Site selection using GIS, Simulated Annealing, AHP and EVAMIX approaches under Fuzzy Set Theory: A case study of landfill siting in Blantyre City, Malawi.

Rodney Godfrey Tsiko

Department Geoinformatics and Surveying, University of Zimbabwe, P.O Box MP167, Mount Pleasant, Harare, Zimbabwe.
tsikoruz@yahoo.com

Abstract: This paper explores the applicability and capability of solving site location problems using an integrated approach that includes the Markov-chain based Simulated Annealing (SA) algorithm, Geographic Information System (GIS), fuzzy Analytical Hierarchy Process (AHP) and Evaluation of Mixed Data (EVAMIX). The novelty of the proposed methodology is its capability to combine the flexibility of fuzzy logic in handling uncertainty with the simplicity, easy implementation and independence offered by simulated annealing in solving non-linear optimization problems, thus providing good site location solutions without losing consistency. In addition, the EVAMIX approach provides complete flexibility in using both quantitative and qualitative criteria. The approach is applied to a multiple objective decision problem of selecting the best location for a new landfill site, which will serve Blantyre City, an urban agglomeration in Malawi.
[Tsiko RG. **Site selection using GIS, Simulated Annealing, AHP and EVAMIX approaches under Fuzzy Set Theory: A case study of landfill siting in Blantyre City, Malawi.** *J Am Sci* 2016;12(10):16-33]. ISSN 1545-1003 (print); ISSN 2375-7264 (online). http://www.jofamericanscience.org. 4. doi:10.7537/marsjas121016.04.

Keywords: Simulated Annealing; fuzzy logic; GIS; Analytical Hierarchy Process; EVAMIX

1. Introduction

Decisions regarding site location are often plagued by vagueness, fuzziness, contradictory multiple objectives and a myriad of quantitative and qualitative factors. These decisions have an inverse relationship with our ability to define and model them in a precise way. As the degree of complexity of the decision making process increases, the methods and tools available to resolve them become rare. Hence, there is need for more sophisticated tools to aid decision making vis-à-vis multi-objective site location problems, which include some form of uncertainty.

In this paper, the proposed solving tool is an integration of simulated annealing (van Laarhoven and Aarts, 1987; Kirkpatrick et al., 1983; Cerny, 1985), geographic information system (GIS) (Burrough and McDonnell, 1998; Heywood et al., 2006; Longley et al., 2010), fuzzy Analytical Hierarchy Process (AHP) (Bellman and Zadeh, 1970; Saaty, 1980; Saaty, 1986; Chang, 1996) and Evaluation of Mixed Data (EVAMIX) (Voogd, 1982; Nijkamp et al., 1990; Martel and Matarazzo, 2005; Hajkowicz and Higgins, 2008). Simulated Annealing (SA), a heuristic search algorithm, when used in conjunction with a GIS, yields optimum and close to optimum site locations that satisfy multiple objective functions or cost criteria within the search domain. Fuzzy AHP provides a useful approach to address the problems associated with the intrinsic imprecision, uncertainty and subjectivity of decision makers when faced with multi-attribute data. EVAMIX provides a way to rank and select the best site location by making use of both quantitative (cardinal) and qualitative (ordinal) criteria within the same evaluation matrix. The algorithm behind EVAMIX maintains the essential characteristics of cardinal and ordinal criteria, yet it is designed to eventually combine the results in a single appraisal score. This unique feature gives EVAMIX much greater flexibility and differentiates it from other Multi-Criteria Decision Analysis (MCDA) methods such as weighted summation, range of value method, PROMETHEE II (Brans et al., 1986; Figueira et al., 2005) etc., which are incorrectly applied to ordinal data by treating it as though they were at a cardinal measurement scale.

The approach is applied to a multiple objective decision problem of selecting the best location for a new landfill site, which will serve Blantyre City, an urban agglomeration in Malawi. The example shows how decision makers can use this approach to treat landfill siting as a combinatorial optimization problem within a GIS environment, and use mixed criteria (qualitative and quantitative) to select the best possible landfill site, at the same time accounting for the intrinsic imprecision and ambiguity associated with decision makers when confronted with multi-criteria data.

The rest of the paper is organized as follows. Section 2 explores the three dominant themes of this study: simulated annealing, fuzzy AHP and EVAMIX. In Section 3, a real world case study is presented, the results are discussed and sensitivity analysis is conducted. The paper concludes in Section 4.

2. Heuristic/Optimization Algorithm

Since locating a landfill site in a GIS can be classified as a combinatorial optimization problem, a heuristic search algorithm is required for solving the model. Note that various researchers have developed linear programming models to solve landfill siting or location allocation problems, but encountered limitations related to the spatial area that could be optimized (Arthur and Nalle, 1997; Aerts et al., 2003; Cova and Church, 2000). This problem arises because each map layer in a GIS by itself can consist of complex information on attribute values, and spatial relationships between attributes. When a large number of layers are involved in GIS analysis (as is normally the case), relationships between layers, and within themselves have to be considered. This often leads to a large solution space from which linear programming solvers find the spatial area too large to explore. In consequence, this exploration is time consuming and, in worse circumstances, linear programming models can get trapped in local minima. Heuristic algorithms, however, are robust, fast, capable of solving large combinatorial problems and do not get trapped in local minima. Application of such algorithms for landfill sitting or location allocation problems include simulated annealing, tabu search, genetic algorithms and greedy growing algorithms (Lockwood and Moore, 1993; Murray and Church, 1995; Muttiah et al., 1996; Brookes, 1997; Boston and Bettinger, 1999; Karaganis and Mimis, 2011). In this study the focus is on using simulated annealing as part of a model for solving a location siting problem.

2.1 Simulated Annealing

In general, Simulated Annealing (SA) is an iterative meta-heuristic search algorithm capable of escaping from local optima. Its ease of implementation, convergence properties and its use of hill-climbing moves to escape local optima have made it a popular technique over the past three decades. The origins and theoretical development of SA are well reviewed within the literature, therefore will not be repeated here. However, a good overview of simulated annealing's theoretical development and domains of application can be found in (Pincus, 1970; Kirkpatrick et al., 1983; Cerncy, 1985; Eglese, 1990; Koulamas et al., 1994; Fleischer, 1995). van Laarhoven and Aarts (1987), Aarts and Korst (1989) also devote entire books to the subject. Aarts and Lenstra (1997) dedicate a chapter to SA in their book on local search algorithms for discrete optimization problems.

Of interest to this research from the reviewed literature is the connection between SA and mathematical optimization, which was first noted by Pincus (1970), but it was the contributions made by Kirkpatrick et al. (1983) and Cerncy (1985), which have given the method its present use in solving a variety of global single objective or multiobjective optimization problems not only in mathematics, but also in other fields such as operations research, geodesy and GIS.

For the purposes of this study a description of the key elements of the SA algorithm are summarised as follows:

Step 1: Define an objective function or cost function f in the form of a mathematical expression to describe the relationship of the parameters that will optimize the location siting process. It is the main aim of the SA algorithm to find an optimum solution that either minimizes or maximizes f .

Step 2: Initialize a starting solution $x_i \in D$, where x_i is either a user provided or random generated initial spatial point solution within a GIS search domain D .

Step 3: Choose an initial control parameter. This parameter allows the algorithm to converge to the optimum solution and every algorithm has one. Since SA originates from thermodynamics, its control parameter is in the form of a temperature (T_0) value. This needs to be large enough to allow the search to traverse a large portion of the study area and overcome infeasible regions.

Step 4: Obtain a new spatial location solution within the search domain by perturbing the previous solution location by a small amount, Δx_i . The increment, Δx_i is random and can be in the form of a function. Among the many functions available, the Gauss distribution density function is one of the most suitable for defining this random movement, and is defined as follows,

$$P(x)=\frac{1}{\sqrt{2\prod\sigma}}e^{-\frac{1}{2}\frac{x^2}{\sigma^2}} \quad (1)$$

With a zero mean and standard deviation σ .

Step 5: Evaluate the objective/cost function to be optimized using the new spatial solution. If there is an improvement in the value of the objective/cost function (i.e., a decrease if the objective/cost function is to be minimized, or an increase if the objective/cost function is to be maximized), then the new spatial location is accepted. However, if the new solution has neither minimized nor maximized the objective/cost function and the algorithm becomes trapped into a local optimum, the SA algorithm is equipped with additional criteria to accept this solution (which is a worse solution) with a certain probability. Pardolas and Romeijn (2002), give a number of alternative ways to overcome this problem, which among others include using a fixed probability (such as 0.2, 0.1, 0.05 or 0.01), the Tsallis' Acceptance Rule and Boltzman's distribution.

Step 6: Decrease the temperature during the search for the optimum solution by making use of a certain function known as the cooling schedule. There are several theoretical and empirical cooling schedules suggested in the literature, but given a sufficiently high initial temperature, T_0, some successful cooling functions are as follows:

$$T(t) = \frac{T_0}{1+\log(t+1)} \qquad (2)$$

$$T(t) = \frac{T_0}{1+t} \qquad (3)$$

$$T(t) = T_0\alpha^t,\ 0<\alpha<1 \qquad (4)$$

Where, t, is the number of iterations the algorithm is allowed to perform at each temperature

According to Berne and Baselga (2003), the final temperature at which to stop the iterations can be stated as a fraction of the initial one, for example, $10^{-4}T_0$, or as the moment when the differences between two consecutive solutions in terms of Δx_i becomes negligible.

Step 7: Return to step 4 until the finish criteria are fulfilled.

2.2 Integrated EVAMIX/ Fuzzy AHP

The EVAMIX approach developed by Voogd (1982, 1983), and described in Nijkamp et al. (1990), Martel and Matarazzo (2005) and Hajkowicz and Higgins (2008) is an outranking method for evaluating alternative options using both quantitative (cardinal) and qualitative (ordinal) criteria within the same evaluation matrix and ranking the options from best to worst. The literature shows some applications of EVAMIX in the field of multi criteria analysis in material selection by Chatterjee et al. (2011), industrial environment by Darji and Rao (2013), spatial ranking of vulnerability proposed by Chung and Lee (2009), analysis of investments in construction by Ustinovichius et al. (2007) and multi criteria analysis of small-scale forestry by Jeffreys (2004). It is very clear from the above listed references that, there exist few applications of the EVAMIX method in science, ecological, industrial, financial and non-financial units, and rare applications in the related fields of multi objective optimization and GIS.

As part of a model for solving a site location problem, this study adopted EVAMIX for evaluating alternative landfill site locations using both quantitative and qualitative multi criteria and rank the sites from best to worst. From a procedural point of view, the EVAMIX method used in this present paper can be summarised in seven steps, which are discussed as follows:

Step 1: Construct an $a \times b$ evaluation matrix E, where a is the number of alternatives and b is the number of evaluation criteria (quantitative and/or qualitative). Thus given a set of evaluation criteria $j(j=1,2,\ldots,b)$ and a finite set of alternatives $i(i=1,2,\ldots,a)$, the evaluation matrix E will be characterized by its ordinal and cardinal components as follows:

$$E = \begin{pmatrix} e_{11} & e_{12} & \cdots & e_{1a} \\ e_{21} & e_{22} & \cdots & e_{2a} \\ \cdots & \cdots & \cdots & \cdots \\ e_{b1} & e_{b2} & \cdots & e_{ba} \end{pmatrix}$$

Then, distinguishing the set of evaluation criteria into two subsets, ordinal and cardinal criteria, we obtain two distinct evaluation matrices: E_o (ordinal criteria) and E_C (cardinal criteria).

Step 2: Standardize the cardinal data set to a common unit using linear normalization. Beneficial criteria are normalized using Eq. (5), whilst non-beneficial criteria are normalized using Eq. (6).

$$r_{ij} = \frac{x_{ij}-\min(x_{ij})}{\max(x_{ij})-\min(x_{ij})} \quad (i=1,2,\ldots,a; j=1,2,\ldots,b) \qquad (5)$$

$$r_{ij} = \frac{\max(x_{ij})-x_{ij}}{\max(x_{ij})-\min(x_{ij})} \quad (i=1,2,\ldots,a; j=1,2,\ldots,b) \qquad (6)$$

According to Eq. (5) and Eq. (6) in the normalized decision matrix, the maximum value will always be 1 and the minimum value equal to 0.

Step 3: Calculate the weights or measure of importance of each criterion (ordinal or cardinal) with respect to the others. This step involves making use of the fuzzy AHP as an integral part of the EVAMIX method. The study concentrates on the fuzzy AHP approach introduced by Chang (1996), in which triangular fuzzy numbers (TFNs) are preferred for the pairwise comparison scale. When using TFNs, an interval is used to define the decision maker's judgement. This is in the form of three numbers or parameters, expressed as (l, m, u), where the lowest possible parameter or value is l, the middle possible parameter or value is m and the upper possible parameter or value is u. The domain of this interval is described by a triangular membership function, which can be represented in both math and graph form using Eq. (7) and Figure 1, respectively (Chang, 1996; Kaufmann and Gupta, 1988) as follows:

$$\mu_A(x)=\begin{cases}0 & x<l\\ (x-l)/(m-l) & l\le x\le m\\ (u-x)/(u-m) & m\le x\le u\\ 0 & x>u\end{cases} \quad ......(7)$$

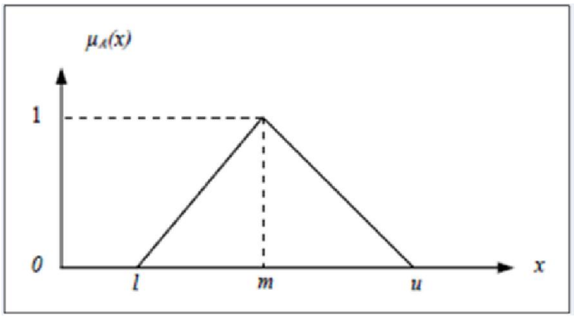

Figure 1: Fuzzy Triangular Number

In order to utilize TFNs in a fuzzy environment, their basic algebraic operations should be understood. An extensive discussion on these operations can be found in Chang (1996), Kaufmann and Gupta (1988) and Zadeh (1965). However, for the purposes of this research only three operations are illustrated. Consider that $\tilde{M}_1$ and $\tilde{M}_2$ are two TFNs were $\tilde{M}_1=(l_1,m_1,u_1)$ and $\tilde{M}_2=(l_2,m_2,u_2)$. The three basic operations used in this study are:

$$\tilde{M}_1\oplus\tilde{M}_2=(l_1+l_2,m_1+m_2,u_1+u_2) \quad ......(8)$$

$$\tilde{M}_1\otimes\tilde{M}_2=(l_1l_2,m_1m_2,u_1u_2) \quad ......(9)$$

$$\tilde{M}_1^{-1}\approx\left(\frac{1}{u_1},\frac{1}{m_1},\frac{1}{l_1}\right) \quad ......(10)$$

Using these three algebraic operations the process of calculating the weights of the evaluation criteria is as follows:

- Construct the fuzzy AHP comparison matrix. The aim of the matrix is to elucidate an order of preference given a number of evaluation criteria. Central to this is a series of pairwise comparisons, indicating the relative preferences between pairs of evaluation criteria. Since decision makers use linguistic terms or verbal judgements during pairwise comparison, a measurement scale is required to convert these pairwise comparisons into fuzzy numbers. One such scale showing the proposed TFNs, membership functions and matching linguistic variables or verbal judgements was provided by Saaty (1980) and is shown in Table 1.

By using TFNs for pairwise comparison of the evaluation criteria a fuzzy comparison matrix $\tilde{A}$ is created and is of the form:

$$\tilde{A}=(\tilde{a}_{pq})_{b\times b}=\begin{bmatrix}(1,1,1) & (l_{12},m_{12},u_{12}) & \cdots & (l_{1b},m_{1b},u_{1b})\\ (l_{21},m_{21},u_{21}) & (1,1,1) & \cdots & (l_{2b},m_{2b},u_{2b})\\ \vdots & \vdots & \vdots & \vdots\\ (l_{b1},m_{b1},u_{b1}) & (l_{b2},m_{b2},u_{b2}) & \cdots & (1,1,1)\end{bmatrix}$$

$$\text{where } \tilde{a}_{pq}=(l_{pq},m_{pq},u_{pq}) \text{ and } \tilde{a}_{pq}^{-1}=(1/u_{qp},1/m_{qp},1/l_{qp}) \text{ for } p,q=1,\ldots,b \text{ and } p\neq q$$

The number of comparisons for each matrix is $b(b-1)/2$, where b is the total number of evaluation criteria.

- Compute the normalized value of row sums (i.e. the fuzzy synthetic extent, $\tilde{S}_j$) for the fuzzy comparison matrix $\tilde{A}$ by making use of fuzzy arithmetic operations from Eq. (8-10) such that:

$$\tilde{S}_j=\sum_{k=1}^{b}\tilde{a}_{jk}\otimes\left[\sum_{j=1}^{b}\sum_{k=1}^{b}\tilde{a}_{jk}\right]^{-1} \quad ......(11)$$

To obtain, $\sum_{k=1}^{b}\tilde{a}_{jk}$, the fuzzy addition operation (Eq. (8)) is applied to the fuzzy judgement matrices, such that,

$$\sum_{k=1}^{b}\tilde{a}_{jk}=\left(\sum_{k=1}^{b}l_{jk},\sum_{k=1}^{b}m_{jk},\sum_{k=1}^{b}u_{jk}\right) \quad ......(12)$$

The second part of Eq. (11) is obtained by applying the fuzzy addition operation (Eq. (8)) to column values in the matrix obtained from Eq. (12), followed by using Eq. (10) to compute the inverse of the resulting vector such that,

$$\left[\sum_{j=1}^{b}\sum_{k=1}^{b}\tilde{a}_{jk}\right]^{-1}=\left(\frac{1}{\sum_{j=1}^{b}\sum_{k=1}^{b}u_{jk}},\frac{1}{\sum_{j=1}^{b}\sum_{k=1}^{b}m_{jk}},\frac{1}{\sum_{j=1}^{b}\sum_{k=1}^{b}l_{jk}}\right) \quad ......(13)$$

- Using the normalized TFNs of two criteria obtained from Eq. (11) determine the degree of possibility of one criteria fuzzy number's being greater than or equal to the other criteria fuzzy number's $\tilde{S}_{j1}\ge\tilde{S}_{j2}$ (see Figure 2). This can be represented by Eq. (14) and Figure 2.

Table 1: Proposed TFN, linguistic variables and membership functions

Saaty's scale of relative importance	Definition	Membership function	Domain	TFNs scale (l,m,u)	Linguistic variables
	Just equal			(1.0,1.0,1.0)	Just equal
1	Equal importance	$\mu_A(x)=(3-x)/(3-1)$	$1\le x\le 3$	(1.0,1.0,3.0)	Least importance
3	Moderate importance of one over another	$\mu_A(x)=(x-1)/(3-1)$	$1\le x\le 3$	(1.0,3.0,5.0)	Moderate importance
		$\mu_A(x)=(5-x)/(5-3)$	$3\le x\le 5$		
5	Essential or strong importance	$\mu_A(x)=(x-3)/(5-3)$	$3\le x\le 5$	(3.0,5.0,7.0)	Essential importance
		$\mu_A(x)=(7-x)/(7-5)$	$5\le x\le 7$		
7	Demonstrated importance	$\mu_A(x)=(x-5)/(7-5)$	$5\le x\le 7$	(5.0,7.0,9.0)	Demonstrate importance
		$\mu_A(x)=(9-x)/(9-7)$	$7\le x\le 9$		
9	Extreme importance	$\mu_A(x)=(x-7)/(9-7)$	$7\le x\le 9$	(7.0,9.0,9.0)	Extreme importance
Reciprocals of above non-zero numbers	If an activity has one of the above numbers (e.g., 3) compared with a second activity, then the second activity has the reciprocal value (i.e., 1/3) when compared to the first.			Reciprocals of above; $A_1^{-1}\approx(1/u_1,1/m_1,1/l_1)$	

Adapted from Saaty (1980)

$$V(\tilde{S}_{j1}\ge\tilde{S}_{j2})=\begin{cases}1 & if \quad m_{j1}\ge m_{j2}\\ \dfrac{l_{j2}-u_{j1}}{(m_{j1}-u_{j1})-(m_{j2}-l_{j2})} & if \quad l_{j2}\ge u_{j1}\\ 0 & otherwise\end{cases} \quad \text{..........(14)}$$

Where, $\tilde{S}_{j1}=(l_{j1},m_{j1},u_{j1})$ *and* $\tilde{S}_{j2}=(l_{j2},m_{j2},u_{j2})$

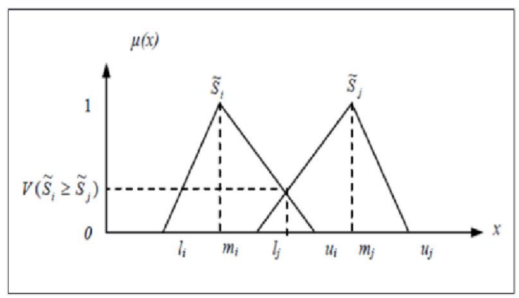

Figure 2: Degree of possibility

- Extend the basic principles from above and determine the degree of possibility of one criterion fuzzy numbers being greater than all the other $(b-1)$ criteria fuzzy numbers. This can be defined as follows,

$$V(\tilde{S}_j\ge\tilde{S}_{j1},\tilde{S}_{j2},...,S_{jb}=V[(\tilde{S}_j\ge\tilde{S}_{j1})$$
$$and\ (\tilde{S}_j\ge\tilde{S}_{j2})\ and\ ...$$
$$and\ (\tilde{S}_j\ge\tilde{S}_{jb})]\text{..................(15)}$$

By taking the minimum values in the degree of possibility sets created from Eq. (15), it is possible to determine a weight vector, W, as

$$w=(d^*(A_1),d^*(A_2),...,d^*(A_b))^T,$$
$$where\quad d^*(A_b)=\min V(\tilde{S}_j\ge\tilde{S}_{jb}\mid j=1,...,b,).$$
$$\text{.....................(16)}$$

The normalized weight vectors for each fuzzy comparison matrix, $\tilde{A}$, at each level of the hierarchy is then determined by normalizing the weight vector, w. This is known as de-fuzzification and involves dividing each value in the weight vector, w, by their total sum as follows,

$$W_i = \frac{V(\tilde{S}_b \geq \tilde{S}_{jb} | j = 1,...b)}{\sum_{k=1}^{b} V(\tilde{S}_k \geq \tilde{S}_j | j = 1,...b, j \neq k)}, i = 1,..b........(17)$$

- Determine whether the process of assigning weights was consistent by calculating the Fuzzy Consistency Ratio (FCR). For this research the algorithm used to calculate the FCR is that proposed by Modarres et al. (2010), which is based on the preference ration concept. The algorithm is as follows:
  - Define a fuzzy matrix $\tilde{h}$, such that, $\tilde{h}_{jk} = w_j * \tilde{a}_{jk}$ ....(18), where w_j is the weight for the j^{th} criterion for $j = 1,2,...,b$ and $\tilde{a}_{jk}$ are the TFN's in the fuzzy judgement matrix.
  - Sum the values of each j^{th} row of the matrix $\tilde{h}$, that is, $\tilde{s}_j = \sum_{j=1}^{b} \sum_{k=1}^{b} \tilde{h}_{jk}$ ....(19)
  - For $j = 1 \text{ to } b$ calculate $\tilde{\lambda}_j$ such that $\tilde{\lambda}_j = \frac{\tilde{s}_j}{w_j}$ ....(20)
  - Calculate the Consistency Index (CI) as follows: $CI = \frac{\frac{1}{b}\sum(\tilde{\lambda}_j - b)}{b-1}$ ....(21). The smaller the CI, the smaller the deviation from the consistency.
  - Obtain the random index (RI) for the number of criteria from Table 2.

Table 2: Random Indices

Order of Matrix	Random Index
2	0
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45
10	1.51

- Calculate the FCR using, $FCR = \frac{CI}{RI}$ ....(22). However, since TFNs were used to represent uncertainty in the judgment matrix, the FCR values obtained from Eq. (22) will be in the form of a set with 3 values. As such, determine the FCR as a preference ratio, such that it is defined as the percentage of the i^{th} fuzzy number within a set being the most preferred. This ratio according to Modarres and Sadi-Nezhad (2001) can be expressed as $R(i) = \frac{|\Omega_i|}{|\Omega|}$ ....(23), where Ω_i and Ω are values in FCR set obtained from Eq. (22). Usually a CR of 0.1 or less is considered as acceptable, and it reflects an informed judgement attributable to the knowledge of the analysts regarding the problem under study.

Step 4: Use the criteria and their associated weights (W_j) to compare the alternative options by computing the dominance scores of each alternative option pair, (i,i'). For all ordinal criteria use Eq. (24), otherwise use Eq. (25) for all cardinal criteria.

$$\alpha_{ii'} = \left[\sum_{j \in O} \{W_j \operatorname{sgn}(r_{ij} - r_{i'j})\}^c\right]^{\frac{1}{c}}, c = 1,3,5..........................(24)$$

$$\gamma_{ii'} = \left[\sum_{j \in C} \{W_j \operatorname{sgn}(r_{ij} - r_{i'j})\}^c\right]^{\frac{1}{C}} ....(25)$$

Where $\operatorname{sgn}(r_{ij} - r_{i'j}) = \begin{cases} +1 & if\ r_{ij} > r_{i'j} \\ 0 & if\ r_{ij} = r_{i'j}. \\ -1 & if\ r_{ij} < r_{i'j} \end{cases}$

and C is a subset of criteria with a cardinal measurement scale.

Step 5: Calculate standardized dominance scores for all ordinal and cardinal criteria. The dominance scores $\alpha_{ii'}$ and $\gamma_{ii'}$ are standardized into the same measurement unit in order to make them comparable. If Z is a standardization function, then the standardized dominance measures for all ordinal $(\delta_{ii'})$ and cardinal $(d_{ii'})$ criteria have the following expressions:

$\delta_{ii'} = Z(\alpha_{ii'})$ *and* $d_{ii'} = Z(\gamma_{ii'})$

The standardized dominance scores can be obtained using three different approaches, i.e., (a) subtractive summation, (b) subtracted shifted interval, and (c) additive interval technique. For this study, the additive interval technique will be used and is represented as follows:

$$(\delta_{ii'}) = \left(\frac{\alpha_{ii'} - \alpha^-}{\alpha^+ - \alpha^-} \right) \qquad (26)$$

Where $\alpha^+(\alpha^-)$ is the highest (lowest) ordinal dominance score for the alternative pair, (i,i').

$$(d_{ii'}) = \left(\frac{\gamma_{ii'} - \gamma^-}{\gamma^+ - \gamma^-} \right) \qquad (27)$$

Where $\gamma^+(\gamma^-)$ is the highest (lowest) cardinal dominance score for the alternative pair, (i,i').

Step 6: Assuming that weights W_j have quantitative properties, the overall dominance measure $D_{ii'}$ for each pair of alternatives landfill sites (i,i') is:

$$D_{ii'} = w_O \delta_{ii'} + w_C d_{ii'} \qquad (28)$$

Where w_O is the sum of the weights for the ordinal criteria $(w_O = \sum_{j \in O} W_j)$ and w_C is the sum of the weights for the cardinal criteria $(w_C = \sum_{j \in C} W_j)$. This overall dominance score reflects the degree to which option a_i dominates option $a_{i'}$ for the given set of criteria and weights.

Step 7: For each option, its final appraisal score is given by,

$$S_i = \left[\sum_{i'}^{a} \frac{D_{i'i}}{D_{ii'}} \right]^{-1} \qquad (29)$$

Where $D_{ii'} + D_{i'i} = 1$. The higher the appraisal score the better is the performance of the option. Thus, the best option is the one with the highest appraisal score.

3.0 Empirical study of landfill site selection

3.1 Study Area

The study is applied in Blantyre City, which is the second largest urban center in Malawi and is located 311km south east on the national capital, Lilongwe (Figure 3).

Situated at an altitude of approximately 1,150 meters above sea level, Blantyre City covers an area of over 220 square kilometers. According to the National Statistical Office the population of Blantyre City was estimated at 661,256 in 2008, with an average population density of 3,006 people per square kilometer. This accounted for a 5.1 per cent share on the national population. However, the population is projected to rise to 1,068,681 by 2015.

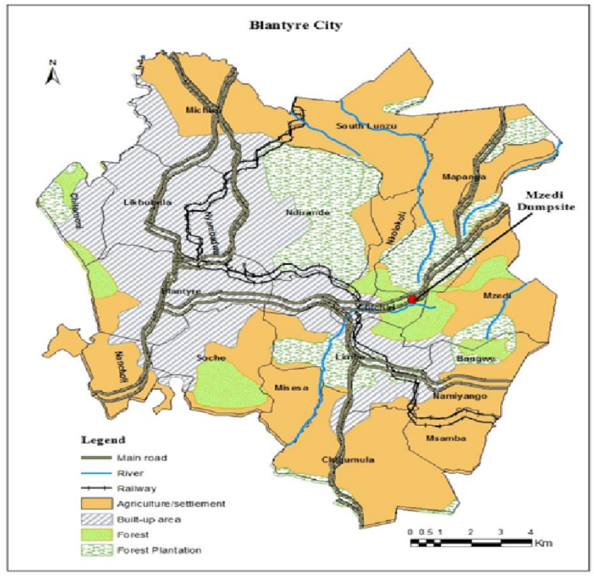

Figure 3: Study Area

3.2 State of solid waste management in Blantyre City

According to Matope (2000), it is estimated that more than a decade ago the city of Blantyre generated waste totalling 0.37 kg per capita per day, the equivalent of 192 tons a day, three-quarters of which was classed as domestic waste. By the year 2006, 540 metric tons were being generated in the city as solid waste. More recently, the average amount of waste being produced is 0.9 kg per capita per day, which amounts to 647 metric tons per day – 81 per cent of which is organic and biodegradable (Matope, 2000; Berman, 2011). A breakdown of the waste according to the city's different economic backgrounds is shown in Table 3. According to UN-HABITAT (2011), only a third of the solid waste produced is collected and disposed properly.

Table 3: Breakdown of waste by economic background

Component of the waste stream	Waste Fraction (% by mass)		
	High Income	Middle Income	Low Income
Paper	30	20	7
Plastics	30	20	8
Glass	18	10	2
Metals	5	3	2
Textiles	4	3	1
Garden waste	1	1	0
Builder's rubble	1.5	1	0
Ash	0.5	20	25
Putrescibles	10	28	55

Blantyre City Council (BCC) Health and Engineering Department, which is responsible for solid waste management as mandated by the Local Government Act of 1980 contributes to this escalating problem by offering selective waste collection services. Waste collection services are not consistently made available to all areas, with high-income areas getting more services than low-income areas. This is a common practice by local authorities in developing countries to allocate resources to areas with higher tax yields. In high-income areas collection of waste is carried out six days a week and in the city's busier markets it is done every day. There are also a number of private entities which provide waste collection services within the city. These include private trash collectors who focus on commercial or industrial companies. In addition, the privately owned Malawi Housing Corporation focuses on collecting waste from high-income residential areas. In other residential areas and market places, the BCC has skips in place, but collection is irregular. In peri-urban areas, informal settlements and Traditional Housing Areas (THAs), regular solid waste collection is non-existent, partly due to the shortage of access roads. Consequently, household solid waste is often indiscriminately discarded or dumped in the streets, pits, or drains.

The situation is compounded by the frequent fuel shortages and breaking down or shortage of equipment. According the BCC, the city owns 100 skips, of which 58 are hired out to private companies while the rest are placed in markets and unplanned areas. This leaves 42 skips to service the city markets, THAs and informal sectors. As a result, only 125 tonnes of waste is collected from these areas meaning a further 475 tonnes of waste go uncollected by the council. In addition, poor maintenance and misuse such as setting garbage loaded skips on fire are contributing to their deteriorated status. With an average working life of 3 years for a skip, poor replacement ratio is compounding this situation. Furthermore, the 14 refuse trucks and 3 tractors used to collect these skips frequently break down due to poor maintenance, which is not helped by the fact that the majority of them are overused and have not been replaced in the past decade, thus have succumbed to natural wear and tear.

As of 1993, the official disposal site for solid waste from Blantyre City is located on the slopes of Mzedi Mountain along the eastern boundary of the city (UN-HABITAT, 2011). The site is more of an open dump than a proper landfill. The site is not fenced or walled and lacks a leachate or gas management system, which are basic requirements of a landfill. On a monthly basis it receives roughly 3,000 tonnes of waste. The majority of the waste is general waste from households, industry, hospitals and educational institutions, however, there are sightings of restricted materials such as hazardous (batteries, paint etc.) and medical waste being dumped at the site illegally. To create room waste is piled up in heaps then eventually spread level then covered in earth before being compacted at the site. This is done twice a year by a private company hired by the BCC. The process is neither sustainable nor cost-effective as the BCC spends an estimated MK 10 million (US35, 335) just to bulldoze the Mzedi dump site each year. As no impervious soil is placed on the ground before compaction, leachate from the waste is more likely to percolate into the ground and eventually contaminate the city's groundwater systems. The BCC also often resorts to burning the waste to curb the nuisance produced by flying litter and to create room for more waste. At least 150 scavengers (mostly women and children) who visit the site on a daily basis run the risk of contracting respiratory diseases as they inhale the smoke or are exposed continuously to the strong foul odours emanating from the decomposing waste. The dumpsite also creates ideal conditions for vectors such as mosquitos to thrive. During the rainy season, nearby streams transport the malaria vectors in the city's water system thus an upsurge of Malaria.

In light of the above, the Mzedi dump site has outlived its lifespan and is dilapidated. It is also a long distance from the city center, influencing the cost involved in collecting and disposing of waste. The BCC has determined that a second site for building a proper landfill is needed and should be located closer to the city. However, they do not expect it to be operational before 2015. In addition, the continuous increase in population and subsequent waste generation within Blantyre City calls for an urgent determination of the best location for a new landfill.

3.3 Implementation and Results

The combined methodology of the study in shown in Figure 4. After determining the problem area, landfill siting was treated as a combinatorial optimization problem. As such, simulated annealing was used to identify optimum sites for locating a landfill. For ranking and selecting the alternative landfill sites an integrated EVAMIX/ Fuzzy AHP approach was employed. The study was implemented under the setting of fuzzy set theory to accommodate the inherent uncertainty or ambiguity associated with decision makers when faced with complex multi-attribute decision making problems such as landfill siting.

3.3.1 Optimizing landfill siting

In this study, the process of finding alternative sites for locating a landfill in Blantyre City was treated as a combinatorial optimization model with all the main elements of a complex problem, that is, multiple objectives, multi-facilities, and non-linearity. The optimization problem was approached as a continuous model and the alternative landfill sites were considered to be entering an existing network. Formally, the problem was formulated by assuming that $D \subseteq R^2$ is the region of interest (i.e., the study area) and $x_1, x_2, ..., x_n \in D$, where $x_1, x_2, ..., x_n$ are the spatial locations of n landfill sites. As discussed in step 1 of section 2.1, an objective function or cost function in the form of a mathematical expression is required to describe the relationship of the parameters that will optimize the landfill siting process. Within the context of this research, the objective functions were modelled by assuming that the aim was to maximize the distance between, (i) new and the existing Mzedi disposal site, and (ii) new landfill sites only. In that respect, two objective functions represented by Eq. (30) and Eq. (31), were defined. As a distance function the Euclidean distance was used as comparisons by Love et al. (1998) and Apparicio et al. (2008) have shown that it is close to reality than the Manhattan or other distance functions.

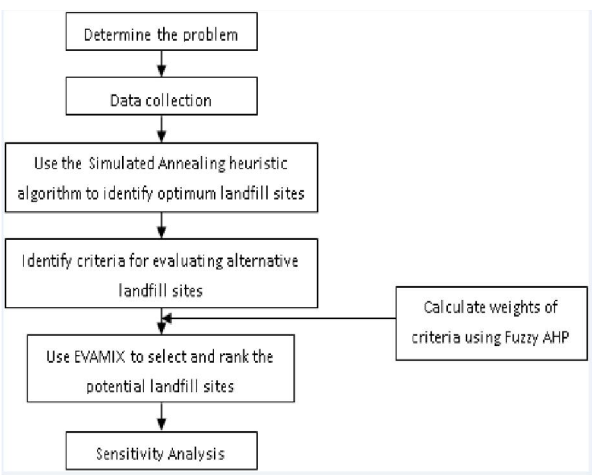

Figure 4: The Proposed Landfill Siting Model

$$\max \int_S \left\| x_i - x_j \right\|^2 dx \qquad (30)$$

$$\max \sum_{g=1}^{m} \sum_{i=1}^{n} \int_S \left\| x_i - x_g \right\|^2 dx \qquad (31)$$

Where, m is the number of existing landfill sites (in this study its one, the Mzedi landfill), n is the number of new landfill sites, $\left\| x_i - x_j \right\|$, and $\left\| x_i - x_g \right\|$ are the Euclidean distances between; (1) any two new landfills x_i and x_j, and (2) a new landfill x_i and an existing landfill x_g.

Having defined the objective functions a pseudo-computer code of the simulated annealing algorithm used for this research was developed as shown in Figure 5. The approach was implemented by tight coupling, with the SA algorithm for continuous non-linear optimization being executed within a Geographical Information System (GIS) environment where with the usage of the associated spatial information, the objective functions were evaluated. Python was used as the scripting language in combination with ArcPy, a new site package in ArcGIS 10 that builds on (and is a successor) to the successful arcgisscripting module. Using ArcPy it was possible to access and work with numerous Python modules from different sources but not necessarily part of ArcGIS 10.

```
1.  Function Simulated_Annealing
2.  set starting landfill location, x ∈ D;
3.  set starting/initial temperature (T_0 > 0);
4.  set N_T (number of trials per temperature);
5.  set t = 0, the temperature change counter;
6.  while stopping condition is not satisfied do
7.      For b ← 1 to N_T do
8.          generate a random trial landfill location x', within the neighbourhood, N(x);
9.          Calculate δ = x' − x;
10.         #Maximization
11.         if δ > 0 then
12.            x := x';
13.         else if random(0,1) < Tsallis' acceptance probability then
14.            x := x';
15.     t := t + 1;
16.     T ← α × T;
17. end while
18. end Function Simulated_Annealing
```

Figure 5: Simulated Annealing Algorithm

Line 2 of the pseudo-computer code initializes a starting point $x \in D$, where x represents the x, y coordinates of a landfill site within the search space or domain D, which is the study area. In this case it was user provided as the centroid of Blantyre City. Line 3 chooses a high enough temperature (T_0), such that almost any trial landfill location x' is accepted. Here, an initial temperature of 100,000 was considered to be high enough for the SA algorithm to converge to the optimal location of a landfill site. At each temperature the SA algorithm in executed N_T times (see Line 4) using Eq. (32) as follows:

$$N_T = \varsigma(10v + m) \quad and \quad \varsigma = 10(v + m)......................(32)$$

Where, v, is the number of variables and m is the number of constraints. Eq. (32) is a by-product of the heuristic rule (Corana et al. (1987). It should be noted that for each landfill site, the problem variables are two, one for each coordinate. Line 6 terminates the algorithm if the current temperature, T, is small enough (i.e., less or equal than 0.0001). Line 8 generates a random trial landfill site, x', within the neighbourhood, $\mathrm{N}(x)$, from the current location x in search space D. The neighbour landfill sites were generated through the use of polar coordinates that guarantee the necessary randomness of the selection process for the new locations considered. Thus, if Δx represents a shift in the location of a site, then

$$\Delta x_1 = \rho \times \cos\theta_1 \qquad \theta_1 \in [0, 2\pi]$$

$$\Delta x_2 = \rho \times \sin\theta_1 \times \cos\theta_2 \qquad \theta_2 \in [0, \pi]$$

$$\ldots$$

$$\Delta x_{n-1} = \rho \times \sin\theta_1 \times \ldots \times \sin\theta_{n-2} \times \cos\theta_{n-1} \qquad \theta_{n-1} \in [0, \pi]$$

$$\Delta x_n = \rho \times \sin\theta_1 \times \ldots \times \sin\theta_{n-2} \times \sin\theta_{n-1} \times \sin\theta_n \qquad \theta_n \in [0, \pi]$$

Where

$$\rho = \begin{cases} 1 & if \quad t < 100 \\ 2 & if \quad 100 \le t < 150 \\ 3 & if \quad 150 \le t < 250 \\ 5 & if \quad 250 \le t < 350 \\ 15 & if \quad t \ge 350 \end{cases}$$

t is the number of iterations

The new landfill site, in the neighbourhood of the previous location is then given by:

$$x' = x + \Delta x$$

$$(x'_1, \ldots, x'_n) = (x_1, \ldots, x_n) + (\Delta x_1, \ldots, \Delta x_n) \qquad (33)$$

Line 9 calculates the difference between the new and previous cost/objective functions. In this case it's the Euclidean distances between, (1) new and already existing Mzedi landfill site and (2) new landfill sites. These are represented by variable δ. Line 11 checks if the new location has maximized or minimized the objective functions shown in Eq. (30) and Eq. (31). This location is now accepted as the new site for the landfill. If the new location has neither minimized nor maximized the objective functions (failure to move to a smaller energy state) and the algorithm becomes trapped into a local optimum, Lines 13 and 14 accepts this location (which gives a worse solution) with a certain probability, which in this study is given by the Tsallis' Acceptance Rule as in Eq. (34). This helps the SA algorithm to escape from local minima and be able to provide a global optimum solution.

$$Tsallis'\,Acceptance\,Rule = \begin{cases} 1 & if \quad \delta \le 0 \\ [1-(1-q)\dfrac{\delta}{A_\mu T}]^{\frac{1}{(1-q)}} & if \quad \delta > 0 \; and \; (1-q)\dfrac{\delta}{A_\mu T} < 1 \\ 0 & if \quad \delta > 0 \; and \; (1-q)\dfrac{\delta}{A_\mu T} \ge 1 \end{cases} \qquad (34)$$

q is known as the Tsallis' parameter and starts from 2 and decreases exponentially to 1 as T decreases.

Finally, Line 16 reduces T after looping N_T times of generating x' and accepting them with probability given by the Tsallis' rule. T was reduced using a geometric cooling schedule, $T = \alpha \times T$, where α is a constant known as the cooling rate and is smaller than 1 (typically between 0.5 and 0.99). At high T, any trial landfill site was accepted with high probabilities, allowing the search to traverse a large portion of the study area and overcome infeasible regions. As T was gradually reduced, its acceptance probability decreased, and at very low temperatures the algorithm behaved like a local search.

At the end of the SA algorithm one candidate landfill site was generated. However, in order to give decision makers more candidate sites to choose from, the SA algorithm was run again six more times thus generating six more alternative sites. During each run, a random spatial location within the study area was used as the starting point for the algorithm. As a result, the SA algorithm provided a good set of seven (7) alternative candidate landfill sites (denoted as L_1, L_2, L_3, L_4, L_5, L_6 and L_7 respectively) for further evaluation based on criteria that had not been considered early on in the research and are difficult to account for by a numerical scheme. Figure 6 shows the location of the alternative sites in relation to the transport network, water sources, land cover type and waste generation centers as well as the existing Mzedi disposal site. Because a high temperature of 100,000 degrees Celsius was used to initialize the search area, it allowed the SA algorithm to traverse a large portion of Blantyre City and overcome infeasible regions hence the location of these sites is such that they are fairly distributed around the study area. The sites are also located such that the Euclidean distances between them maximizes the objective functions set out in Eq. (30) and Eq. (31).

3.3.2 Selecting and ranking landfill sites

For identifying the selection criteria to rank and select the seven candidate sites, a number of qualitative and quantitative factors affecting the location evaluation procedure were considered from a long list of criteria discussed in Zeiss and Lefsrud (1995), Wang et al. (2009), Siddiqui et al. (1996), Sener et al. (2006) and Kontos et al. (2003). The following seven criteria were adopted: land cover type, slope, elevation, soil, distance from transport network, distance from waste generation centers and distance from water sources. Table 4 is a summary of the essential characteristics of each alternative landfill site with respect to the chosen criteria. Among the criteria, two are qualitative (ordinal) in nature and five are treated as quantitative (cardinal).

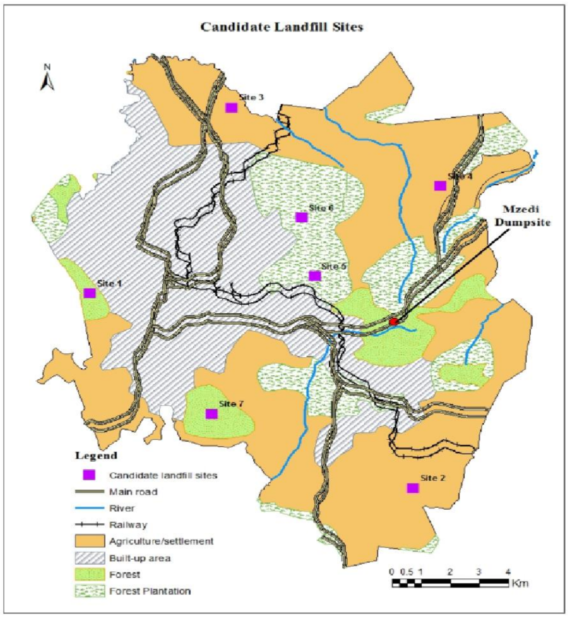

Figure 6: Candidate Landfill Sites

Using the EVAMIX method, the original decision matrix, as shown in Table 4, was separated into ordinal (Landcover and Soil) and cardinal criteria. The decision matrix was then normalized using Eq. (5) and Eq. (6) respectively for beneficial and non-beneficial attributes. This normalized decision matrix is shown in Table 5. Distances between landfill sites and waste generation areas (residential, industrial and commercial areas) need to be maximized such that aesthetics and odour do not become a public concern. At the same time, locating landfills further away from any water body not only reduces their pollution, it also protects the aquatic life in these water bodies and reduces the likely human health impacts of leachate from landfills. Thus for this study, distance from waste generation sources and distance from water sources criteria were treated as beneficial criteria. On the other hand, steeper slopes and high ground mean higher excavation costs. Furthermore, sites located on higher slopes or elevated land increases the chances of runoff of pollutants from the landfill, and as such contaminates the wider environment (Gemitzi et al., 2007; Lin and Kao, 1999). Moreover, landfills should be located as close as possible to transport networks to allow accessibility to the site and minimize transport costs. Thus, slope, elevation and distance from transport networks criteria were treated as non-beneficial criteria were lower values are more desirable.

In order to determine the measure of importance of each criterion (ordinal or cardinal) with respect to the others, weights were calculated using fuzzy AHP. Fuzzy set theory was utilized to accommodate the inherent uncertainty or ambiguity associated with decision makers when faced with complex multi-attribute decision making problems. First, the criteria were divided into four main groups; environmental, geomorphological, economic and hydrological, to form the data hierarchy shown in Figure 7.

Using fuzzy AHP, criteria shown in Figure 7 were compared via pairwise comparison. The resulting matrices shown in Tables 6a-f use TFNs to tackle ambiguities involved in comparison judgements using linguistic variables as set out in Table 1. For determining weights of criteria in these fuzzy comparison matrices, the Fuzzy Extent Analysis (FEA) proposed by Chang (1996) and discussed in step 3 of section 2.2 was applied. First, the fuzzy synthetic extent values were obtained from Eq. (11) with the help of Eq. (12) and Eq. (13). Eq. (15), which extends the basic principles of Eq. (14), was then used to express the degree of synthetic extent values. Using Eq. (16), a weight vector was obtained. This weight vector was then normalized using Eq. (17) to obtain priority weight vectors of criteria as presented in the last column of the matrices.

Table 4: Quantitative and Qualitative data for landfill siting

Landfill site	Criteria						
	Landcover type	Slope (%)	Elevation (m)	Soil type	Distance from transport network (m)	Distance from waste generation centers (m)	Distance from water sources (m)
L_1	Forest (3)	9	1011	Eutric Ca (2)	264	295	8258
L_2	Agriculture/Settlement (1)	1	1110	Chromic L (1)	740	2360	4333
L_3	Agriculture/Settlement (1)	1	902	Eutric Ca (2)	723	1474	1587
L_4	Agriculture/Settlement (1)	3	1100	Chromic L (1)	477	5393	1153
L_5	Forest Plantation (2)	6	1117	Eutric Ca (2)	482	430	2488
L_6	Forest Plantation (2)	19	1200	Eutric Ca (2)	208	669	2551
L_7	Forest (3)	42	1402	Leptosols (3)	748	930	3169

Table 5: Normalized decision matrix

Landfill Site	Criteria						
	Landcover Type	Soil Type	Slope (%)	Elevation (m)	Distance from transport network (m)	Distance from waste generation centers (m)	Distance from water sources (m)
L_1	1.0000	0.3333	0.8049	0.7820	0.8963	0.0000	1.0000
L_2	0.0000	0.0000	1.0000	0.5840	0.0148	0.4051	0.4476
L_3	0.0000	0.3333	1.0000	1.0000	0.0463	0.2313	0.0611
L_4	0.0000	0.0000	0.9512	0.6040	0.5019	1.0000	0.0000
L_5	0.5000	0.3333	0.8780	0.5700	0.4926	0.0265	0.1879
L_6	0.5000	0.3333	0.5610	0.4040	1.0000	0.0734	0.1968
L_7	1.0000	1.0000	0.0000	0.0000	0.0000	0.1246	0.2837

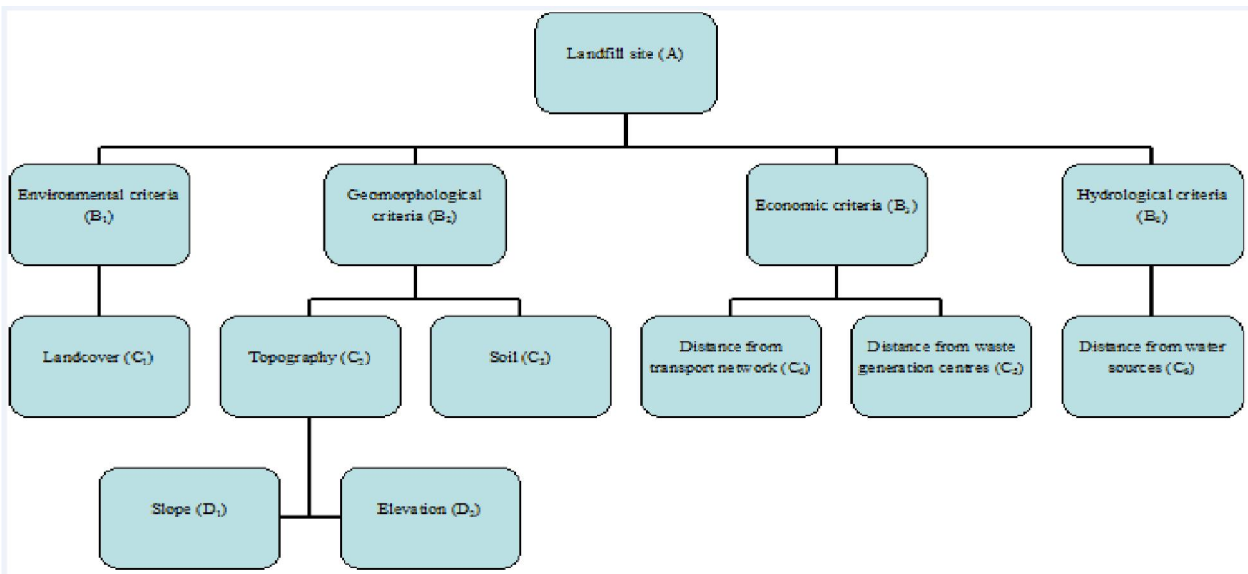

Figure 7: Criterion hierarchy for landfill siting

Table 6a: The pairwise comparison matrix A-B_{1-4}

A	**B_1**	**B_2**	**B_3**	**B_4**	**W**
B_1	(1,1,1)	(5.0,7.0,9.0)	(1.0,3.0,5.0)	(3.0,5.0,7.0)	**0.4197**
B_2	(1/9,1/7,1/5)	(1,1,1)	(1/7,1/5,1/3)	(1/5,1/3,1)	**0.1257**
B_3	(1/5,1/3,1)	(3.0,5.0,7.0)	(1,1,1)	(5.0,7.0,9.0)	**0.3716**
B_4	(1/7,1/5,1/3)	(1.0,3.0,5.0)	(1/9,1/7,1/5)	(1,1,1)	**0.0830**

FCR =0.014, A = Landfill site suitability, B_1 = Environmental Criteria, B_2 = Geomorphological criteria, B_3 = Economic criteria, B_4 = Hydrological criteria, W is the weight of B_1, B_2, B_3 and B_4 to A.
$V(S_{B1} \geq S_{B2}, S_{B3}, S_{B4}) = 1$; $V(S_{B2} \geq S_{B1}, S_{B3}, S_{B4}) = 0.2995$; $V(S_{B3} \geq S_{B1}, S_{B2}, S_{B4}) = 0.8852$; $V(S_{B4} \geq S_{B1}, S_{B2}, S_{B3}) = 0.1976$

Table 6b: The pairwise comparison matrix B_1-C_1

B_1	**C_1**	**W**
C_1	(1,1,1)	**1**

FCR =0.000, B_1 = Environmental Criteria, C_1 = Landcover type, W is the weight of C_1 to B_1.

Table 6c: The pairwise comparison matrix B_2-C_{2-3}

B_2	**C_2**	**C_3**	**W**
C_2	(1,1,1)	(1/7,1/5,1/3)	**0.2541**
C_3	(3.0,5.0,7.0)	(1,1,1)	**0.7459**

FCR = 0.012, B_2 =Geomorphological Criteria, C_2 =Topography, C_3 = Soil, W is the weight of C_2 and C_3 to B_2.

$V(S_{C2} \geq S_{C3}) = 0.3407$; $V(S_{C3} \geq S_{C2}) = 1$

Table 6d: The pairwise comparison matrix B_3-C_{4-5}

B_3	**C_4**	**C_5**	**W**
C_4	(1,1,1)	(5.0,7.0,9.0)	**0.5107**
C_5	(1/9,1/7,1/5)	(1,1,1)	**0.4893**

FCR =0.016, B_3 =Economic Criteria, C_4 =Distance from transport network, C_5 = Distance from waste generation centers, W is the weight of C_4 and C_5 to B_3.
$V(S_{C4} \geq S_{C5}) = 1$; $V(S_{C5} \geq S_{C4}) = 0.9581$

Table 6e: The pairwise comparison matrix B_4-C_6

B_4	C_6	W
C_6	(1,1,1)	1

FCR =0.000, B4= Hydrological Criteria, C_6= Distance from water sources, W is the weight of C_6 to B_4.

Table 6f: The pairwise comparison matrix C_2-D_{1-2}

C_2	D_1	D_2	W
D_1	(1,1,1)	(3.0,5.0,7.0)	**0.7459**
D_2	(1/7,1/5,1/3)	(1,1,1)	**0.2541**

FCR =0.000, C2 =Topography, D_1 =Slope, D_2 = Elevation, W is the weight of D_1 and D_2 to C_2.
$V(S_{D1} \geq S_{D2}) = 1$; $V(S_{D2} \geq S_{D1}) = 0.3407$

To determine whether consistency was maintained in assigning the weights, a Fuzzy Consistency Ratio (FCR) was calculated for each matrix using the algorithm outlined in step 3 of section 2.2, which is based on the preference ratio concept. Since the FCRs obtained for each matrix were less than 0.1, the weights were deemed acceptable. Because the criteria are in the form of a hierarchy, their final weight is dependent on that of other criteria either in the lower or upper level of the hierarchy. Hence, the final weight of a criterion in the lower level of the hierarchy was obtained by multiplying its weight by those of elements in the upper level as long as they are directly related in the hierarchical structure. For example, the final weight of the elevation criterion (represented by D_2 in the hierarchy) was obtained as follows:

Final weight of D_2 = Weight of D_2 to C_2 * Weight of C_2 to B_2 * Weight of B2 to Objective A

This was done for all criteria and the results are shown in Table 7. The sum of the final weights is 1, a requirement which must be fulfilled during the process of assigning weights.

Table 7: Final weights of criteria

Goal A	Hierarchy B	Hierarchy C	Hierarchy D	W_f
A	B_1	C_1		0.4197
	B_2	C_2	D_1	0.0238
	B_2	C_2	D_2	0.0081
	B_2	C_3		0.0938
	B_3	C_4		0.1898
	B_3	C_5		0.1818
	B_4	C_6		0.0830

From the normalized decision matrix (Table 5), the evaluative differences of i^{th} alternative for each ordinal and cardinal criteria with respect to all the other alternatives were calculated. Using the weight of each criterion obtained in Table 7 and the evaluative differences, dominance scores for each pair of landfill site alternatives for all the ordinal and cardinal criteria were estimated by applying Eq. (24) and Eq. (25) respectively, and are given in Table 8. While calculating the dominance scores for all pairs of landfill site alternatives, the value of c (see Eq. (24) and Eq. (25)) was taken as 1. Table 8 also exhibits the standardized dominance scores computed by employing Eq. (26) and Eq. (27) respectively for ordinal and cardinal criteria.

Table 8: Dominance and standardized dominance scores of each alternative landfill site pair

Site pair	$\alpha_{ii'}$	$\gamma_{ii'}$	$\delta_{ii'}$	$d_{ii'}$	Site pair	$\alpha_{ii'}$	$\gamma_{ii'}$	$\delta_{ii'}$	$d_{ii'}$
L_1,L_2	0.5135	0.0752	1.0000	0.5773	L_4,L_5	-0.5135	0.3206	0.0000	0.8295
L_1,L_3	0.4197	0.0590	0.9087	0.5606	L_4,L_6	-0.5135	-0.0590	0.0000	0.4394
L_1,L_4	0.5135	0.0752	1.0000	0.5773	L_4,L_7	-0.3260	0.3206	0.1826	0.8295
L_1,L_5	0.4197	0.0752	0.9087	0.5773	L_5,L_1	-0.4197	-0.0752	0.0913	0.4227
L_1,L_6	0.4197	-0.2567	0.9087	0.2362	L_5,L_2	0.5135	-0.1070	1.0000	0.3901
L_1,L_7	-0.0938	0.1229	0.4087	0.6263	L_5,L_3	0.4197	0.0590	0.9087	0.5606
L_2,L_1	-0.5135	-0.0752	0.0000	0.4227	L_5,L_4	0.5135	-0.3206	1.0000	0.1705
L_2,L_3	-0.0938	0.0669	0.4087	0.5687	L_5,L_6	0.0000	-0.4226	0.5000	0.0657
L_2,L_4	0.0000	-0.2729	0.5000	0.2195	L_5,L_7	-0.5135	-0.0431	0.0000	0.4557
L_2,L_5	-0.5135	0.1070	0.0000	0.6099	L_6,L_1	-0.4197	0.2567	0.0913	0.7638
L_2,L_6	-0.5135	0.1070	0.0000	0.6099	L_6,L_2	0.5135	-0.1070	1.0000	0.3901
L_2,L_7	-0.5135	0.4865	0.0000	1.0000	L_6,L_3	0.4197	0.0590	0.9087	0.5606
L_3,L_1	-0.4197	-0.0590	0.0913	0.4394	L_6,L_4	0.5135	0.0590	1.0000	0.5606
L_3,L_2	0.0938	-0.0669	0.5913	0.4313	L_6,L_5	0.0000	0.4226	0.5000	0.9343
L_3,L_4	0.0938	-0.2567	0.5913	0.2362	L_6,L_7	-0.5135	-0.0431	0.0000	0.4557
L_3,L_5	-0.4197	-0.0590	0.0913	0.4394	L_7,L_1	0.0938	-0.1229	0.5913	0.3737
L_3,L_6	-0.4197	-0.0590	0.0913	0.4394	L_7,L_2	0.5135	-0.4865	1.0000	0.0000
L_3,L_7	-0.5135	0.3206	0.0000	0.8295	L_7,L_3	0.5135	-0.3206	1.0000	0.1705
L_4,L_1	-0.5135	-0.0752	0.0000	0.4227	L_7,L_4	0.3260	-0.3206	0.8174	0.1705
L_4,L_2	0.0000	0.2729	0.5000	0.7805	L_7,L_5	0.5135	0.0431	1.0000	0.5443
L_4,L_3	-0.0938	0.2567	0.4087	0.7638	L_7,L_6	0.5135	0.0431	1.0000	0.5443

The overall dominance scores for each alternative site pair were calculated using Eq. (28), and they reflect the degree to which alternative a_i dominates alternative $a_{i'}$ for the given set of criteria and weights. These overall dominance scores for all the site pairs are given in Table 9. Finally, using Eq. (29), the appraisal score for each landfill site alternative was calculated, as shown in Table 10. Based on the descending values of the scores, the best alternative site for locating a landfill in Blantyre City was L_1.

3.3.3 Sensitivity Analysis

In all site selection processes, it is necessary to assess the reliability of the technique used in identifying the best candidate site. Tayyebi et al. (2010, Onut et al. (2010) and Onut and Doner (2008) suggest that a small perturbation in the decision weights may have a significant impact on the choice of the final candidate site. The disadvantage with these techniques is that only a few different configurations of weights can be permutated since the total is governed by the number of criteria, which in most research is rarely more than 10, and the author deems this as not enough. As a result, this study adopted Stochastic Multicriteria Acceptability Analysis (SMAA) methods to carry out sensitivity analysis and determine the probability of changes in the ranking of the final seven landfill sites. SMAA methods use at least 10,000 simulations, which are enough to get a uniform distribution on the weight space, and gives sufficient accuracy to the final rankings of alternative sites. SMAA methods have been used successfully in siting problems, see e.g. Hokkanen et al. (1999) and Lahdelma et al. (2002). For a full description of the SMAA methods, see Tervonen and Figueira (2008), and for the actual algorithms, Tervonen and Lahdelma (2007). These problems have included environmental and/or socio-economic criteria that are also present in this study.

Table 9: Overall dominance scores

Site pair	$D_{ii'}$	Site pair	$D_{ii'}$	Site pair	$D_{ii'}$	Site pair	$D_{ii'}$	Site pair	$D_{ii'}$
L_1,L_2	0.7944	L_2,L_5	0.2967	L_4,L_1	0.2056	L_5,L_4	0.5965	L_7,L_1	0.4855
L_1,L_3	0.7393	L_2,L_6	0.2967	L_4,L_2	0.6365	L_5,L_6	0.2887	L_7,L_2	0.5135
L_1,L_4	0.7944	L_2,L_7	0.4865	L_4,L_3	0.5815	L_5,L_7	0.2217	L_7,L_3	0.5965
L_1,L_5	0.7475	L_3,L_1	0.2607	L_4,L_5	0.4035	L_6,L_1	0.4185	L_7,L_4	0.5027
L_1,L_6	0.5815	L_3,L_2	0.5134	L_4,L_6	0.2138	L_6,L_2	0.7033	L_7,L_5	0.7783
L_1,L_7	0.5145	L_3,L_4	0.4185	L_4,L_7	0.4973	L_6,L_3	0.7393	L_7,L_6	0.7783
L_2,L_1	0.2056	L_3,L_5	0.2607	L_5,L_1	0.2525	L_6,L_4	0.7862		
L_2,L_3	0.4866	L_3,L_6	0.2607	L_5,L_2	0.7033	L_6,L_5	0.7113		
L_2,L_4	0.3635	L_3,L_7	0.4035	L_5,L_3	0.7393	L_6,L_7	0.2217		

Table 10: Appraisal score and rank of each landfill site

Site	L_1	L_2	L_3	L_4	L_5	L_6	L_7
S_i	0.3483	0.0802	0.0811	0.0883	0.0963	0.1574	0.2357
Rank	1	7	6	5	4	3	2
Latitude	15°47'24''S	15°51'45''S	15°43'11''S	15°44'55''S	15°46'59''S	15°45'40''S	15°50'07''S
Longitude	34°58'57''E	35°05'13''E	35°01'38''E	35°05'39''E	35°03'16''E	35°02'59''E	35°01'18''E

In this paper, only the SMAA-2 model was used, more specifically emphasis was on one of its descriptive measures, the Rank Acceptability Index (Lahdelma and Salminen, 2001). It measures the probability that an alternative acquires a certain position or rank given a set of alternatives, which are considered the best. This is achieved by calculating what proportion of weights will grant an alternative a certain rank. Since it is a probability, the rank acceptability indices range between 0 and 1. An index value of 0 for a certain rank or position indicates that the alternative will never obtain that rank, whilst an index value of 1 indicates that its very highly likely that alternative will always obtain that rank or position given any selected weights. Hence, the rank acceptability indices are considered a measure of robustness as far as the choice of an alternative in concerned (Lahdelma et al., 2002; Tervonen and Figueira, 2008; Tervonen and Lahdelma, 2007; Lahdelma and Salminen, 2001). When applied to this study the Rank Acceptability Indices are shown numerically in Table 11 and illustrated graphically by Figure 8.

Table 11: Rank Acceptability Indices

Landfill Site	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5	Rank 6	Rank 7
L_1	1.000	0.000	0.000	0.000	0.000	0.000	0.000
L_2	0.000	0.000	0.434	0.000	0.000	0.000	0.566
L_3	0.000	0.000	0.002	0.000	0.822	0.018	0.158
L_4	0.000	0.433	0.000	0.003	0.000	0.564	0.000
L_5	0.000	0.000	0.000	0.840	0.148	0.000	0.012
L_6	0.000	0.000	0.564	0.157	0.030	0.012	0.237
L_7	0.000	0.567	0.000	0.000	0.000	0.406	0.027

As can be seen from Table 11 and Figure 8, the resulting indices gave high first rank acceptability to landfill site L_1. Landfill sites L_7, L_6 and L_5 had high acceptability's as rank 2, 3 and 4 respectively. These first four ranks were the same as those obtained from the EVAMIX approach (see Table 10). Landfill sites L_3 and L_4 were ranked 5^{th} and 6^{th} according to their acceptability indices, which was a direct swap to the rankings obtained by the EVAMIX approach. Finally, landfill site L_2 was ranked 7^{th}, the same in Table 11 as in Table 10. The fact that 10,000 simulations had a small impact on the ranking of the sites other than a direct swap of rankings for two sites reveals that the degree of domination of landfill site L_1 and the subsequent ranking of the other sites was independent of changes in the weights associated with the selected criteria.

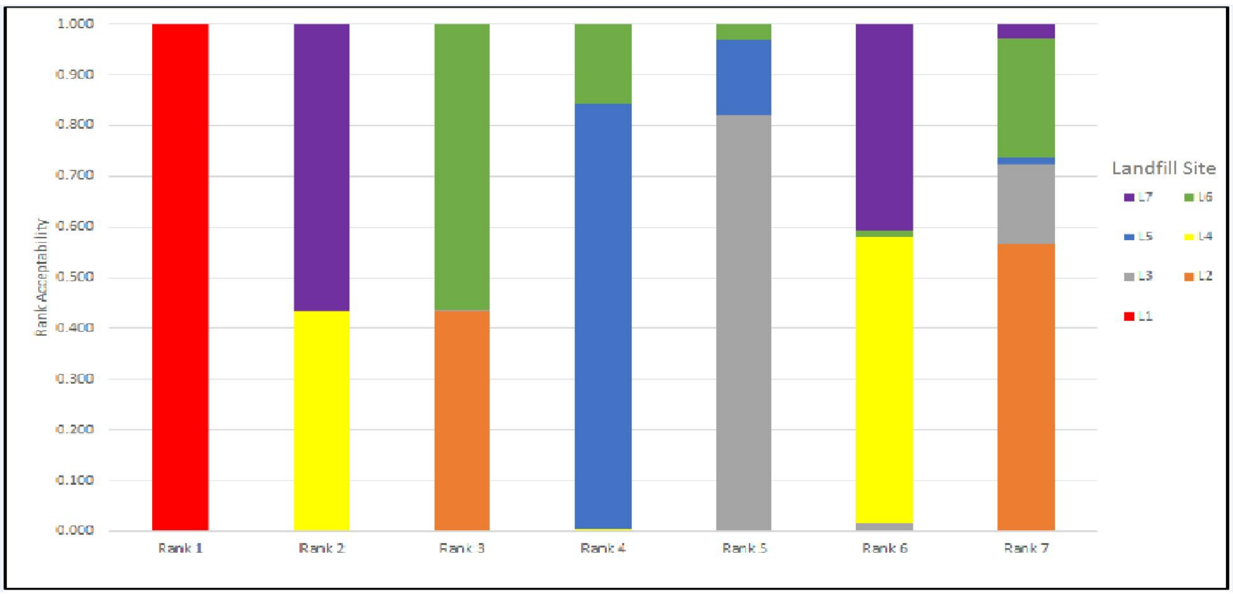

Figure 8: Graphic illustration of Rank Acceptability Indices

4.0 Conclusions

The main goal of this research was to investigate whether GIS, SA, fuzzy AHP combined with the EVAMIX approach, is an attractive alternative for solving multi-objective site location problems. SA set the tone for this research by providing a platform on which one can solve a problem with more than one objective, which was highlighted within the case study by the two objectives represented by Eq. (30) and Eq. (31). By using SA, issues related to optimization and large spatial areas were also addressed. Regarding this, previous research utilizing optimization algorithms such as linear programming tools to solve landfill siting or location allocation problems (Arthur and Nalle, 1997; Aerts et al., 2003; Cova and Church, 2000) have encountered limitations related to the spatial area that could be optimized. This problem arises because each map layer in a GIS by itself can consist of complex information on attribute values, and spatial relationships between attributes. When a large number of layers are involved in GIS analysis (as is normally the case), relationships between layers, and within themselves have to be considered. This often leads to a large solution space from which linear programming solvers find the spatial area too large to explore. In consequence, this exploration is time consuming and, in worse circumstances, linear programming models can get trapped in local minima.

SA, however, addresses these problems by allowing the user to set a large enough initial control temperature, for example 100,000, to be large enough such that the algorithm can traverse a large solution space and still find an optimum solution. In addition, SA offers options for not getting trapped in local minima e.g. fixed probabilities, Tsallis' Acceptance Rule and Boltz-man's distribution, hence is a much faster and robust optimization algorithm. Since decision makers use during pairwise comparison, a measurement scale is thus required.

Fuzzy AHP provided the author with a way to take into account the intrinsic imprecision, uncertainty and subjectivity of linguistic terms or verbal judgements used by decision makers when faced with multi-attribute data during pairwise comparison. Using the EVAMIX approach allowed the author to be able to use both quantitative (cardinal) and qualitative (ordinal) criteria within the same model. This unique feature gives EVAMIX much greater flexibility and differentiates it from other MCDA methods such as weighted summation, range of value method, PROMETHEE II etc., which are incorrectly applied to ordinal data by treating it as though they were at a cardinal measurement scale.

The above reasons and the fact that not one single MCDA method on its own includes all these attributes help to explain why this integrated approach is useful not only for the case study but also for other site location problems. In addition, the results from the methodology were further supported during sensitivity analysis using the SMAA-2 model.

The case study in Blantyre City, Malawi to which the methodology was applied, clearly shows the potential of the approach in a decision support setting. The methodology was able to provide 7 alternative landfill sites ranked form best to worst.

Corresponding Author:
Rodney Godfrey Tsiko
Department of Geoinformatics and Surveying
University of Zimbabwe
P.O Box MP167 Mount Pleasant
Harare, Zimbabwe
E-mail: tsikoruz@yahoo.com

References

1. van Laarhoven PJM, Aarts EHL. *Simulated Annealing: Theory and Applications*. Dordrecht, Boston, Norwell, Massachusetts: D. Reidel, Kluwer Academic Publishers; 1987.
2. Kirkpatrick S, Gelatt CD Jr., Vecchi MP. Optimization by simulated annealing. *Science 1983*, 220 (4598), 671-680.
3. Cerny V. Thermodynamical approach to the travelling salesman problem: an efficient simulation algorithm. *Journal of Optimization Theory Applications 1985*, 45 (1), 41-51.
4. Burrough PA, McDonnell RA. *Principles of Geographical Information Systems*. Oxford University Press; 1998.
5. Heywood I, Cornelius S, Carver S. *An Introduction to Geographical Information Systems,* 2nd ed. Toronto: Pearson Education Limited; 2006.
6. Longley PA, Goodchild MF, Maguire DJ, Rhind DW. *Geographical Information Systems and Science*, 3rd ed. Toronto: John Wiley and Sons; 2010.
7. Bellman RE, Zadeh LA. Decision-making in a fuzzy environment. *Management Science 1970*, 17, 141-164.
8. Saaty TL. *The Analytical Hierarchy Process: Planning, Priority Setting, Resource Allocation*. New York: McGraw-Hill; 1980.
9. Saaty TL. Axiomatic Foundation of the Analytic Hierarchy Process. *Management Science 1986*, 32(7), 841-855.
10. Chang DY. Applications of the extent analysis method on fuzzy AHP. *European Journal of Operational Research 1996*, 95, 649-655.
11. Voogd H. Multicriterion evaluation with mixed qualitative and quantitative data. *Environmental and Planning Bulletin 1982*, 9, 221-236.
12. Nijkamp P, Rietveld P, Voogd H. *Multicriteria Evaluation in Physical Planning*. North Holland, Amsterdam; 1990, pp. 65-100.
13. Martel JM, Matarazzo B. *Other outranking approaches.* In: Figueira, J., Salvatore, G., Ehrgott, M. (Eds), Multiple Criteria Decision Analysis: State of the Art Surveys. New York: Springer; 2005, pp. 197-262.
14. Hajkowicz S, Higgins A. A Comparison of Multiple Criteria Analysis Techniques for Water Resource Management. *European Journal of Operations Research 2008*, 184 (1), 255-265.
15. Brans JP, Mareschal B, Vincke PH. How to select and how to rank projects: The PROMETHEE method. *European Journal of Operations Research 1986*, 24(2), 228-238.
16. Figueira J, Greco S, Ehrgott M. *Multiple Criteria Decision Analysis: State of the Art Surveys*. Springer Verlag; 2005.
17. Arthur JL, Nalle DJ. Clarification on the Use of Linear Programming and GIS for Land Use Modelling. *International Journal of Geographical Information Science 1997*, 11, 397-402.
18. Aarts JCJH, Eisinger E, Heuvelink GBM and Stewart TJ. Using Linear Integer Programming for Multi-Site Land Use Allocation. *Geographical Analysis 2003*, 35 (2), 148-169.

19. Cova T J, Church RL. Contiguity Constraints for Single-Region Site Search Problems. *Geographical Analysis 2000*, 32, 306-329.
20. Lockwood C, Moore T. Harvest scheduling with spatial constraints a Simulated Annealing approach. *Canadian Journal of Forest Reserve 1993*, 23, 468-478.
21. Murray AT, Church RL. Measuring the efficacy of adjacency constraints structure in forest planning models. *Canadian Journal of Forest Reserve 1995*, 25, 1416-1424.
22. Muttiah RS, Engel BA, Jones DD. Waste Disposal Site Selection Using GIS-based Simulated Annealing. *Computers & Geosciences 1996*, 22 (9), 1013-1017.
23. Brookes CJ. A parameterized region-growing program for site allocation on raster suitability maps. *International Journal of Geographical Information Science 1997*, 11, 375-396.
24. Boston K, Bettinger P. An Analysis of Monte Carlo Integer Programming, Simulated Annealing, and Tabu Search Heuristics for Solving Spatial Harvest Scheduling Problems. *Forest Science 1999*, 45, 292-301.
25. Karaganis A, Mimis A. A Geographical Information System Framework for Evaluating the Optimum Location of Point-Like Facilities. *Asian Journal of Information Technology 2011*, 10 (4), 129-135.
26. Pincus M. A Monte Carlo Method for the Approximate Solution of Certain Types of Constrained Optimization Problems. *Operations Research 1970*, 18, 1225-1228.
27. Eglese RW. Simulated annealing: a tool for operational research. *European Journal of Operational Research 1990*, 46, 271-281.
28. Koulamas C, Antony SR, Jaen R. A survey of simulated annealing applications to operations-research problems. *OMEGA- International Journal of Management Science 1994*, 22, 41-56.
29. Fleischer MA. Simulated annealing: past, present and future. In: C Alexopoulos, K Kang, WR Lilegdon and D Goldsman (eds.), *Proceedings of the 1995 Winter Simulation Conference,* IEEE Press; 1995, pp. 155-161.
30. Aarts EHL, Korst J. *Simulated Annealing and Boltzmann Machines: A Stochastic Approach to Combinatorial Optimization and Neural Computing*. Chichester, England: John Wiley & Sons; 1989.
31. Aarts EHL, Lenstra, JK. *Local Search in Combinatorial Optimization*. Chichester, England: John Wiley & Sons; 1997.
32. Pardolas PM, Romeijn HE. (Eds). *Handbook of global optimization volume II.* Kluwer Academic, Dordrecht; 2002.
33. Berne JL, Baselga S. First – order design of geodetic networks using the simulated annealing method. *Journal of Geodesy 2003*, (78), 47-54.
34. Voogd H. *Multicriteria Evaluation for Urban and Regional Planning*. London: Pion Limited; 1983.
35. Chatterjee P, Athawale VM, Chakraborty S. Material Selection Using Complex Proportional Assessment and Evaluation of Mixed Data Methods. *Material and Design 2011*, 32(2), 851-860.
36. Darji VP, Rao RV. Application of AHP/EVAMIX Method for Decision Making in the Industrial Environment. *American Journal of Operations Research 2013*, 3, 542-569.
37. Chung ES, Lee KS. Identification of Spatial Ranking of Hydrological Vulnerability Using Multi-Criteria Decision Making Techniques: Case Study of Korea. *Water Resource Management 2009*, 23(12), 2395-2416.
38. Ustinovichius L, Zavadskas EK, Podvezko V. Application of a Quantitative Multiple Criteria Decision Making (MCDM-1) Approach to the Analysis of Investments in Construction. *Control and Cybernetics 2007*, 36(1), 251-268.
39. Jeffreys I. The Use of Compensatory and Non-Compensatory Multi-Criteria Analysis for Small-Scale Forestry. *Small-Scale Forest Ecol Manage Policy 2004*, 3(1), 99-117.
40. Kaufmann A, Gupta MM. *Fuzzy Mathematical Models in Engineering and Management Science*. New York, USA: Elsevier Science Inc.; 1988.
41. Zadeh LA. Fuzzy Sets. *Information Control 1965*, 8, 338-353.
42. Modarres M, Sadi-nezhad S, Aarbi F. Fuzzy analytical hierarchy process using preference ration: A case study for selecting management short course in a business school. *International Journal of Industrial Engineering Computations 1 2010*, 173-184.
43. Modarres M, Sadi-Nezhad S. Ranking fuzzy numbers by preference ratio. *Fuzzy Sets and Systems 2001*, 118, 429-436.
44. Matope J. *Blantyre City Environmental Profile*. UNDP, UN-HABITAT; 2000.
45. Berman S. *Living with Garbage* [online]. The Star. 2011. Available from: http://thestar.blogs.com/africa/author-sarah-berman/. [Accessed 3 February 2014].
46. UN-HABITAT. *Malawi: Blantyre Urban Profile*. Nairobi: UN-HABITAT; 2011.
47. Love RR, Morris JG, Wesolowsky GO. *Facilities Location, Models and Methods.* North Holland; 1998.

48. Apparicio P, Abdelmajid M, Riva M, Shearmur R. Comparing alternative approaches to measuring the geographical accessibility of urban health services: Distance types and aggregation-error issues. *Int. J. Health Geographics 2008*, 7, 7-7.
49. Corana A, Marchesi C, Martini C, Ridella S. Minimizing multimodal functions of continuous variables with the simulated annealing algorithm. *ACM Transactions on Mathematical Software 1987*, 13(3), 262-280.
50. Zeiss C, Lefsrud L. "Analytical Framework for Facility Waste Siting." *Journal of Urban Planning and Development – ASCE 1995* 121(4), 115–145.
51. Wang G, Qin L, Li G, Chen, L. "Landfill Site Selection Using Spatial Information Technologies and AHP: A Case Study in Beijing, China." *Journal of Environmental Management 2009,* 90, 2414–2421.
52. Siddiqui MZ, Everoett JW, Vieux BE. Landfill Siting Geographical Information Systems: A Demonstration. *Journal of Environmental Engineering –ASCE 1996,* 122(6), 515–523.
53. Sener B, Suzen L, Doyuran V. "Landfill Site Selection by Using Geographic Information Systems." *Environmental Geology 2006,* 49, 376–388.
54. Kontos TD, Komilis DP, Halvadakis, CP. "Siting MSW Landfills on Lesvos Island with a GIS-Based Methodology." *Waste Management Resources 2003,* 21, 262–278.
55. Gemitzi A, Tsihrintzis VA, Voudrias V, Petalas C, Stravodimos G. Combining Geographic Information Systems, Multi-Criteria Evaluation Technique and Fuzzy Logic in Siting MSW Landfills. *Environmental Geology 2007*, 51, 797-811.
56. Lin HY, Kao JJ. Enhanced Spatial Model for Landfill Siting Analysis. *Environmental Engineering – ASCE 1999*, 125(9), 845-851.
57. Tayyebi AH, Delavar MR, Tayyebi A, Golobi M. Combining Multi criteria decision making and dempster shafer theory for landfill site selection. *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Science 2010*, 38 (8), 1073-1078.
58. Onut S, Efendigil T, Kara SS. A combined fuzzy MCDM approach for selecting shopping centre site; An example from Istanbul, Turkey. *Expert Systems with Applications 2010*, 37, 1973-1980.
59. Onut D, Soner S. Transhipment site selection using the AHP and TOPSIS approaches under fuzzy environment. *Waste Management 2008*, 28, 1552-1559.
60. Hokkanen J, Lahdelma R, Salminen P. A multiple criteria decision model for analysing and choosing among different development patterns for the Helsinki cargo harbour. *Socio-Economic Sciences 1999*, 33, 1-23.
61. Lahdelma R, Salminen P, Hokkanen J. Locating a waste treatment facility by using stochastic multicriteria acceptability analysis with ordinal criteria. *European Journal of Operational Research 2002*, 142, 345-356.
62. Tervonen T, Figueira JR. A survey on Stochastic Multicriteria Acceptability Analysis Methods. *Journal of Multi-Criteria Decision Analysis 2008*, 15, 1-14.
63. Tervonen T, Lahdelma R. Implementing stochastic multicriteria acceptability analysis. *European Journal of Operational Research 2007*, 178(2), 500-513.
64. Lahdelma R, Salminen P. SMAA-2: stochastic multicriteria acceptability analysis for group decision making. *Operations Research 2001*, 49(3), 444-454.

10/13/2016