policies either drive the economy to its equilibrium steady path in case of instability, or sustain stability in case of existence equilibrium. With concern of stability policies policy

makers attempt to promote smooth steady path to national income by tackling anti cyclical measures. These measures balance income growth to growths in population, capital as well as technology. If policy makers could able to manage the balance between income growth and production factors growths the economy would persisting excellent performance. In effect, the rate of unemployment has to be in at an appropriate equilibrium level if the rate of inflation is to be stable.

7. model Evaluation (Gunner Bardsen, etal., 2005. Arnold, 2004) :

In order to evaluate an economic model it is valuable to investigate the transmission channels, testing exogeniety and invariance as well as model performance.

As far as transmission channels or mechanism quantitative picture of how changes in nominal variables bring about real effects in macroeconomic model. This procedure is main objective of the whole modeling exercise, eg, how changes in exchange rate or interest rate or even price level can affect the sub-models and finally national income. So the transmission mechanism can be seen as a complex system where different groups of agents interact through markets. An attractive feature of macroeconomic is that it represents the different linkages in a consistent framework.

In our derived model first we determine the equilibrium level price, accordingly exchange rate and interest rate calculate respectively, then we incorporate these equilibrium rates into sub models or markets in order to determine their equilibrium quantities. Finally how these values feedback the national income capacity level.

As far as testing exogeneity and invariance macroeconomic model must involves econometric approach. The concept weak exogeneity and parameter invariance refer to different questions concerning valid conditioning in the context of estimation and valid policy analysis respectively. Weak exogeneity of the conditional variables for the parameters of the model implies that these parameters are free to vary with respect to the parameters of marginal models of national income. Valid policy analysis involves as a necessary condition that the coefficients of the model are invariant to the interaction occurring in the marginal models.

Finally in evaluating the model the performance of estimated model gives insight about feasibility of future model implementations. Here the tracking performance under simulation is the best technique in comparing actual quantities versus estimated ones. Many suggestion concerning estimation and simulation are available like full information maximum likelihood. According to the model performance checks the forecast power of the model apparently becomes judgable.

8. Derivation of the approach's equations:-

Algebra of IS curve   (Product market)
Government expenditure   (Constant)

$$G \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(1)$$

Investment

$$I = d - ei + \upsilon y \quad \dots\dots\dots\dots\dots\dots\dots(2)$$

where;

I ≡ Investment
d ≡ Constant
e ≡ Elasticity of investment for interest rate
i ≡ interest rate
v ≡ Accelerator coefficient

Saving

$$S = -a + sY \quad \dots\dots\dots\dots\dots\dots\dots\dots(3)$$

where;

S ≡ Saving
a ≡ Constant
s ≡ Marginal propensity to save
Y ≡ Income

Tax

$$T = t_0 + tY \quad \dots\dots\dots\dots\dots\dots\dots\dots(4)$$

where;

T ≡ Total tax
t_0 ≡ Indirect tax
t ≡ Direct tax rate
Y ≡ Income

IS curve is derived by the following rule:-

$$I + G = S + T \quad \dots\dots\dots\dots\dots\dots\dots\dots(5)$$

By substituting equations (1) to (4) in rule (5) we get

$$G + d - ei + \upsilon y = -a + sy + t_0 + ty$$

$$(G + d + a - t_0) - ei = (s + t - \upsilon)y$$

$$\left(\frac{G + d + a - t_0}{s + t - \upsilon}\right) - \left(\frac{e}{s + t - \upsilon}\right)i = y \quad \dots(6)$$

Algebra of (LM) curve (Money Market)
Demand for money

$$M^d = f - hi + ky \quad \dots\dots\dots\dots\dots\dots(7)$$

where;

M^d ≡ Demand for money
f ≡ Constant
h ≡ demand for money elasticity for interest rate
i ≡ Interest rate
k ≡ Percentage of income in form of money
Y ≡ Income

Supply of money

$$M^s = n + ci - jp \quad \dots\dots\dots\dots\dots\dots(8)$$

Where;

$M^s \equiv$ Money supply

$n \equiv$ Constant

$c \equiv$ Supply of money elasticity for interest rate

$i \equiv$ Interest rate

$j \equiv$ Supply of money elasticity for price level

$p \equiv$ Price level

(LM) curve is derived by the following rule:-

$$M^d = M^s \dots\dots\dots\dots\dots\dots\dots\dots\dots(9)$$

By substituting equations (7) and (8) in rule (9) we get

$$f - hi + ky = n + ci - jp$$
$$ky = n - f + hi + ci - jp$$
$$y = \left(\frac{n-f}{k}\right) + \left(\frac{h+c}{k}\right)i - \left(\frac{j}{c}\right)p \quad \dots(10)$$

By assuming fixed price level, from equation (10) the term (j/c) equal zero therefore (LM) cure only function in interest rate

$$y = \left(\frac{n-f}{k}\right) + \left(\frac{h+c}{k}\right)i \dots\dots\dots\dots(11)$$

9. Derivation of (AD) curve:-

Aggregate demand is a relation between income and price level and is derived from the equilibrium of product market and money market. The adaptation with price level happens in money market equation (10). As far as equilibrium level of interest rate is get from product market, IS curve, equation (6) as follows:-

$$\left(\frac{G + d + a - t_0}{s + t - \upsilon}\right) - y = \left(\frac{e}{s + t - \upsilon}\right)i$$

$$i = \left(\frac{G + d + a - t_0}{e}\right) - \left(\frac{s + t - \upsilon}{e}\right)y \quad \dots(12)$$

By substituting equation (12) in equation (10) we get:-

$$y = \left(\frac{n-f}{k}\right) + \left(\frac{h+c}{k}\right)\left[\left(\frac{G+d+a-t_0}{e}\right) - \left(\frac{s+t-\upsilon}{e}\right)y\right] - \left(\frac{j}{c}\right)p$$
$$\dots\dots\dots\dots\dots\dots\dots(13)$$

By tracing the derivation we get aggregate demand curve from equation (13) in its final form as follows:-

$$y = \left[\left(\frac{n-f}{c}\right)\left(\frac{k}{h+c}\right)\left(\frac{e}{s+t-\upsilon}\right)\right] - \left[\left(\frac{k}{h+c}\right)\left(\frac{e}{s+t-\upsilon}\right)\left(\frac{j}{c}\right)\right]p$$
$$\dots\dots\dots\dots\dots\dots\dots(14)$$

10. Derivation of (AS) curve:-

Demand for labour

$$L^d = o - z\omega \dots\dots\dots\dots\dots\dots\dots(15)$$

where;

$L^d \equiv$ Demand for labour

$O \equiv$ constant

$z \equiv$ labour demand elasticity for wage

$\omega \equiv$ real wage

Nominal wage curve is (Hyperbola) one of its properties is:-

$$p\omega = x \dots\dots\dots\dots\dots\dots\dots\dots\dots(16)$$

Where;

$p \equiv$ Price level

$\omega \equiv$ Real wage

$x \equiv$ Constant

From equation (16) equilibrium real wage equal:-

$$\omega = \left(\frac{x}{p}\right) \dots\dots\dots\dots\dots\dots\dots(17)$$

By substituting equation (17) in equation (15) we get the equilibrium quantity of labour as follows:-

$$L^* = o - zx\left(\frac{1}{p}\right) \dots\dots\dots\dots\dots\dots(18)$$

The production function if capital and technological state are fixed is:-

$$y = \alpha + \lambda L^* \dots\dots\dots\dots\dots\dots\dots(19)$$

Where;

$y \equiv$ Income

$\alpha \equiv$ Constant count for capital and technological state

$\lambda \equiv$ Coefficient of labour

$L^* \equiv$ Equilibrium labour quantity

By substituting equation (18) in equation (19) we get aggregate supply curve (AS) as follows:-

$$y = \alpha + \lambda\left[o - zx\left(\frac{1}{p}\right)\right]$$

$$y = \alpha + \lambda o - \lambda zx\left(\frac{1}{p}\right) \dots\dots\dots\dots\dots(20)$$

As the aggregate supply function in prices level, we notice that the prices level in equation (20) in form of (1/p), therefore the product of subtraction is positive and hence the relationship between the prices level and income is direct despite the negative sign in the right side.

11. Extension to open economy:

In this case we add the foreign exchange market in order to count the effect of international exchange. Here we add export plus capital inflow as a demand side and import plus capital outflow as a supply side. Then we subtract supply from demand and hence the final result is foreign exchange surplus. We this surplus (in term of prices level and interest rate) to the aggregate demand equation in hope of derivation the final equation of aggregate demand with the effect of

international exchange. Here below the equations of foreign exchange market:-

Export

$$X = \sigma - \delta P + \varepsilon Y - \phi E \qquad \text{…………(21)}$$

Where;

X ≡ export

σ ≡ constant

δ ≡ elasticity of export for prices level

ε ≡ elasticity of export for income

ϕ ≡ elasticity of export for exchange rate

Import

$$M = \theta + \rho P + \mu Y + \chi E \qquad \text{…………(22)}$$

Where;

M ≡ Import

θ ≡ constant

ρ ≡ elasticity of import for prices level

μ ≡ elasticity of import for income

χ ≡ elasticity of import for exchange rate

Capital inflow

$$C_{\text{inf}} = M + \psi i \qquad \text{……………….………(23)}$$

Where;

C_{inf} ≡ capital inflow

m ≡ constant

ψ ≡ elasticity of capital inflow for interest rate

Capital outflow

$$C_{outf} = N - \varpi i \qquad \text{…………………………..(24)}$$

Where;

C_{outf} ≡ capital outflow

$$Y = \left[\left(\frac{n-f}{c}\right)\left(\frac{k}{h+c}\right)\left(\frac{e}{s+t-\upsilon}\right)\right] -$$

$$\left[\left(\frac{k}{h+c}\right)\left(\frac{e}{s+t-\upsilon}\right)\left(\frac{j}{c}\right)\right]P +$$

$$\times \left[\left(\frac{G+d+a-t_0}{e}\right)-\left(\frac{s+t-\upsilon}{e}\right)Y\right]$$

The final equation of aggregate demand is;

$$Y = \frac{\left[\left(\frac{n-f}{c}\right)\left(\frac{k}{h+c}\right)\left(\frac{e}{s+t-\upsilon}\right) + (\sigma-\theta) - \phi\varphi + \chi\varphi + (M+N) + [(\phi-\chi)\eta + (\psi-\varpi)]\left[\frac{G+d+a-t_0}{e}\right]\right]}{\left[1-(\varepsilon-\mu)+(\phi-\chi)\eta+(\psi-\varpi)\left(\frac{s+t-\upsilon}{e}\right)\right]}$$

$$-\frac{\left[\left(\frac{k}{h+c}\right)\left(\frac{e}{s+t-\upsilon}\right)\left(\frac{j}{c}\right) + (\delta+\rho)\right]P}{\left[1-(\varepsilon-\mu)+(\phi-\chi)\eta+(\psi-\varpi)\left(\frac{s+t-\upsilon}{e}\right)\right]}$$

$$\text{…(29)}$$

12. Results &Policy Implications

The first result is determining the equilibrium values of income and price level, then we substitute

n ≡ constant

ϖ ≡ elasticity of capital outflow for interest rate

In order to put the equations of the foreign exchange market only in term of prices level and interest rate we treat exchange rate in term of interest rate as follows:-

Exchange rate

$$E = \varphi + \eta I \qquad \text{……………………………..(25)}$$

Where;

E ≡ exchange rate

φ ≡ constant

η ≡ elasticity of exchange rate for interest rate

The foreign exchange market surplus

$$(\sigma-\theta)-(\delta+\rho)P+(\varepsilon-\mu)Y-\phi\varphi+\chi\varphi+(M+N)+[(\phi-\chi)\eta+(\psi-\varpi)]i$$
$$\text{….(26)}$$

(for detailed equation derivation see Appendix 1)

From equation (12) the interest rate is;

$$I = \left(\frac{G+d+a-t_0}{e}\right)-\left(\frac{s+t-\upsilon}{e}\right)Y$$

The foreign exchange surplus only in term of prices level and interest rate is;

$$(\sigma-\theta)-(\delta+\rho)P+(\varepsilon-\mu)Y-\phi\varphi+\chi\varphi+(M+N)+[(\phi-\chi)\eta+(\psi-\varpi)]$$

$$\times\left[\left(\frac{G+d+a-t_0}{e}\right)-\left(\frac{s+t-\upsilon}{e}\right)Y\right] \text{…………...(27)}$$

We add the surplus to the equation (14)

$$(\sigma-\theta)-(\delta+\rho)P+(\varepsilon-\mu)Y$$
$$-\phi\varphi+\chi\varphi+(M+N)+[(\phi-\chi)\eta+(\psi-\varpi)]$$
$$\text{…………(28)}$$

these values in foreign market in order to find the equilibrium values of interest rate and exchange rate.

The second result is substitution different equilibrium prices and income in each market to find equilibrium level of investment, tax, saving, export, import, consumption, labor, and nominal wage.

After estimating the different functions and inserting the parameters in modeling equations, we deduct:-

(a) From equations (6) and (16 & 29) we can get knowledge about existence and stability of

equilibrium. Here we can use analytical geometry to determine the degree of flatness and steepness of (AD) and (AS) curves

(b) From equations (20) and (29) we can be able to determine the shapes of (AD) and (AS) curves. The graphic representation of the two curves explains the state of the economy from the point of full employment, in addition, we can select the suitable demand side or supply side polices according to the state of the economy.

(c) From different estimated parameters it could be able to count the multipliers of fiscal and monetary policy. Such multipliers clarify the limits of effectiveness of fiscal and monetary policies in the presence of crowding out mechanism.

(d) AD & AS equations tell us about the capacity of the economy which determine the full employment of the income.

(e) AD & AS equations provide dynamic analysis of income from both side of economy (Demand & Supply) here we use the elasticity of AD & AS to determine the stability characteristics of the economy.

(f) From IS & LM equations we can determine the effectiveness of fiscal and monetary policies and the effectiveness of supply side policies from the state of AS

13. Acknowledgment:

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Appendixes
Appendix 1
For equation 26

$$(\sigma - \delta P + \varepsilon Y - \phi E + M + \psi i) - (\theta + \rho P + \mu Y + \chi E + N - \varpi i) \quad \text{...........................(1)}$$

$$(\sigma - \theta) - (\delta + \rho)P + (\varepsilon - \mu)Y - (\phi - \chi)E + (M + N) + (\psi - \varpi)i \quad \text{...................(2)}$$

$$(\sigma - \theta) - (\delta + \rho)P + (\varepsilon - \mu)Y - (\phi - \chi)[\varphi - \eta I] + M + N + (\psi - \varpi)i \quad \text{...............(3)}$$

$$(\sigma - \theta) - (\delta + \rho)P + (\varepsilon - \mu)Y - \phi\varphi + \chi\varphi + (M + N) + [\phi - \chi]\eta I + (\psi - \varpi)i \quad \text{.....(4)}$$

Thus, it could be rewritten as:

$$(\sigma - \theta) - (\delta + \rho)P + (\varepsilon - \mu)Y - \phi\varphi + \chi\varphi + (M + N) + [(\phi - \chi)\eta + (\psi - \varpi)]i$$

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