

## Numerical study for dynamic vibration absorber using Coriolis force for pendulum system

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**Abstract:** The present work is concerned with the vibration reduction of pendulum system with dynamic vibration absorber using Coriolis force. The proposed dynamic vibration absorber DVA can attenuate the bi-directional vibration of the pendulum system by one mass, which attaches to the main system through an additional spring and damper. The mass of the DVA moves in the radial direction and it provides Coriolis force in the circumference direction to reduce the swing of main system. The response of the main system is evaluated numerically under free and forced excitations. The damping ratio for free vibration is evaluated for different mass ratios, position ratios, initial amplitudes and DVA natural frequency. The main system response is evaluated in frequency domain for different DVA design parameters. The results indicate that the proposed DVA installed with suitable parameters in the system can effectively attenuate the main system vibration over a broad frequency range, especially for large vibration. The DVA natural frequency should be tuned to twice as that of the pendulum.

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**Keywords:** vibration control, dynamic vibration absorber, spherical pendulum, ropeway carrier

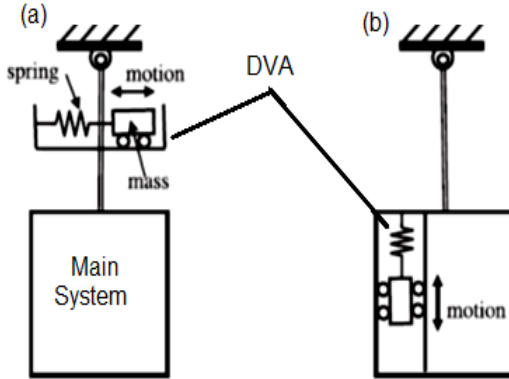
### 1. Introduction

The problem of unwanted vibration reduction has been known many years and it has become more attractive nowadays. A dynamic vibration absorber (DVA) is a device, which is designed to attenuate the unwanted vibration of a main system. DVA has been widely used in many engineering applications to passively attenuate vibration such as, vibration of the fuselage, to control of buildings with undesired vibration, to reduce chatter in turning operations, to reduce pump vibration and to control the motion of power transmission line towers under the wind action [1, 2, 3, and 4]. DVA consisting of a mass, spring, and damper attached to a primary system for reducing undesirable vibration. Frahm in 1909 proposed undamped DVA, and later in 1956, Den Hartog proposed DVA with viscous damper, they modeled the primary system as a single degree of freedom system [5]. After that, the designs of multi-DVAs for continuous structures and multi-degrees of freedom structures have gained attention of many researchers. Varpasuo [6] studied the response characteristics of stochastic vibration absorber. He evaluated the effectiveness of the vibration absorber for different stiffness and damping values. Jang et.al. [7] studied the design of a cantilever type multi-degree of freedom DVA to reduce resonant vibrations of the main structure. Ghosh and Basu [8] presented a closed-form expression for optimal reduction ratio of DVA. Viana et al. [9] used a probabilistic technique to evaluate the optimal parameters of a DVA. Doubrava Filho et al. [10] presented a several optimal DVA designs for viscoelastic vibration absorber. Singhose

and Vaughan [11, 12], designed a multi-mode input shaper to attenuate double-pendulum oscillations for the multi-hoist cranes. Huang and Lin [13] presented a vibration absorber called periodic vibration absorber for mechanical systems subjected to periodic excitation. Matsuhisa et al. [14] proposed a new DVA that uses Coriolis force and moves in the radial direction of the pendulum-type system. Sun et al. [15] made a comparison between the conventional DVA, the state-switched absorber, and dual-DVA their results clarified that dual-DVA had very good performance in multifrequency vibration suppression. Blajer et al. [16] have presented governing equations without consideration of the Coriolis inertia force for rotary crane to execute the prescribed load trajectory. Shyh and Lin [17] studied the performance of conventional dynamic absorbers with ropeway gondola to maximize their location. They concluded that DVA should be as high as possible. Brzeski et al. [18] proposed pendulum dynamic vibration absorber consists of lumped mass suspended to primary system by massless rod as simplified configuration

The planar pendulum has been used as model for many engineering applications such as, machinery, transportation, civil engineering, ropeway gondola, crane and ships in waves [19-21]. Several previous studies have used DVA to reduce the swing of pendulum structures this types of DVA has two main types Fig.1. The first type of DVA moving in a circumference direction. The second type the DVA moving in the radial direction and a Coriolis damping force is produced by this DVA type. The DVA with

Coriolis damping force has a bidirectional vibration control nature, whereas the first type can only reduce planar vibration.



**Fig. 1. Dynamic vibration absorbers for pendulum system,**  
(a) Conventional type , (b) Coriolis type

In the present paper, we focused on the analysis of the dynamic behavior of a pendulum system under the influence of DVA with Coriolis damping to reduce the sway motion of a pendulum. The equation of motion expressed in the dimensionless form and solved numerically. The different design parameters of the DVA have been tested in order to maximizing the damping characteristic of the main system. These include the mass ratio, natural frequency of DVA, damping ratio of DVA and position ratio. The analysis performed in free and harmonic excitation. The effect of initial amplitude of main system and excitation frequency of external force on the system response are considered.

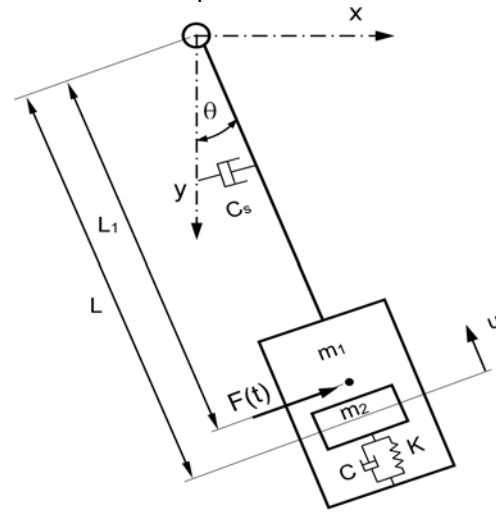
### Notation

|            |  |
|------------|--|
| C          | Damping constant of the DVA.                     |
| K          | Stiffness of absorber spring.                    |
| L          | Distance of the DVA from the pivot.              |
| $L_1$      | Distance of the main system mass from the pivot  |
| $m_1$      | Mass of the main system.                         |
| $m_2$      | Mass of the DVA.                                 |
| r          | Frequency ratio $\frac{\omega}{\omega_n}$ .      |
| $r_a$      | Frequency ratio $\frac{\omega_a}{\omega_n}$ .    |
| u          | The displacement of the DVA in radial direction. |
| $f_0$      | Amplitude of the excitation force.               |
| F          | External force function.                         |
| $\theta$   | Angular displacement of main system.             |
| $\theta_0$ | Initial angular displacement of main system.     |
| $\mu$      | Mass ratio.                                      |
| $\gamma$   | Relative position of the DVA.                    |
| $\varphi$  | Relative displacement of the DVA.                |
| $\xi$      | Damping ratio of the main system.                |
| $\xi_a$    | Damping ratio of the DVA.                        |

|            |                                       |
|------------|---------------------------------------|
| $\omega_a$ | Natural frequency of the DVA.         |
| $\omega_n$ | Natural frequency of the main system. |
| $\omega$   | Frequency of excitation force.        |

## 2. Problem formulation

The concept of the dynamic vibration absorber DVA for reducing pendulum sway motion is shown in Fig. 2. When the structure is in sway motion, the centrifugal force acting on the DVA changes with time and the DVA is in radial motion. The radial motion in turn produces the Coriolis damping that acts on the sway motion of the main structure and attenuates it. The pendulum mass is denoted by  $m_1$ , the rotational angle is denoted by  $\theta$  measured in the x y plane. The x and y-axes are perpendicular to each other; the y-axis is vertical to the direction of gravity where g is the acceleration of gravity. The distance from the pendulum mass to the pivot O is denoted by  $L_1$ . The DVA mass is denoted by  $m_2$ , K is the spring constant and C is the damping constant. The distance between the pivot and the DVA in the static position is denoted by L. The DVA has a linear motion in radial direction with the displacement u.



**Fig. 2. Dynamic model of a pendulum system with a DVA**

## 3. Equation of motion

Consider the coordinate system as shown in Fig. 2, the locations of the main system mass ( $x_1, y_1$ ) and the DVA mass ( $x_2, y_2$ ) can be expressed as:

$$x_1 = L_1 \sin \theta, \quad y_1 = L_1 \cos \theta \quad (1)$$

$$x_2 = (L - u) \sin \theta, \quad y_2 = (L - u) \cos \theta \quad (2)$$

In addition, the velocities of both will be

$$\dot{x}_1 = L_1 \dot{\theta} \cos(\theta), \quad \dot{y}_1 = -L_1 \dot{\theta} \sin(\theta) \quad (3)$$

$$\dot{x}_2 = (L - u) \dot{\theta} \cos(\theta) - \dot{u} \sin(\theta), \quad \dot{y}_2 = -(L - u) \dot{\theta} \sin(\theta) - \dot{u} \cos(\theta) \quad (4)$$

The kinetic energy K.E. is given by:

$$K.E. = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) \quad (5)$$

Using Eqs. 3 and 4 gives:

$$K.E. = \frac{1}{2}m_1L_1^2\dot{\theta}^2 + \frac{1}{2}m_2(\dot{\theta}^2(L-u)^2 + \dot{u}^2) \quad (6)$$

The potential energy P.E. is given by:

$$P.E. = g(1 - \cos\theta)(m_1L_1 + m_2L - m_2u) + \frac{1}{2}Ku^2 \quad (7)$$

The dissipation energy function D.E. is given by

$$D.E. = \frac{1}{2}C\dot{u}^2 + \frac{1}{2}C_s\dot{\theta}^2 \quad (8)$$

The equation of motion of the system can be obtained using Lagrange equations. The system has two degree of freedom with  $\theta$  and  $u$ , the Lagrange equations become:

$$\frac{d}{dt}\left(\frac{\partial(K.E.-P.E.)}{\partial\dot{\theta}}\right) - \frac{\partial(K.E.-P.E.)}{\partial\theta} + \frac{\partial D}{\partial\dot{\theta}} = Q \quad (9)$$

$$\frac{d}{dt}\left(\frac{\partial(K.E.-P.E.)}{\partial\dot{u}}\right) - \frac{\partial(K.E.-P.E.)}{\partial u} + \frac{\partial D}{\partial\dot{u}} = 0 \quad (10)$$

Where  $Q$  is the external moment acting on the main system.

Using Eqs. 6, 7 and 8 gives:

$$\ddot{\theta}(m_1L_1^2 + m_2(L-u)^2) - 2m_2\dot{\theta}\dot{u}(L-u) + C_s\dot{\theta} + g(m_1L_1 + m_2L - m_2u)\sin\theta = f_oL_1\sin(\omega t) \quad (11)$$

$$m_2\ddot{u} + m_2\dot{\theta}^2(L-u) - m_2g(1 - \cos\theta) + Ku + C\dot{u} = 0 \quad (12)$$

To write the Eqs. 11 and 12 in non-dimensional forms, some parameters are introduced:

$$\mu = \frac{m_2}{m_1}, \gamma = \frac{L}{L_1}, \varphi = \frac{u}{L_1}, \xi_a = \frac{C}{2m_2\omega_a},$$

$$\omega_a = \sqrt{\frac{K}{m_2}}, \omega_n = \sqrt{\frac{g}{L_1}}, \xi_s = \frac{C_s}{2m_1L_1^2\omega_n}$$

In which,  $\mu$  is the mass ratio,  $\gamma$  is the relative position of the DVA,  $\varphi$  is the relative displacement of the DVA,  $\xi_a$  is the damping ratio of the DVA and  $\omega_a$  is natural frequency of the DVA.

Then the equations of motion now have the form:

$$\ddot{\theta}(1 + \mu(\gamma - \varphi)^2) - \frac{2\mu\dot{\theta}\dot{u}}{L_1}(\gamma - \varphi) + \omega_n^2(1 + \mu(\gamma - \varphi))\sin\theta + 2\xi_s\omega_n\dot{\theta} = \frac{f_o}{m_1L_1}\sin(\omega t) \quad (13)$$

$$\ddot{u} + L_1\dot{\theta}^2(\gamma - \varphi) - g(1 - \cos\theta) + \omega_a^2u + 2\xi_a\omega_a\dot{u} = 0 \quad (14)$$

These equations are used in the numerical calculations. In Eq. 13, the Coriolis force  $-\frac{2\mu\dot{\theta}\dot{u}}{L_1}(\gamma - \varphi)$  effect as a nonlinear damping force to attenuate the structure response. In Eq. 14 the second term  $L_1\dot{\theta}^2(\gamma - \varphi)$  and third term  $g(1 - \cos\theta)$  are the driving force on the DVA.

#### 4. Results

To illustrate the performance of the proposed DVA, a numerical simulation was carried out. The numerical simulation is performed by solving the full nonlinear Eqs. 13 and 14.

#### 4.1. Free vibration

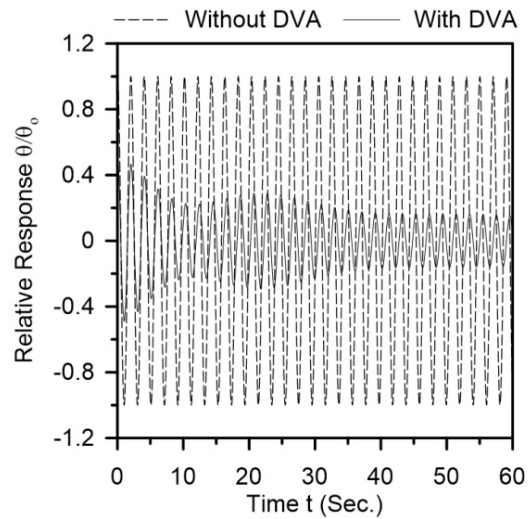
In order to determine the damping coefficient of the system, a free vibration study is performed; an average-damping ratio  $\xi$  of the pendulum system with DVA is estimated herein using the logarithmic decrement  $\delta$ . Where;

$$\delta = \frac{2\pi\xi}{\sqrt{1 - \xi^2}} = \frac{1}{N}\ln\left(\frac{\theta_i}{\theta_{i+N}}\right)$$

$\theta_i$  and  $\theta_{i+N}$  are the amplitudes of the main system at the beginning and end of the cycles.  $N$  is the number of the cycles

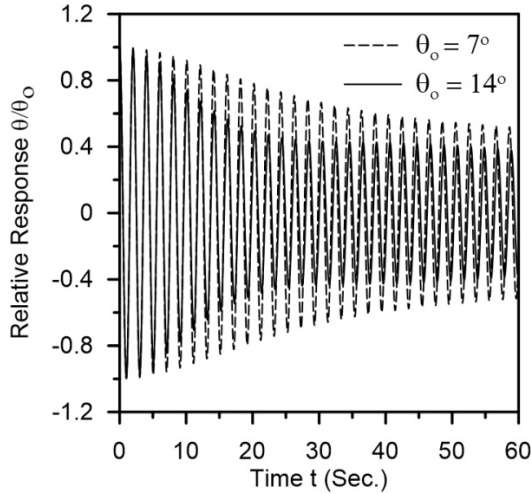
For undamped pendulum system, the damping is only due to the DVA effect. Different initial angular displacements, mass ratios, frequency ratios and relative positions are considered.

Figs. 3 and 4, show non-dimensional responses for the main system acquired past the equilibrium position for free-vibration mode. The results show that the DVA works better for the larger initial angle because the motion of the proposed DVA is proportional to second order of the pendulum amplitude. The effect of mass ratios on the magnitude of the damping ratio  $\zeta$  is illustrated in Fig. 5, for four initial angles  $\theta_o$  of 7°, 14°, 28° and 40°.

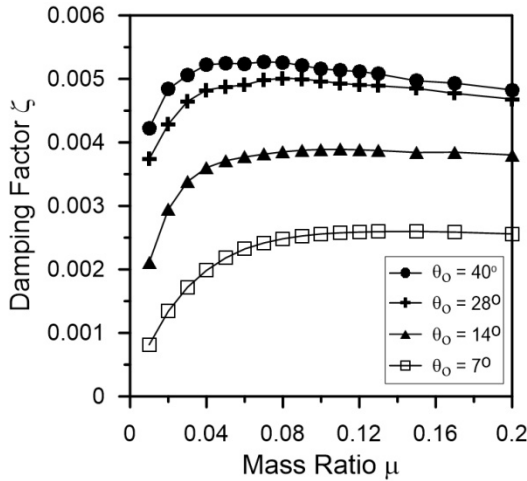


**Fig.3. Free responses of main structure with and without DVA**

The damping ratio is increasing with high initial angles; these results prove that this type of DVA has a good efficiency in case of a large vibration. The increment of mass ratio increases the damping ratio, however, it can also increase the energy stored in DVA, and the large mass ratio does not perform the required damping ratio due to the increasing of total stored energy.



**Fig.4.** Free responses main structure with DVA for different initial displacements



**Fig.5.** Effect of mass ratio on damping factor

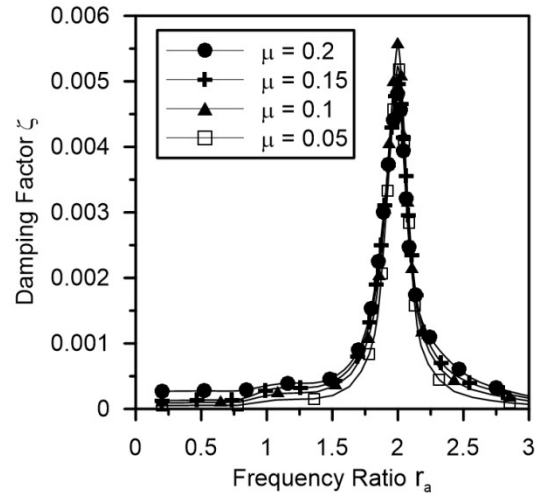
Fig. 6 shows the damping ratio for each period of oscillation for four different mass ratios: 0.05, 0.1, 0.15 and 0.2. This figure shows that when the DVA frequency ratio is close to 2 the maximum damping is achieved for any value of mass ratios. Therefore, in order to increase the DVA efficiency its natural frequency should be tune to twice of the pendulum system to increase the velocity of the DVA. The effect of the DVA damping constant on damping ratio  $\zeta$  is illustrated in Fig. 7, for three relative positions  $\gamma = 0.8, 1.1$  and  $1.4$ . The effectiveness of the absorber is proportional to its location. Because motion of the absorber is caused by the centrifugal force, the absorber must be attached to a place far from the center of swing.

**4.2 Forced vibration**

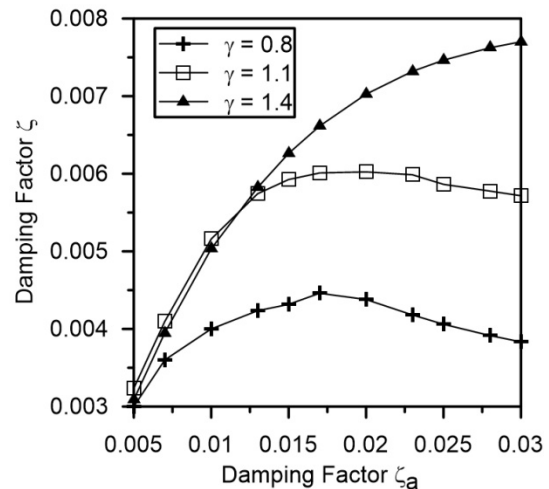
The calculated frequency responses for various mass ratios  $\mu$ , with parameters  $\gamma = 1.1, \xi_s = 0.1,$

$\xi_a = 0.08,$  are shown in Fig. 8. The peak amplitude of the structure is significantly reduced by the DVA, its value depends on mass ratios and in general, the optimum value of mass ratio is between 0.1 and 0.2.

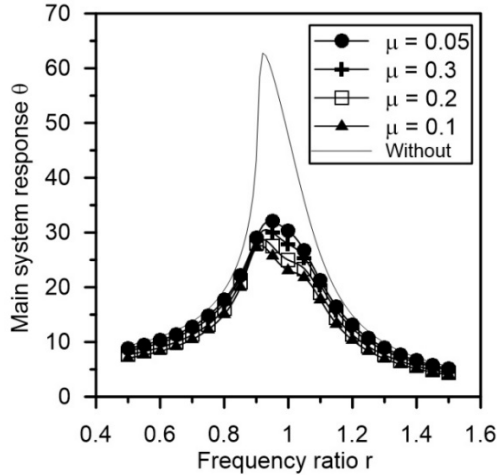
Fig. 9, shows the variation of system responses in frequency domain for mass ratio  $\mu = 0.1$  and relative position  $\gamma = 1.1$ . The DVA frequency ratio  $r_a$  is a parameter. It can be concluded that the maximum contribution of the DVA occurs when its natural frequency becomes twice from the system natural frequency. The system response in frequency domain for different damping ratios of the DVA are illustrated in Fig. 10. For excitation frequencies near resonance, the system response reaches to its peak value, this value depends on the value DVA's damping ratio. Adjust the DVA's damping ratio near 0.05 gives an acceptable system's response.



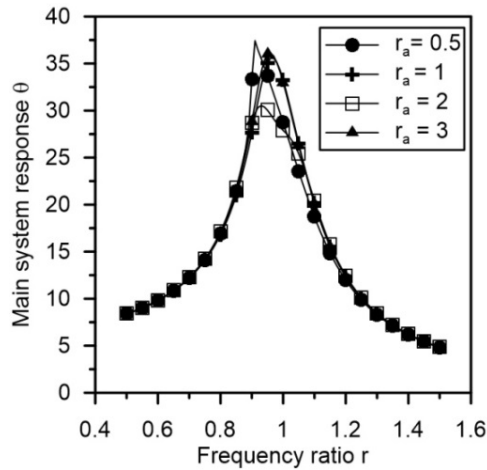
**Fig.6.** Variation of damping factor with DVA frequency ratio for different mass ratios.



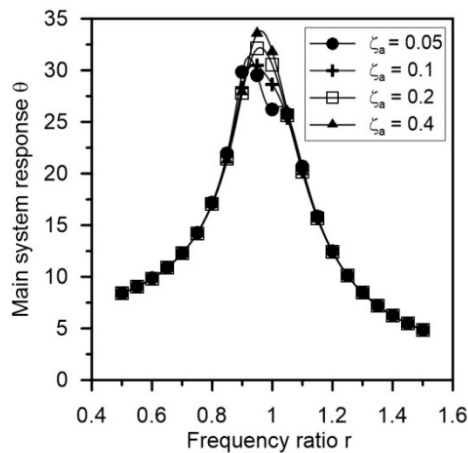
**Fig.7.** Effect of DVA damping ratio on the system damping ratio for different DVA positions



**Fig.8.** Variation of main system response with excitation frequency for different mass ratios



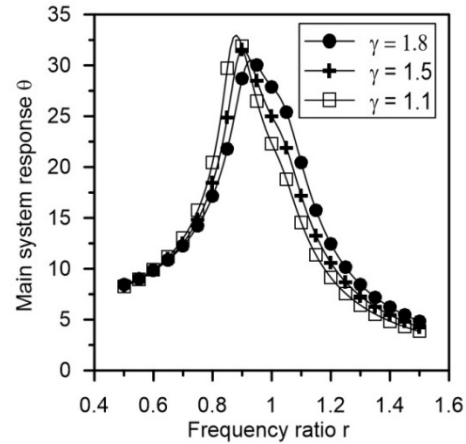
**Fig.9.** Variation of main system response with excitation frequency for different DVA frequency ratio



**Fig.10.** system response in frequency domain for different DVA damping ratio

Fig. 11 shows the main system response in frequency domain with parameters  $\mu = 0.1$ ,  $\xi_s = 0.1$ ,  $\xi_a$

$= 0.08$ , and  $\gamma = 1.1, 1.5$  and  $1.8$ . it can be noticed that the position of the DVA has a high influence on its effectiveness. Due to excitation of the DVA is caused by the centrifugal force; the DVA must be located to the position far from the center of swing.



**Fig.11.** Variation of main system response in frequency domain for different relative position

**5. Conclusions**

In this paper, a passive dynamic vibration absorber that uses Coriolis force has been proposed to reduce the vibration of the pendulum systems. The numerical demonstration of the free and forced vibration of a pendulum systems shows the good effect of Coriolis DVA in case of large vibration that can be considered as a safety device to limit the large sway motion but it has very little effect if the vibration is too small. The absorber should be fixed as far as possible from the center of oscillation of the main system. The optimal DVA frequency ratio should be equal to two, whereas the optimal DVA damping ratio should be within 1% - 5%. Unlike the conventional absorber, the proposed DVA only works for pendulum type system but not for spring-mass system. Because the Coriolis force is proportional to the angular velocity of swing and the velocity of absorber’s mass in the radial direction. The proposed DVA is applicable to many systems such as gondola, boats and cranes. It has the advantage of a bidirectional vibration control by one mass.

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