The Machine Interference Model with Bulk Arrivals and Hyperexponential Service Time Distribution: M^X/H_r /1/K/N with Balking and Reneging

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Abstract: The objective of this paper is to derive the analytical solution of the machine interference model: $M^X/H_r/1/k/N$ with balking and reneging considering the discipline FIFO. Some measures of effectiveness are deduced and some special cases are also obtained.

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1. Introduction

Morse [10] discussed the steady –state queueing system in which the service channel consists of two branches, the units arrive singly and the capacity of the waiting space is infinite. Gupta and Goyal [5] studied the queue: $M/H_r/1/k$ by using the generating functions. Kleinrock [8] treated the system: M/M/c/k/N for machine interference without balking and reneging, Gross and Harris [4] discussed the queue: M/M/c/k/k with spares only and Medhi [9] treated the system M/M/c/k/k without any concept. Habib [7] and Gupta and Goyal [6] treated the system: $M^X/H_r/1$. White *et al.* [15] solved the system: M/H/2/2 numerically. All the previous studies are without balking and reneging while Shawky and El-Paoumy [14] studied the queue: $M/H_r/1/k$ with balking and reneging, Shawky [11] treated the system: M/M/c/k/N with balking, reneging and spares, in [12] discussed the queue: $H_rM/c/k/N$ with balking and reneging, and Al-Seedy and Al-Ibraheem [1] studied the system: $H_2/M/1/m+Y/m+Y$ with the concepts of balking, reneging, state-dependent, spares, and an additional server for longer queues.

In the present system, it is assumed that the units arrive at the system in batches of random size X, i.e., at each moment of arrival, there is a probability $c_j=Pr(X=j)$ that j units arrive simultaneously, and the interarrival times of batches follow a negative exponential distribution with parameter λ. It is also assumed that the arriving batches are served in order of their arrival at a service channel consisting of r independent branches with different rates of service (see, Badr [2]).

The service channel is busy if a unit is present in any one of the branches and in case the service channel is busy. The moment the service channel disposes of the unit being served, the unit at the head of the queue, if there be any, enters at random any one of the r branches of the service channel. The probability that it goes to the i^{th} branch is $\sigma_i, \sum_{i=1}^{r}\sigma_i = 1.$ The service time distribution in the i^{th} branch is negative exponential with rate μ_i. We assume that we have a finite source (population) of N customers, one server (repairman) is available and the system has finite storage room such that the total number of customers (machines) in the system is no more than k. The queue discipline is assumed to be first come, first served (FCFS).

Consider the balk concept with probability:

β = prob.{a unit joins the queue},

where $0\le\beta<1$ if m = 1(1)N and $\beta=1$ if m=0, m is a number of units in the system.

It is also assumed that the units may renege according to an exponential distribution,

f(t) = $\alpha e^{-\alpha t}, t > 0,$

with parameter α. The probability of reneging in a short period of time Δt is given by $r_m = (m-1)\alpha\,\Delta t$, for $1 < m \le N$ and $r_m = 0,$ for m= 0, 1.

The steady-state equations and their solution

Let $P_{m,i}$ denote the equilibrium probability that there are m units in the system and the unit in service occupies i^{th} branch, m =1(1)k, i = 1(1)r,

P_0 denote the equilibrium probability that there are no units in the system.

Then, the steady-state probability difference equations, in the presence of balking and reneging, are:

$$N\lambda P_0 = \sum_{s=1}^{r} \mu_s P_{1,s}, \qquad m=0 \tag{1}$$

$$\left[(N-1)\beta\lambda + \mu_i\right]P_{1,i} = \sigma_i \sum_{s=1}^{r}(\mu_s + \alpha)P_{2,s} + N\lambda\sigma_i c_1 P_0, \qquad m=1 \tag{2}$$

$$\left[(N-m)\beta\lambda + \mu_i + (m-1)\alpha\right]P_{m,i} = \sigma_i \sum_{s=1}^{r}(\mu_s + m\alpha)P_{m+1,s} + \beta\lambda\sum_{j=1}^{m-1}(N-m+j)c_j P_{m-j,i} + N\lambda\sigma_i c_m P_0, \qquad m=2(1)k-1 \tag{3}$$

$$\left[(N-k)\beta\lambda + \mu_i + (k-1)\alpha\right]P_{k,i} = \beta\lambda\sum_{j=1}^{k-1}(N-k+j)c_j P_{k-j,i} + \beta\lambda \sum_{j=1}^{k}\sum_{n=k-j+1}^{k}(N-n)c_j P_{n,i} + N\lambda\sigma_i c_k P_o, \qquad m=k \tag{4}$$

where i = 1(1)r.

Summing (2) over i and using (1),

$$\sum_{i=1}^{r}(\mu_i + \alpha)P_{2,i} = (N-1)\beta\lambda\sum_{i=1}^{r}P_{1,i} + N\lambda(1-c_1)P_o. \tag{5}$$

Also, summing (3) over i and using (5),

$$\sum_{i=1}^{r}(\mu_i + m\alpha)P_{m+1,i} = \beta\lambda\sum_{j=1}^{m}\sum_{i=1}^{r}(N-j)P_{j,i} - \beta\lambda\sum_{n=1}^{m-1}\sum_{j=1}^{m-n}\sum_{i=1}^{r}(N-j)c_n P_{j,i} - N\lambda P_0\sum_{i=1}^{m}c_i + N\lambda P_0, \qquad m=2(1)k-1. \tag{6}$$

From (2) and (5),

$$\left[B(1,i) - (N-1)\beta\lambda\sigma_i\right]P_{1,i} - (N-1)\beta\lambda\sigma_i\sum_{s\neq i}^{r}P_{1,s} = N\lambda\sigma_i P_0, \qquad i=1(1)r, \tag{7}$$

where

$B(m,i) = (N-m)\beta\lambda + \mu_i + (m-1)\alpha$, m = 1(1)k, i = 1(1)r,

which can be written in the matrix form as

$B_1 P_1 = N\lambda P_0 S$, (8)

where

$B_m = \left[b_{ij}(m)\right]$,

such that

$$b_{ij}(m) = -(N-m)\beta\lambda\sigma_i \quad, i \neq j$$

$$b_{ii}(m) = B(m,i) - (N-m))\beta\lambda\sigma_i$$

$$P_m^T = [P_{m,1}, P_{m,2}, \ldots, P_{m,r}],\ m = 1(1)k-1$$

and

$S^T = [\sigma_1, \sigma_2, \ldots, \sigma_r]$,

where T denotes the transpose of a matrix.

Now, the inverse matrix of B_1 is given by

$$B_1^{-1} = [b_{ij}^*(1)],$$

where

$$b_{ij}^{*}(m)=\frac{(N-m)\beta\lambda\sigma_i}{B(m,i)B(m,j)D_m},\quad i\neq j$$

$$b_{ii}^{*}(m)=\frac{1}{B(m,i)}+\frac{(N-m)\beta\lambda\sigma_i}{B^2(m,i)D_m},$$

such that

$$D_m=1-(N-m)\beta\lambda\sum_{i=1}^{r}\frac{\sigma_i}{B(m,i)},\quad m=1(1)k.$$

Using this value of B_1^{-1} in (8), we have

$$P_{1,i}=\frac{N\lambda\sigma_i}{B(1,i)D_1}P_0,\quad i=1(1)r. \tag{9}$$

Similarly, from (3) and (6),

$$[B(m,i)-(N-m)\beta\lambda\sigma_i]P_{m,i}-(N-m)\beta\lambda\sigma_i\sum_{s\neq i}^{r}P_{m,s}=\beta\lambda\sum_{j=1}^{m-1}(N-m+j)c_jP_{m-j,i}-R(m)\sigma_i$$

where

$$R(m)=-\beta\lambda\sum_{j=1}^{m-1}\sum_{s=1}^{r}(N-j)P_{j,s}+\beta\lambda\sum_{n=1}^{m-1}\sum_{j=1}^{m-n}\sum_{s=1}^{r}(N-j)c_nP_{j,s}+N\lambda P_0\sum_{j=1}^{m-1}c_j-N\lambda P_0,$$

which can be written in the matrix form as:

$B_m P_m = E - R(m) S$ (10)

where

$B_m=[b_{ij}(m)]$, m = 2(1)k-1

$$E^T=[\beta\lambda\sum_{j=1}^{m-1}(N-m+j)c_jP_{m-j,1},\beta\lambda\sum_{j=1}^{m-1}(N-m+j)c_jP_{m-j,2},\ldots,\beta\lambda\sum_{j=1}^{m-1}(N-m+j)c_jP_{m-j,r}].$$

Now, the inverse matrix of B_m is given by

$$B_m^{-1}=[b_{ij}^{*}(m)].$$

Using this value of B_m^{-1} in (10), we get

$$P_{m,i}=\frac{\beta\lambda}{B(m,i)}\left\{\sum_{j=1}^{m-1}(N-m+j)c_jP_{m-j,i}+\frac{(N-m)\beta\lambda\sigma_i}{D_m}\sum_{s=1}^{r}\sum_{j=1}^{m-1}\frac{(N-m+j)c_jP_{m-j,s}}{B(m,s)}\right\}-\frac{R(m)\sigma_i}{B(m,i)D_m},\quad i=1(1)r,\ m=2(1)k-1. \tag{11}$$

From (4),

$$P_{k,i}=\frac{\beta\lambda}{\mu_i+(k-1)\alpha}\left\{\beta\lambda\sum_{j=1}^{k-1}(N-k+j)c_jP_{k-j,i}+N\lambda c_k\sigma_iP_o+\beta\lambda\sum_{j=2}^{k}\sum_{n=k-j+1}^{k-1}(N-n)c_jP_{n,i}\right\},$$

m = k. (12)

Equations (9), (11) and (12) are the required recurrence relations, that give all the probabilities in terms of P_0, which itself may now be determined by using the normalizing condition:

$$P_0 + \sum_{m=1}^{k}\sum_{s=1}^{r} P_{m,s} = 1, \tag{13}$$

hence all the probabilities are completely known in terms of the queue parameters.

The following example illustrates the method discussed above.

Example:

In the above model: $M^X/H_r/1/k/N$ with balking and reneging, letting r = 3, k=5 and N = 8, i.e., the model: $M^X/H_3/1/5/8$ with balking and reneging, the results are:

$$P_{1,1} = a_1 P_0, \quad P_{1,2} = a_2 P_0, \quad P_{1,3} = a_3 P_0, \quad P_{2,1} = b_1 P_0, \quad P_{2,2} = b_2 P_0,$$

$$P_{2,3} = b_3 P_0, \quad P_{3,1} = d_1 P_0, \quad P_{3,2} = d_2 P_0, \quad P_{3,3} = d_3 P_0, \quad P_{4,1} = e_1 P_0,$$

$$P_{4,2} = e_2 P_0, \quad P_{4,3} = e_3 P_0, \quad P_{5,1} = f_1 P_0, \quad P_{5,2} = f_2 P_0, \quad P_{5,3} = f_3 P_0,$$

where

$$a_1 = \frac{8\lambda\sigma_1}{(7\beta\lambda+\mu_1)D_1}, \quad a_2 = \frac{8\lambda\sigma_2}{(7\beta\lambda+\mu_2)D_1}, \quad a_3 = \frac{8\lambda\sigma_3}{(7\beta\lambda+\mu_3)D_1}$$

$$b_1 = \frac{\beta\lambda}{6\beta\lambda+\mu_1+\alpha}\left\{7c_1a_1 + \frac{42\beta\lambda c_1\sigma_1}{D_2}\left[\frac{a_1}{6\beta\lambda+\mu_1+\alpha} + \frac{a_2}{6\beta\lambda+\mu_2+\alpha} + \frac{a_3}{6\beta\lambda+\mu_3+\alpha}\right]\right\}$$
$$+ \frac{\sigma_1(1-c_1)[7\beta\lambda(a_1+a_2+a_3)+8\lambda]}{(6\beta\lambda+\mu_1+\alpha)D_2}$$

$$b_2 = \frac{\beta\lambda}{6\beta\lambda+\mu_2+\alpha}\left\{7c_1a_2 + \frac{42\beta\lambda c_1\sigma_2}{D_2}\left[\frac{a_1}{6\beta\lambda+\mu_1+\alpha} + \frac{a_2}{6\beta\lambda+\mu_2+\alpha} + \frac{a_3}{6\beta\lambda+\mu_3+\alpha}\right]\right\}$$
$$+ \frac{\sigma_2(1-c_1)[7\beta\lambda(a_1+a_2+a_3)+8\lambda]}{(6\beta\lambda+\mu_2+\alpha)D_2}$$

$$b_3 = \frac{\beta\lambda}{6\beta\lambda+\mu_3+\alpha}\left\{7c_1a_3 + \frac{42\beta\lambda c_1\sigma_3}{D_2}\left[\frac{a_1}{6\beta\lambda+\mu_1+\alpha} + \frac{a_2}{6\beta\lambda+\mu_2+\alpha} + \frac{a_3}{6\beta\lambda+\mu_3+\alpha}\right]\right\}$$
$$+ \frac{\sigma_3(1-c_1)[7\beta\lambda(a_1+a_2+a_3)+8\lambda]}{(6\beta\lambda+\mu_3+\alpha)D_2}$$

$$d_1 = \frac{\beta\lambda}{5\beta\lambda+\mu_1+2\alpha}\left\{6c_1b_1 + 7c_2a_1 + \frac{5\beta\lambda\sigma_1}{D_3}\left[\frac{6c_1b_1+7c_2a_1}{5\beta\lambda+\mu_1+2\alpha} + \frac{6c_1b_2+7c_2a_2}{5\beta\lambda+\mu_2+2\alpha} + \frac{6c_1b_3+7c_2a_3}{5\beta\lambda+\mu_3+2\alpha}\right]\right\}$$
$$-\sigma_1\frac{7\beta\lambda(a_1+a_2+a_3)(c_1+c_2-1) + 6\beta\lambda(b_1+b_2+b_3)(c_1-1) + 8\lambda(c_1+c_2-1)}{(5\beta\lambda+\mu_1+2\alpha)D_3}$$

$$d_2 = \frac{\beta\lambda}{5\beta\lambda+\mu_2+2\alpha}\left\{6c_1b_2 + 7c_2a_2 + \frac{5\beta\lambda\sigma_2}{D_3}\left[\frac{6c_1b_1+7c_2a_1}{5\beta\lambda+\mu_1+2\alpha} + \frac{6c_1b_2+7c_2a_2}{5\beta\lambda+\mu_2+2\alpha} + \frac{6c_1b_3+7c_2a_3}{5\beta\lambda+\mu_3+2\alpha}\right]\right\}$$
$$-\sigma_2\frac{7\beta\lambda(a_1+a_2+a_3)(c_1+c_2-1) + 6\beta\lambda(b_1+b_2+b_3)(c_1-1) + 8\lambda(c_1+c_2-1)}{(5\beta\lambda+\mu_2+2\alpha)D_3}$$

$$d_3 = \frac{\beta\lambda}{5\beta\lambda+\mu_3+2\alpha}\left\{6c_1b_3 + 7c_2a_3 + \frac{5\beta\lambda\sigma_3}{D_3}\left[\frac{6c_1b_1+7c_2a_1}{5\beta\lambda+\mu_1+2\alpha} + \frac{6c_1b_2+7c_2a_2}{5\beta\lambda+\mu_2+2\alpha} + \frac{6c_1b_3+7c_2a_3}{5\beta\lambda+\mu_3+2\alpha}\right]\right\}$$

$$-\sigma_3 \frac{7\beta\lambda(a_1+a_2+a_3)(c_1+c_2-1)+6\beta\lambda(b_1+b_2+b_3)(c_1-1)+8\lambda(c_1+c_2-1)}{(5\beta\lambda+\mu_3+2\alpha)D_3}$$

$$e_1=\frac{\beta\lambda}{4\beta\lambda+\mu_1+3\alpha}\left\{5c_1d_1+6c_2b_1+7c_3a_1+\frac{4\beta\lambda\sigma_1}{D_4}\left[\frac{5c_1d_1+6c_2b_1+7c_3a_1}{4\beta\lambda+\mu_1+3\alpha}\right.\right.$$
$$\left.\left.+\frac{5c_1d_2+6c_2b_2+7c_3a_2}{4\beta\lambda+\mu_2+3\alpha}+\frac{5c_1d_3+6c_2b_3+7c_3a_3}{4\beta\lambda+\mu_3+3\alpha}\right]\right\}-\frac{Y\sigma_1}{(4\beta\lambda+\mu_1+3\alpha)D_4}$$

$$e_2=\frac{\beta\lambda}{4\beta\lambda+\mu_2+3\alpha}\left\{5c_1d_2+6c_2b_2+7c_3a_2+\frac{4\beta\lambda\sigma_2}{D_4}\left[\frac{5c_1d_1+6c_2b_1+7c_3a_1}{4\beta\lambda+\mu_1+3\alpha}\right.\right.$$
$$\left.\left.+\frac{5c_1d_2+6c_2b_2+7c_3a_2}{4\beta\lambda+\mu_2+3\alpha}+\frac{5c_1d_3+6c_2b_3+7c_3a_3}{4\beta\lambda+\mu_3+3\alpha}\right]\right\}-\frac{Y\sigma_2}{(4\beta\lambda+\mu_2+3\alpha)D_4}$$

$$e_3=\frac{\beta\lambda}{4\beta\lambda+\mu_3+3\alpha}\left\{5c_1d_3+6c_2b_3+7c_3a_3+\frac{4\beta\lambda\sigma_3}{D_4}\left[\frac{5c_1d_1+6c_2b_1+7c_3a_1}{4\beta\lambda+\mu_1+3\alpha}\right.\right.$$
$$\left.\left.+\frac{5c_1d_2+6c_2b_2+7c_3a_2}{4\beta\lambda+\mu_2+3\alpha}+\frac{5c_1d_3+6c_2b_3+7c_3a_3}{4\beta\lambda+\mu_3+3\alpha}\right]\right\}-\frac{Y\sigma_3}{(4\beta\lambda+\mu_3+3\alpha)D_4}$$

$$f_1=\frac{1}{\mu_1+4\alpha}\{\beta\lambda[4e_1+5d_1(1-c_1)+6b_1(c_3+c_4+c_5)+7a_1(c_4+c_5)]+8\lambda c_5\sigma_1\}$$

$$f_2=\frac{1}{\mu_2+4\alpha}\{\beta\lambda[4e_2+5d_2(1-c_1)+6b_2(c_3+c_4+c_5)+7a_2(c_4+c_5)]+8\lambda c_5\sigma_2\}$$

$$f_3=\frac{1}{\mu_3+4\alpha}\{\beta\lambda[4e_3+5d_3(1-c_1)+6b_3(c_3+c_4+c_5)+7a_3(c_4+c_5)]+8\lambda c_5\sigma_3\}$$

$$D_1=1-7\beta\lambda\left[\frac{\sigma_1}{7\beta\lambda+\mu_1}+\frac{\sigma_2}{7\beta\lambda+\mu_2}+\frac{\sigma_3}{7\beta\lambda+\mu_3}\right]$$

$$D_2=1-6\beta\lambda\left[\frac{\sigma_1}{6\beta\lambda+\mu_1+\alpha}+\frac{\sigma_2}{6\beta\lambda+\mu_2+\alpha}+\frac{\sigma_3}{6\beta\lambda+\mu_3+\alpha}\right]$$

$$D_3=1-5\beta\lambda\left[\frac{\sigma_1}{5\beta\lambda+\mu_1+2\alpha}+\frac{\sigma_2}{5\beta\lambda+\mu_2+2\alpha}+\frac{\sigma_3}{5\beta\lambda+\mu_3+2\alpha}\right]$$

$$D_4=1-4\beta\lambda\left[\frac{\sigma_1}{4\beta\lambda+\mu_1+3\alpha}+\frac{\sigma_2}{4\beta\lambda+\mu_2+3\alpha}+\frac{\sigma_3}{4\beta\lambda+\mu_3+3\alpha}\right]$$

$$Y=7\beta\lambda(a_1+a_2+a_3)(c_1+c_2+c_3-1)+6\beta\lambda(b_1+b_2+b_3)(c_1+c_2-1)$$
$$+5\beta\lambda(d_1+d_2+d_3)(c_1-1)+8\lambda(c_1+c_2+c_3-1).$$

Now, from (13),

$$P_0=1/(1+a_1+a_2+a_3+b_1+b_2+b_3+d_1+d_2+d_3+e_1+e_2+e_3+f_1+f_2+f_3)$$

Therefore, the expected number of units in the system and in the queue are, respectively,

$$L = \sum_{m=1}^{5}\sum_{i=1}^{3} m\,P_{m,i}$$

$$= \{a_1 + a_2 + a_3 + 2(b_1 + b_2 + b_3) + 3(d_1 + d_2 + d_3) + 4(e_1 + e_2 + e_3) + 5(f_1 + f_2 + f_3)\}P_0$$

$$L_q = \sum_{m=2}^{5}\sum_{i=1}^{3}(m-1)P_{m,i} = L + P_0 - 1$$

$$= \{b_1 + b_2 + b_3 + 2(d_1 + d_2 + d_3) + 3(e_1 + e_2 + e_3) + 4(f_1 + f_2 + f_3)\}P_0.$$

The machine availability (rate of production per machine) is
M. A. = 1- L/5.
The operative efficiency (utilization) is
O. E. = 1- P_0.
Moreover, if we put σ_1= 0.3, σ_2 = 0.2, σ_3 = 0.5, μ_1= 4, μ_2 =3, μ_3 =2, λ =6, β=0.3, α = 0.6, c_1=0.2, c_2=0.1, c_3=0.3, c_4= 0.15 and c_5=0.25, we get:

Special Cases

Some modeling systems can be obtained as special cases of this system:

(i) Let $\sigma_r = \delta_{rs}$ and $\mu_s = \mu$ where δ_{rs} is the Kronecker delta function, then we get the Markovian machine interference system: M^X/M/1/k/N with bulk arrivals, balking and reneging, and the results are:

$$P_1 = \frac{N\lambda}{\mu}P_0,$$

$$P_m = \frac{\beta\lambda}{\mu + (m-1)\alpha}\sum_{j=1}^{m-1}(N - m + j)c_j P_{m-j} - \frac{R(m)}{\mu + (m-1)\alpha}, \quad m = 2(1)k-1,$$

$$P_k = \frac{\lambda}{\mu + (k-1)\alpha}\left\{\beta\lambda\sum_{j=1}^{k-1}(N - k + j)c_j P_{k-j} + N\lambda c_k P_o + \beta\lambda\sum_{j=2}^{k}\sum_{n=k-j+1}^{k-1}(N - n)c_j P_n\right\}$$

where

$$R(m) = -\beta\lambda\sum_{j=1}^{m-1}(N - j)P_j + \beta\lambda\sum_{n=1}^{m-1}\sum_{j=1}^{m-n}(N - j)c_n P_j + N\lambda P_0\sum_{j=1}^{m-1}c_j - N\lambda P_0.$$

Moreover, if we put $c_j = \delta_{j1}$, we get the system: M/M/1/k/N with balking and reneging which discussed by Shawky [11] while, if β =1 and α = 0, we have the system: M/M/1/k/N without balking and reneging which treated by Kleinrock [8], also, when N = k, the model becomes M/M/1/k/k without assumptions of balking and reneging which had been studied by White *et al.* [15], Medhi [9], Gross and Harris [4] and Bunday [3].

(ii) If we put $c_j = \delta_{j1}$, we get the system: M/H_r/1/k/N with balking and reneging which studied by Shawky [13].

References

1. Al-Seeedy, R.O. and Al-Ibraheem, F. M.,"An interaarrival hyperexponential machine interference with balking, reneging, state-dependent, spares and an additional server for longer queues". IJMMS 12, 737-747, 2001.

2. Badr, M.M., “The truncated queues with bulk arrivals and hyperexponential service time distribution with reneging”. Acta Scientia et Intellectus, 2015.
3. Bunday, B. D., "An introduction to queueing theory". John Wiley, New York, 1996.
4. Gross, D. and Harris, C. M., "Fundamentals of queueing theory". John Wiley, New York, 1974.
5. Gupta, S. K. and Goyal, J. K., "Queues with Poisson input and hyperexponential output with finite waiting space". Oper. Res. 14, 75–81, 1964.
6. Gupta, S. K. and Goyal, J. K., "Queues with batch Poisson arrivals and hyperexponential service time distribution". Metrika 10, 171–178, 1966.
7. Habib, M. E., "Queues with bulk arrivals and hyperexponential service time distribution". Tamkang J. Manag. Sci. 7, 31–36, 1986.
8. Kleinrock, L., "Queueing systems". Vol.1, John Wiley, New York, 1975.
9. Medhi, J., "Stochastic models in queueing theory". Academic Press, New York, 1991.
10. Morse, P. M., "Queues, Inventories and Maintenance". John Wiley, New York, 1958.
11. Shawky, A. I., “The machine interference model: M/M/C/K/N with balking, reneging and spares". Opsearch 37, 25-35, 2000.
12. Shawky, A. I., "An interarrival hyperexponential machine interference model: H_r /M/c/k/N with balking and reneging". Comm. Korean Math. Soc. 16, 659-666, 2001.
13. Shawky, A. I., "The service hyperexponential machine interference model: M/H_r /1/k/N with balking and reneging". International Journal of Applied Mathematics, 16(2), 167-174, 2004.
14. Shawky, A. I. and El - Paoumy, M. S., " The truncated hyperexponential service queues: M/H_k/1/N with balking and reneging". Far East J. Theo. Stat. 6, 73–80, 2002.
15. White, J. A., Schmidt, J. W. and Bennett, G. K., "Analysis of queueing systems". Academic Press, New York, 1975.

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