Structural Efficiency of Prestressing for Concrete Bridges in Ultimate Stage

Maged A. Hashad, Fathy A. Saad and Khaled H. Riad

Department of Structural Engineering, Faculty of Engineering, Ain Shams University, Cairo, Egypt
Email: Maged.Hashad@dargroup.com

Abstract: The codes describe the behavior of the prestressed beams in the ultimate stage as the prestressed concrete beam starts to behave like a reinforced concrete beam when the value of the flexural moment is well beyond the cracking moment and the total service load moment. The ultimate theory in flexure and the principles and concepts underlying it are thus equally applicable to prestressed concrete. The same fundamental format of equations, modified to reflect the characteristics of the different reinforcing materials and the geometry peculiar to prestressed concrete. The code equations aims to calculate the ultimate moment capacity (ultimate moment of resistance) while neglecting the beneficial effect of prestressing normal force and the induced prestressing moment on the ultimate moment then The capacity is compared with the demand at ultimate load considering the corresponding factor of safety. This study presents the beneficial effect of prestressing normal force and the induced moment on the ultimate moment for twenty five internally bonded prestressed beams.

[Maged A. Hashad, Fathy A. Saad and Khaled H. Riad. **Structural Efficiency of Prestressing for Concrete Bridges in Ultimate Stage.** *J Am Sci* 2015;11(6):237-241]. (ISSN: 1545-1003). http://www.jofamericanscience.org. 27. doi:10.7537/marsjas110615.27

Keywords: Prestressed; Bonded tendons; Ultimate stage Behavior; Strain Compatibility

1. Introduction

The analysis of prestressed concrete sections at ultimate capacity is a simple procedure if one follows the ACI or the Eurocode. These codes equations aims to calculate the ultimate moment capacity (ultimate moment of resistance) while neglecting the beneficial effect of prestressing normal force and the induced moment on the ultimate moment. The capacity is compared with the demand at ultimate load considering the corresponding factor of safety.

These specifications provide a number of simplifying assumptions, namely, that the state of strain in the concrete compressive zone at ultimate is known, that the force in the concrete can be approximated from the equivalent rectangular stress block and that the stress in the prestressing steel can be approximated from materials and section properties.

It has been shown that these assumptions lead to under estimated predictions of ultimate moments for fully prestressed and somewhat partially prestressed normal weight concrete beams. Partial prestressing implies the use in combination with prestressing steel of non-prestressed conventional reinforcement in the tensile and/or compressive zone of the section.

Prestressed concrete is today being used in combination with substantially large amounts of non prestressed reinforcement. Thus, there is an increasing need in these types of applications for a tool to predict flexural capacity of the section and more importantly to predict with enough accuracy, curvatures, rotations and deflections at ultimate. It should be based on a more accurate analysis in which the actual stress-strain properties of the materials involved are taken into consideration.

Such a procedure (referred to as "strain compatibility") is suggested by the ACI specifications in order to determine the stress in the prestressing steel (fps) at ultimate behavior, it is accepted in all cases in lieu of the more approximate Code formula for fps is required when the steel stress-strain curve does not conform with specified ASTM standards. Furthermore, the use of a more accurate analysis may lead to substantial savings in the amount of prestressing steel required which more than offset the additional cost in design.

The purpose of this study is to present a simplified procedure to analyze the behavior at ultimate of bonded prestressed and partially prestressed concrete structural elements in which the non-linear behavior of the prestressing steel is fully accounted for.

For given conditions of reinforcement we lead to the values of stress and strain in the prestressing steel at ultimate, the ultimate moment capacity and the corresponding curvature of the section and other relevant information.

We allow a quantitative assessment of the influence on ultimate behavior of important parameters such as amount of non-prestressed reinforcement, effective prestress, type of prestressing steel, ultimate compressive strain of the concrete, and stress block dimensional factors.

2. Proposal for Statically determinate Beams

The design assumptions of the ACI concerning the linear strain distribution and the concrete stress block at ultimate are followed and could be visualized by referring to Fig. 1.1. It is also assumed that the stress strain relation of the prestressing steel is known either graphically or numerically. It is further assumed that the strain in the top fiber of the concrete section under effective prestress alone is negligible.

The determination of the actual stress and strain in the prestressing steel at ultimate requires also the knowledge of a relation between stress and strain as derived from compatibility of strain and equilibrium of the section.

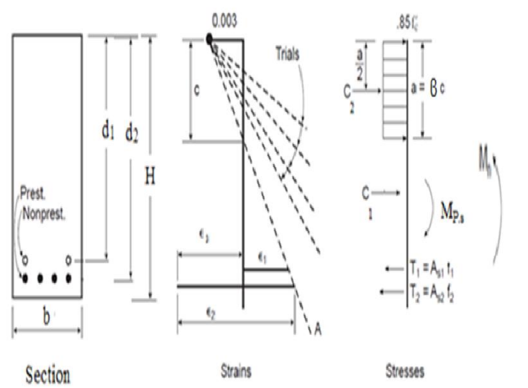

Fig. 1.1: Possible strain distributions in the ultimate limit state in case of adding the prestressing normal force to the ACI equations

2.1 Stress-Strain Relation as Derived from Compatibility and Equilibrium and ACI Equations

Relating the stress fps and the strain Eps in the prestressing steel at ultimate capacity of the section has been primarily derived from considerations of equilibrium and linear strain distribution in the concrete section so actual values of fps and Eps must also satisfy the stress-strain relation of the prestressing steel.

Referring to the strain diagram in the concrete at ultimate capacity (Fig. 1.1), the effective strain in prestressing steel ε3 is calculated

$\varepsilon 3 = (0.75fpu\text{-}losses)/Eps$ (1.1)

It can be shown that the distance from the top fiber to the neutral axis c is firstly assumed

$c/ \varepsilon cu = (d-c)\, \varepsilon 1$ (1.2)

Note, that for a given beam cross section, Eq. (1.1) is a relation between c and Eps as all other terms are known. An additional equation translating the equilibrium of tensile and compressive forces in the section at ultimate capacity (Fig. 1.1) can be written, but its form depends on whether the section behaves as a rectangular section or as a T section.

If the section behaves at ultimate as a rectangular section the equilibrium condition leads to the following equation

$C1+C2 = T$ (1.3)

Where:

C1: Normal force due to pestressing.

C2: Compression force of concrete stress block.

$C2=0.85f'c\, b\, a$ (1.4)

$a=\beta\, c$ (1.5)

$\beta = 0.85$ for $f'c \le 27$ Mpa

$\beta = 0.80$ for $f'c = 34$ Mpa

$\beta = 0.85$ for $f'c = 41$ Mpa

$\beta = 0.85$ for $f'c = 48$ Mpa

$\beta = 0.85$ for $f'c \ge 55$ Mpa

$T=T1+T2= Aps\, fps + As\, fts$ (1.6)

Where fps is the tensile stress in the prestressing steel at ultimate moment capacity, fts is the tensile stress in the non-prestressed tensile steel ($fts \le fy$) Generally, fts equals fy at ultimate.

Then check equilibrium using C1+C2 =T. if C1+C2< T increase c or vice versa and repeat above steps until satisfactory convergence is achieved then calculate the nominal moment strength about T2.

$Mn=((d2-a/2)C2-((d2-d1)T1)-MP.s+C1y$ (1.7)

Where:

MP.s: Moment due to prestressing normal force.

Note that this equation can easily be modified to accommodate concretes for which the dimensional factors of the stress block are different.

2.2 Stress-Strain Relation as Derived from Compatibility and Equilibrium and Eurocode Equations

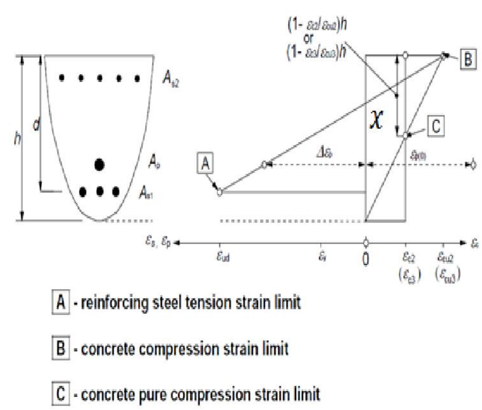

Fig. 1.2: Possible strain distributions in the ultimate limit state in case of adding the prestressing normal force to the Eurocode equations

In Fig 1.2 It can be shown that the distance from the top fiber to the neutral axis x is firstly assumed

$$\frac{x}{\varepsilon cu2-\varepsilon c2}=\frac{d-x}{\varepsilon ud+\varepsilon c2} \quad (1.8)$$

Where

$\varepsilon cu2$: Ultimate concrete strain=0.0035 if $fck \le 50$ Mpa

$\varepsilon c2$:Strain at reaching the maximum strength =0.002 if $fck \le 50$ Mpa

εud: Strain at prestressing steel

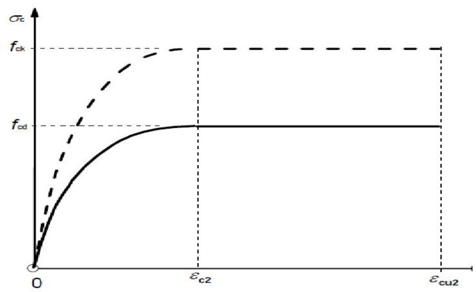

Fig. 1.3: Parabola-rectangle diagram for concrete under compression

Calculate Compression force of concrete stress block

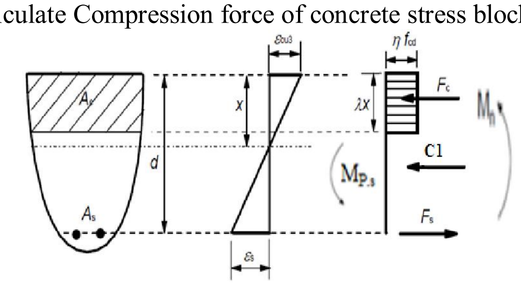

Fig. 1.4: Rectangular stress distribution

Obtain equilibrium of horizontal forces (Tension force =Compression force)

C1+ Fc =T (1.9)

Where:

C1: Normal force due to pestressing.

Fc: Compression force of concrete stress block.

T=T1+T2= Aps fps +As fts

Where f_{ps} is the tensile stress in the prestressing steel at ultimate moment capacity, f_{ts} is the tensile stress in the non-prestressed tensile steel (fts $\leq$ fy) Generally, fts equals fy at ultimate.

$\eta=1$ for $f_{ck} \leq 50$ Mpa

$\eta=1-(f_{ck}-50)/200$ for $50< f_{ck} \leq 90$ Mpa

The design value of compressive strength is

Fc $=\eta$ fcd λx b (1.10)

$\lambda=0.8$ for fck ≤ 50 Mpa

$\lambda=0.8-($ fck-50)/400 for $50<$ fck ≤ 90 Mpa

fcd=α fck/ γc (1.11)

γc=1.5 for persistent & transient design situations

γc=1.2 for accidental design situations

Calculate the section moment capacity Mn

3. Determinate Prestressed Bonded Concrete Beams

3.1 Experimental Program

3.1.1 Specimen Detail

All the post-tensioned beams were rectangular in cross section. The nominal cross-sectional dimensions were 15 by 30 cm. Because the actual dimensions differed slightly from the nominal, the measurements of width and the total depth for each beam are given in table 1.2. The overall length of all beams was 3 m. The beams were cast with a rectangular hole to provide a channel for the single wire reinforcement which extended in a straight line through the length of the beam.

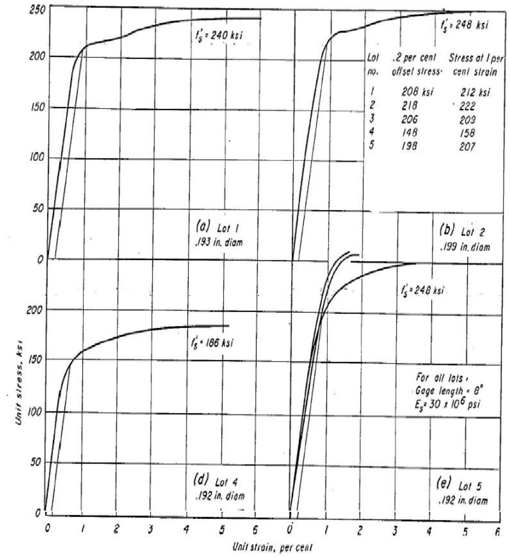

Fig. 1.5: Stress strain relationship for prestressing reinforcement.

Table 1.1 Properties of prestressing reinforcement

Lot	Strength
1	1653.6
2	1708.72
3	1694.94
4	1274.65
5a	1770.73

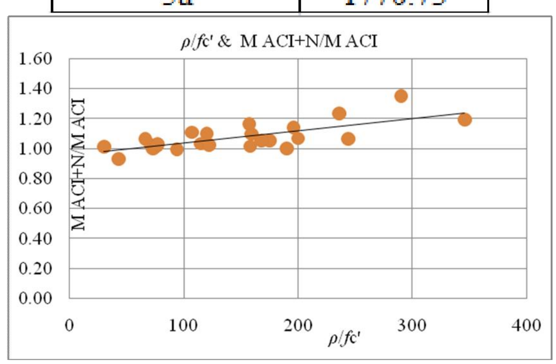

Fig. 1.7: Effect of ρ/f_c' on the increase in ultimate strength due to the prestressing normal force effect.

It is observed from figure 1.7 that the increase in ultimate strength due the normal force increases linearly with the ρ/f_c' ratio.

3.1.2 Test Set-Up and Loading Procedure

The 25 beams were numbered originally according to the order of testing; however they have been regrouped and re-designated according to the major variables. Each beam is designated by two letters and two groups of numerals.e.g.OB.35.153.

Theletters refer to the method of construction of the construction of the beam and to the type of bond, respectively. The of numerals indicate the nominal level of effective prestress and the position of the applied load in terms of twelfth-points of the span, respectively.

The second group of numerals is the ratio ρ/fc' in l0-5 / p.s.i. for a given beam. All the properties of the beams is indicated in table 1.2 and stress strain curve for the prestressing reinforcement is shown in Fig. 1.5 and the properties of prestressing reinforcement is indicated in table 1.1.

4. Results

The table of comparison of ACI and proposed method and experimental results is shown in table 1.3. It is observed from Fig. 1.6 that the increase in ultimate strength due the normal force increases linearly with the reinforcement ratio *ρ*.

Table 1.2 Beams data.

Specimen Code	Concrete strength *fc*' MPA	beam width b cm.	beam depth h cm.	effective depth d cm.	Area of reinforcement As mm^2	Reinforcement ratio %	*ρ*/*fc*'	RFT lot	Effective prestress *f*se MPA
OB.14.030	24.34	15.49	30.48	22.89	38.1	0.107	30	1	131
OB.14.066	43.64	15.49	30.73	23.44	151	0.418	66	1	131.6
OB.14.107	26.96	15.49	30.73	23.39	151	0.418	107	1	140.6
OB.14.157	38.27	15.49	30.73	21.16	283.2	0.87	157	1	140.6
OB.14.175	25.86	15.24	30.73	20.70	207.7	0.656	175	1	146.1
OB.14.244	25.86	15.24	30.73	20.29	283.2	0.916	244	1	139.2
OB.24.168	23.79	15.49	30.99	20.90	187.1	0.579	168	5	689
OB.24.190	17.24	15.75	30.48	20.14	149.7	0.375	190	4	689
OB.34.043	45.23	15.49	30.73	22.99	100.6	0.284	43	2	813
OB.34.071	49.50	15.24	30.73	23.67	183.9	0.51	71	5a	827
OB.34.073	26.34	15.49	30.73	23.55	100.6	0.278	73	2	817.2
OB.34.074	52.61	15.49	30.73	23.19	200.6	0.561	74	3	793.7
OB.34.076	37.85	15.24	30.48	23.14	149.7	0.424	76	3	742.7
OB.34.077	38.96	15.49	30.73	23.70	160.6	0.437	77	2	786.1
OB.34.115	56.54	15.24	30.48	20.83	307.1	0.943	115	2	808.2
OB.34.120	23.72	15.49	30.73	23.34	149.7	0.413	120	3	784.8
OB.34.122	42.20	15.49	30.48	20.93	240.6	0.746	122	2	802
OB.34.159	40.75	15.49	30.73	20.55	301.3	0.942	159	2	777.2
OB.34.196	22.55	15.49	30.48	20.35	200.6	0.641	196	2	788.9
OB.34.200	31.65	15.49	30.73	21.23	301.3	0.92	200	2	813
OB.34.236	20.34	15.49	30.73	20.62	220.6	0.695	236	2	799
OB.34.290	22.61	15.49	30.73	20.29	301.3	0.953	290	2	777.9
OB.34.346	8.76	15.49	30.48	23.55	160.6	0.44	346	2	802.7
OB.44.094	31.58	15.24	30.48	23.09	151	0.429	94	1	1040
OB.44.158	28.27	15.24	30.48	21.06	207.7	0.647	158	1	1025.2

Table 1.3 Comparison between ACI and the proposed method for calculating the ultimate capacity of beams

Specimen Code	M ACI m.t.	Mu cal. m.t.	Mu cal. /M ACI	M EXP m.t.	Reinforcement ratio %	*ρ*/*fc*'	M EXP/Mu cal.
OB.14.030	1.33	1.35	1.01	1.40	0.107	30	1.04
OB.34.043	3.50	3.70	1.06	3.80	0.284	43	1.03
OB.14.066	4.89	5.23	1.07	5.18	0.418	66	0.99
OB.34.071	6.25	6.41	1.03	7.10	0.51	71	1.11
OB.34.073	3.35	3.35	1.00	3.56	0.278	73	1.06
OB.34.074	6.40	6.52	1.02	7.50	0.561	74	1.15
OB.34.076	4.76	4.86	1.02	5.62	0.424	76	1.16
OB.34.077	5.25	5.41	1.03	6.10	0.437	77	1.13
OB.44.094	4.52	4.51	1.00	5.18	0.429	94	1.15
OB.14.107	4.46	4.94	1.11	5.27	0.418	107	1.07
OB.34.115	7.59	7.85	1.03	8.98	0.943	115	1.14
OB.34.120	4.34	4.77	1.10	4.94	0.413	120	1.03
OB.34.122	6.16	6.33	1.03	7.30	0.746	122	1.15
OB.14.157	6.63	7.71	1.16	6.40	0.87	157	0.83
OB.44.158	4.86	4.95	1.02	5.86	0.647	158	1.18
OB.34.159	6.81	7.47	1.10	8.20	0.942	159	1.10
OB.24.168	4.55	4.80	1.05	4.90	0.579	168	1.02
OB.14.175	4.58	4.83	1.05	5.00	0.656	175	1.04
OB.24.190	2.60	2.61	1.00	3.43	0.375	190	1.32
OB.34.196	4.21	4.80	1.14	5.50	0.641	196	1.15
OB.34.200	6.52	6.97	1.07	7.30	0.92	200	1.05
OB.34.236	4.25	5.25	1.24	5.38	0.695	236	1.03
OB.14.244	5.18	5.53	1.07	5.64	0.916	244	1.02
OB.34.290	4.96	6.70	1.35	7.26	0.953	290	1.08
OB.34.346	2.70	3.23	1.19	4.26	0.44	346	1.32
		Average	**1.078**				**1.10**
		STD	**0.089**				**0.10**

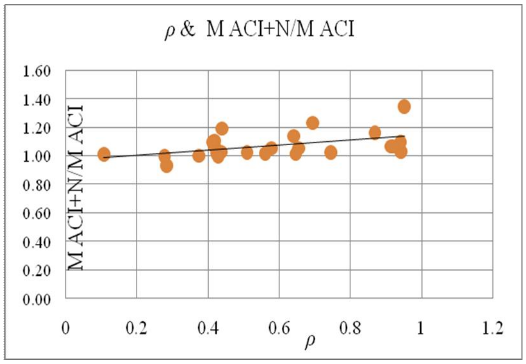

Fig. 1.6: Effect of ρ on the increase in ultimate strength due to the prestressing normal force effect.

5. Conclusion

1. An ultimate strength analysis involving equilibrium and strain compatibility equations and prestressing normal force and induced prestessing moment permitted satisfactory prediction of the effects of all important variables on flexural strength.
2. It was observed that the ultimate capacity of the beams was increased by an average ratio 7.8% by taking the effect of normal force into consideration in the same time the average ratio between the (M ult. experimental/ Mu cal.) was 110% taking into consideration that the results aren't multiplied with the factor of safety *(Φ)* stated in ACI.
3. The increase in ultimate moment capacity due to the effect of normal force increases linearly with the increase in the reinforcement ratio *(ρ)*.
4. The increase in ultimate moment capacity due to the effect of normal force increases linearly with the increase in (ρ/f_c') ratio.

References

1. ACI Committee 318. Building Code Requirements for Structural Concrete (ACI 318–2008) and Commentary (ACI 318–2008). American Concrete Institute, 2008.
2. AASHTO LRFD "Bridge design specifications" 4th Ed, Washington D.C. (2007).
3. California department of transportation (Caltrans). (2006).
4. Eurocode 2 Design of concrete structures - Part 1-1 "General rules and rules for buildings" 2004
5. Eurocode 2 Design of concrete structures "Concrete bridges - Design and detailing rules" 2005.
6. Manual for the design of concrete building structures to Eurocode 2 "The institution of structural engineers 2006".
7. Janney, J.R., Hognestad, E., and McHenry, D. (1956). "Ultimate flexural strength of prestressed and conventionally reinforced concrete beams." *J. Am. Concr. Inst., 2*(1), 601-620. sections." J. Prestress. Concr. Inst.,50(1), 74-93.
8. Baran, E., Schultz, A. E., and French, C. E. (2005). "Analysis of the flexural strength of prestressed concrete flanged sections".

Notations

As = cross-sectional area of concrete
Aps = area of internal prestressing steel
As = area of non-prestressed tensile steel
a= depth of equivalent rectangular stress block
b = width of the member
c = neutral axis depth from the extreme compression fiber in concrete
dp =distance from extreme compression fiber to centroid of prestressed reinforcement
ds = depth of non-prestressing reinforcement from extreme compression fiber
Ec = elastic modulus of concrete in compression
Eps = modulus of elasticity of prestressing reinforcement
Es = modulus of elasticity of nonprestressing steel
fps = stress in prestressing reinforcement at ultimate
fc' = cylinder strength of concrete
fy = yield stress in non-prestressing reinforcement
fpy = yield stress of the prestressing reinforcement
ecu = assumed failure strain of concrete in compression
fpe = effective stress in prestressing tendons (after allowance for all prestress losses)
fpu = tensile strength of prestressed reinforcement
Δfps = tendon stress increase =fps- fse
h = overall height or thickness of member
I = moment of inertia of section about centroidal axis
Mult = Ultimate moment
εce = strain in concrete at level of prestressing steel due to fse
εcu = ultimate concrete compression strain in top fiber.
εps = strain in prestressing steel at ultimate
εpu = ultimate strain in prestressing steel

6/23/2015