

## Network Parameters Errors Identification in Power System State Estimation Using Synchronized Phasor Measurements

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**Abstract:** This paper proposes a technique for identification of simultaneous errors of either measurements or parameters in power system state estimation, at which the traditional methods fails to identify. The strength of data correlation is represented by a correlation coefficient matrix that can be further analyzed to decorrelate dependent data. Synchronized phasor measurement units (PMU) are used to identify these simultaneous dependent errors. The paper extends the discussion of optimal PMU placement, previously utilized to achieve network observability and state estimation, to include the conditions of unidentifiable measurements and parameters. An optimal PMU placement solution can then be utilized for identification of erroneous measurements and parameters.

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**Keywords:** Power System State estimation, Parameter Error Identification, Optimal Meter Placement, Phasor Measurement Units.

### Introduction

Earlier conventional methods for error identification assumed that all errors are referred to network measurements. This assumption was based on that, using conventional techniques, errors in parameters are reflected as measurements errors. The method proposed by [1] used Lagrange multipliers to distinguish between measurements and parameters errors. However, [1] has identified four cases in which this method fails to identify the erroneous quantity;

- 1) When the error occurs in a measurement that is part of a critical measurements k-tuple.
- 2) When the error occurs in a parameter that is part of a critical parameter k-tuple.
- 3) When the error occurs in a measurement or parameter that is part of a critical measurement-parameter k-tuple.
- 4) When the error is in a measurement that is critical for network observability.

The conclusion of [1] recommended the use of PMUs to identify these errors.

This article presents a mathematical approach to detect the correlation between measurements or parameters forming critical k-tuples using Pearson's coefficient [2] and proposes the use of linear Karhunen-Loeve Transform (KLT) algorithm [3] to decorrelate measurements or parameters forming critical k-tuples. Utilizing KLT to decorrelated dependant data helped in identifying errors in the first two cases where the method proposed in [1] failed to do. Thus, the number of PMUs expected to be required to identify unidentifiable error will be less than when applying the recommendations of [1].

Different methods, techniques and algorithms [4]

are used to solve the problem of optimum placement of PMUs. The purpose of these approaches is to maintain network observability and estimating system state.

Integer Linear Programming (ILP), as explained in [5], is considered the most common procedure used to solve the optimum PMU placement problem to achieve network observability. This article utilizes the same procedure. However, new problem constraints are introduced to consider the placement of PMUs to achieve errors identifiability as well.

### Measurements and Parameters Errors Identification using Lagrange Multipliers Method [1]

#### A. Problem Formulation

The mathematical formulation of measurement model is given as;

$$z = h(x, p_e) + e \quad (1)$$

where;

$z$  is the measurements vector

$x$  is the vector of system state variables

$p_e$  is a vector containing network parameter errors

$h(x, p_e)$  is a nonlinear function relating the measurements to system states and network parameter errors

$e$  vector of measurements errors

Zero injection buses can be considered as an equality constraint given by;

$$c(x, p_e) = 0 \quad (2)$$

Network parameter vector can be modeled as;

$$p = p_t + p_e \quad (3)$$

where  $p$  and  $p_t$  respectively represent the assumed and true value of parameters.

Assuming that parameters are error free, the following equality constraint may be used;

$$p_e = 0 \quad (4)$$

Considering the above, the solution of the Weighted Least Squares (WLS) estimation problem as defined by [6] can be formulated as an optimization problem that is subject to constraints as follow;

$$\text{Minimize } J(x) = \frac{1}{2} r^t W r \quad (5)$$

$$\text{Subject to } c(x, p_e) = 0$$

$$p_e = 0$$

where;

$r = z - h(x, p_e)$  is the measurement residual vector and

$W$  is the inverse of the measurement error covariance matrix.

The Lagrange of the optimization problem of equation (5) is given as;

$$L = \frac{1}{2} r^t W r - \mu^t c(x, p_e) - \lambda^t p_e \quad (6)$$

Two Lagrange multipliers are introduced; the first ( $\mu$ ) is related to the constraint of zero injection buses, and the second ( $\lambda$ ) is related to error free parameters constraint.

Optimality conditions can be obtained as;

$$\frac{\partial L}{\partial x} = H_x^t W r + C_x^t \mu = 0 \quad (7)$$

$$\frac{\partial L}{\partial p} = H_p^t W r + C_p^t \mu + \lambda = 0 \quad (8)$$

$$\frac{\partial L}{\partial \mu} = c(x, p_e) = 0 \quad (9)$$

$$\frac{\partial L}{\partial \lambda} = p_e = 0 \quad (10)$$

where

$$H_x = \frac{\partial h(x, p_e)}{\partial x} \quad (11)$$

$$C_x = \frac{\partial c(x, p_e)}{\partial x} \quad (12)$$

$$H_p = \frac{\partial h(x, p_e)}{\partial p_e} \quad (13)$$

$$C_p = \frac{\partial c(x, p_e)}{\partial p_e} \quad (14)$$

Equality constraint (4) allows the substitution  $p_0 = 0$  into  $h(x, p_0)$  and  $c(x, p_0)$  to become  $h(x, 0)$  and  $c(x, 0)$  respectively. These will be denoted as  $h_0(x)$  and  $c_0(x)$ . The measurements model and zero injection equations will become;

$$z = h_0(x) + e \quad (15)$$

$$c_0(x) = 0 \quad (16)$$

Note that equations (15) and (16) do not include parameter errors as explicit variables.  $h_0(x)$  and  $c_0(x)$  will be substituted with their first order Taylor expansion to obtain the following linear equations;

$$H_x \Delta x + r = \Delta z \quad (17)$$

$$C_x \Delta x = -c_0(x_0) \quad (18)$$

where;

$x_0$  is the initial guess of the system state vector,

$\Delta x = x - x_0$  and

$\Delta z = z - h_0(x_0)$ .

Equations (7), (17) and (18) can be reformulated in matrix presentation as;

$$\begin{bmatrix} 0 & H_x^t W & C_x^t \\ H_x & I & 0 \\ C_x & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ r \\ \mu \end{bmatrix} = \begin{bmatrix} 0 \\ \Delta z \\ -c_0(x_0) \end{bmatrix} \quad (19)$$

This system of equations can be solved using the same iterative solution used for WLS state estimation problem. The system solution will provide the estimated values of state variables, measurement residuals and Lagrange multiplier  $\mu$ .

### B. Calculation of Normalized Lagrange Multiplier

Detection of erroneous measurements is carried out using the normalized residual method as explained in [6] and using the residuals vector obtained from equation (19).

Recalling that the gain matrix  $G$  is calculated using

$$G = H_x^t W H_x \quad (20)$$

The hat matrix  $K$  is defined as

$$K = H_x G^{-1} H_x^t W \quad (21)$$

The measurements residual sensitivity matrix  $S_m$  is calculated as

$$S_m = I - K \quad (22)$$

Measurements residuals covariance matrix is then calculated using

$$\Omega = S_m R_z S_m^t \quad (23)$$

where  $R_z$  is the measurement covariance matrix.

The normalized residual of measurement  $i$  can be obtained by

$$r_i^N = \frac{|r_i|}{\sqrt{\Omega_{ii}}} \quad (24)$$

Erroneous parameters, on the other hand, are selected to be those that corresponds to the largest normalized Lagrange multiplier  $\lambda$ .

Equation (8) is used to calculate the Lagrange multiplier  $\lambda$  as follows

$$\lambda = -(W H_p r + C_p \mu)$$

which can be reformulated in matrix form as

$$\lambda = - \begin{bmatrix} W H_p \\ C_p \end{bmatrix}^t \begin{bmatrix} r \\ \mu \end{bmatrix}$$

The sensitivity matrix corresponding to errors in parameters is defined as;

$$S_p = - \begin{bmatrix} W H_p \\ C_p \end{bmatrix}^t \quad (25)$$

Therefore,

$$\lambda = S_p \begin{bmatrix} r \\ \mu \end{bmatrix} \quad (26)$$

The matrix given by equation (19) will be used to calculate the covariance of  $\lambda$  by reformulating the equation as

$$\begin{bmatrix} \Delta x \\ r \\ \mu \end{bmatrix} = \begin{bmatrix} 0 & H_x^t W & C_x^t \\ H_x & I & 0 \\ C_x & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \Delta z \\ -c_0(x_0) \end{bmatrix}$$

Note that, at the solution of (19)  $c_0(x) = 0$ .

The inverse matrix above can be arranged as below

$$\begin{bmatrix} 0 & H_x^t W & C_x^t \\ H_x & I & 0 \\ C_x & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} E_1 & E_2 & E_3 \\ E_4 & E_5 & E_6 \\ E_7 & E_8 & E_9 \end{bmatrix} \quad (27)$$

Therefore,

$$\begin{bmatrix} \Delta x \\ r \\ \mu \end{bmatrix} = \begin{bmatrix} E_1 & E_2 & E_3 \\ E_4 & E_5 & E_6 \\ E_7 & E_8 & E_9 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \Delta z \\ 0 \end{bmatrix}$$

From the above;

$$r = E_5 \Delta z \quad (28)$$

$$\mu = E_8 \Delta z \quad (29)$$

Let  $\Psi = [E_5 \ E_8]^t$ , then

$$u = \Psi \cdot \Delta z \quad (30)$$

Knowing that  $u = [r \ \mu]^t$

$$cov(u) = \Psi \cdot W^{-1} \cdot \Psi^t \quad (31)$$

The covariance of Lagrange multiplier  $\lambda$  can then be calculated as

$$\Lambda = cov(\lambda) = S_p \cdot cov(u) \cdot S_p^t \quad (32)$$

Finally, the normalized Lagrange multiplier for parameter  $i$  can be obtained as

$$\lambda_i^N = \frac{\lambda_i}{\sqrt{\Lambda_{ii}}} \quad (33)$$

### Decorrelating Measurements or Parameters k-Tuples

#### A. Identifying the Correlation between Measurements or Parameters

In this article, data correlation is considered instead of dependence since correlation offers a broader class of statistical relationships that includes dependence. Correlation between data is identified by correlation coefficients. Since linearized model equations are used for error identification, Pearson's coefficient [2] will be used as it is the most commonly used coefficient that is sensitive to linear relationships. Pearson's coefficient is defined as;

$$\rho(x, y) = corr(x, y) = \frac{cov(x, y)}{\sigma_x \sigma_y} \quad (34)$$

$$\text{and } -1 \leq \rho(x, y) \leq 1$$

where  $x$  and  $y$  are two data items the correlation coefficient among which is to be identified.

$cov(x, y)$  is the covariance between data  $x$  and data  $y$ .

$\sigma_x$  and  $\sigma_y$  are the variances of data  $x$  and data  $y$  respectively.

Pearson's coefficient  $\rho(x, y)$  represents the degree of linearity in the relationship between  $x$  and  $y$ . Therefore, a coefficient of +1 indicates a perfect positive (directly proportional) relationship, a coefficient of -1 indicates a perfect negative (inversely proportional) relationship and a coefficient of any value in between represents the linear dependence of data. One important aspect is when the coefficient is

equal to zero that means that there is no relationship between the corresponding data items. Ideally, the correlation coefficients matrix is symmetrical and has its diagonal elements equal to unity.

For a vector of  $n$  data items, the covariance matrix is given as;

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_{nn} \end{bmatrix} \quad (35)$$

Pearson's Coefficient matrix is given as;

$$\Sigma = \begin{bmatrix} \rho_{11} & \rho_{12} & \dots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \dots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \dots & \rho_{nn} \end{bmatrix} \quad (36)$$

$$\text{where } \begin{cases} -1 \leq \rho_{ij} \leq 1 \text{ for } i \neq j \\ \rho_{ij} = 1 \text{ for } i = j \end{cases}$$

The covariance matrix for measurements residuals  $\Omega$  and the covariance matrix for parameters residuals  $\Lambda$  are obtained from equations (23) and (32) respectively. These matrices can be used to calculate the correlation coefficients matrices for measurements and parameters residuals. However, the calculation of correlation between measurements and parameters cannot be directly obtained despite the possibility to calculate the covariances between these quantities, this is due to the fact that correlation coefficient matrices are built based on equal number of data observations (i.e. square covariance matrices). In what follows, an indirect method for calculating the covariance matrix between measurements and parameters will be deduced and used to obtain the correlation coefficients matrix between the two quantities.

Equations (7) and (8) can be re-written as;

$$H_x^t W r = -C_x^t \mu \quad (37)$$

$$H_p^t W r + \lambda = -C_p^t \mu \quad (38)$$

These equations can be expressed in the matrix form as;

$$\begin{bmatrix} H_x^t W & 0 \\ H_p^t & I \end{bmatrix} \begin{bmatrix} r \\ \lambda \end{bmatrix} = \begin{bmatrix} -C_x^t \\ -C_p^t \end{bmatrix} \mu \quad (39)$$

Therefore,  $r$  and  $\lambda$  can be expressed in terms of  $\mu$  as;

$$\begin{bmatrix} r \\ \lambda \end{bmatrix} = \begin{bmatrix} H_x^t W & 0 \\ H_p^t & I \end{bmatrix}^{-1} \begin{bmatrix} -C_x^t \\ -C_p^t \end{bmatrix} \mu \quad (40)$$

Following equation (25);

$$S_{mp} = \begin{bmatrix} H_x^t W & 0 \\ H_p^t & I \end{bmatrix}^{-1} \begin{bmatrix} -C_x^t \\ -C_p^t \end{bmatrix} \quad (41)$$

where  $S_{mp}$  is the sensitivity matrix between measurements and parameters residuals.

From equations (30) and (31),

$$cov(\mu) = E_8 W^{-1} E_8^t \quad (42)$$

where  $E_8$  is calculated from equation (27).

The combined covariance matrix of measurements and parameters  $Z$  can, then be

calculated as;

$$Z = \text{cov} \left( \begin{bmatrix} r \\ \lambda \end{bmatrix} \right) = S_{mp} \text{cov}(\mu) S_{mp}^t \quad (43)$$

Equations (34) and (36) can, then, be used to calculate the correlation coefficients matrix for measurements and parameters. However, it is to be noted that the resulting matrix needs to be utilized in its reduced form by eliminating elements that represent the correlation between parameters and those that represent the correlation between measurements and maintain only those elements that represent the correlation between measurements and parameters.

### B. Test Results for Correlation Coefficients

Correlation coefficients matrices for IEEE 14, 30 and 57-bus test networks are calculated for the test cases presented by [1] where parameters are unidentifiable or having very close values of their normalized Lagrange multipliers.

Table (I) extracts unidentifiable parameters or parameters having very close  $\lambda^N$  values as yielded from the test cases presented in [1] and shows the correlation coefficients of these parameters.

From Table I, it is shown that the inability of the conventional error identification algorithm to identify the erroneous quantity is due to the high correlation between the erroneous quantity and other quantities. The effect of correlation can be mainly visualized when examining the above 14-bus test cases (shown shaded) where error is once introduced to  $x_{6-12}$  and once gain introduced to  $x_{12-13}$ . Error could be identified for the first case since the correlation coefficient between the erroneous quantity and other quantities is low. However, the error in  $x_{12-13}$ , in the second case, could not be identified due to the high correlation coefficient between  $x_{12-13}$  and  $x_{6-12}$ .

**Table I** Coorelation Coefficients between Unidentifiable Parameters or Parameters having Close Values of Normalized Lagrange Multiplier

| Network | Examined Parameters             | Correlation Coefficient |
|---------|---------------------------------|-------------------------|
| 30-Bus  | $x_{5-7}$ *, $x_{6-7}$          | -0.98942                |
| 14-Bus  | $b_{sh_{9-9}}$ *, $q_{9-9}$     | 1.0                     |
| 30-Bus  | $b_{sh_{24-24}}$ *, $q_{24-24}$ | 1.0                     |
| 14-Bus  | $x_{6-12}$ *, $x_{12-13}$       | -0.42067                |
|         | $x_{6-12}$ , $x_{12-13}$ *      | 0.910899                |
|         | $x_{15-18}$ , $x_{18-19}$ *     | 0.989001                |
| 30-Bus  | $x_{18-19}$ *, $x_{10-20}$      | -0.9914                 |
|         | $x_{18-19}$ *, $x_{19-20}$      | -0.9916                 |
| 30-Bus  | $r_{24-25}$ , $x_{25-27}$ *     | 0.982575                |
| 14-Bus  | $x_{10-11}$ , $x_{13-14}$ *     | 0.98453                 |
|         | $x_{13-14}$ , $x_{6-11}$        | -0.93201                |

\* Indicates parameter where error is introduced

### C. Decorrelating Dependant Measurements or Parameters

As explained in section II (B), the main reason behind the presence of critical k-tuples of parameters and the inability to identify the erroneous quantity, either it was a parameter or a measurement, is the presence of strong correlation (high correlation coefficient) between the erroneous quantity and other measurements or parameters. The correlation can also be viewed from the presence of non-zero values in the off-diagonal elements of the covariance and, consequently, the correlation coefficient matrices.

Decorrelation is defined as a process that is used to reduce (or eliminate) the correlation between data. The process has wide application in signal and image processing using linear and non-linear algorithms that are aimed at diagonalizing the correlation coefficient and correlation matrices while preserving the aspects of the decorrelated data. Linear Karhunen-Loève Transform (KLT) [3] is considered the most common algorithm among other linear and non-linear algorithms. This section will illustrate the application of KLT to decorrelate measurements and parameters and will investigate the behavior of the error identification algorithm on decorrelated data.

Considering that R is the generalized correlation coefficients matrix that includes either measurements or parameters, it was shown by [7] that R can be diagonalized to  $R^d$  by multiplying R by another matrix  $\Phi$ , called the KLT matrix, as per the equation;

$$R^d = \Phi^{-1} R \Phi \quad (44)$$

with  $\Phi$  being a matrix having its columns representing the Eigen vectors of R, the diagonalized matrix  $R^d$  is actually a matrix having its diagonal elements representing the Eigen values of R.

KLT matrix  $\Phi$  can, also, be used to transform a set of data to a decorrelated set. The following equations can be used to transform the measurements residuals  $r$  and parameters residuals  $\lambda$  to their corresponding decorrelated sets.

$$r^d = \Phi^{-1} r \quad (45)$$

$$\lambda^d = \Phi^{-1} \lambda \quad (46)$$

Therefore, the covariance and correlation coefficients matrices for  $r^d$  and  $\lambda^d$  will be diagonal (i.e. decorrelated).

A covariance matrix can be diagonalized without calculating the correlation coefficients matrix, as proved by [8] using the equation;

$$X = Y^t \Sigma Y \quad (47)$$

where;  $\Sigma$  is the original (correlated) covariance matrix and X is the diagonalized one.

In the above equation, Y is matrix holding the Eigen vectors of  $\Sigma$  as its columns and can be used to transform a set of data to a decorrelated set. Therefore, the decorrelated measurements and parameters

residuals expressed in equations (45) and (46) can be expressed in terms of Y as below;

$$r^d = Y^t r \quad (48)$$

$$\lambda^d = Y^t \lambda \quad (49)$$

The decorrelated covariance matrices  $\Omega^d$  and  $\Lambda^d$  can be calculated using Equation (47). The decorrelated normalized measurements residuals ( $r^{Nd}$ ) and normalized parameters Lagrange multipliers ( $\lambda^{Nd}$ ) can be calculated using Equations (24) and (33) respectively after substituting  $r, \lambda, \Omega$  and  $\Lambda$  with their decorrelated counterparts  $r^d, \lambda^d, \Omega^d$  and  $\Lambda^d$  respectively. The parameter or measurement in the unidentified suspect set that corresponds to the largest  $r^{Nd}$  or  $\lambda^{Nd}$  will be the erroneous quantity

#### D. Simulation Results for Data Decorrelation

The following test cases were conducting by introducing errors to parameters that for critical k-tuples with other parameters or measurements.

*Test Case 1:* Errors are individually introduced to different parameters of IEEE 14-bus test network as shown by Table (II).

**Table II: Test Case 1: Simulated Errors**

| Sub-Case | Parameter   | True Value | Erroneous Value |
|----------|-------------|------------|-----------------|
| a        | $x_{12-13}$ | 0.1999     | 0.5             |
| b        | $x_{13-14}$ | 0.348      | 0.7             |

Tables III and IV show the values of the highest normalized residuals for different parameters and measurements, resulting from errors introduced by test cases 1(a) and 1(b), before and after applying data decorrelation.

**Table III: Test Case 1(a): Error Identification Results**

| Measurement/Parameter | $r^N/\lambda^N$           | $r^{Nd}/\lambda^{Nd}$     |
|-----------------------|---------------------------|---------------------------|
|                       | (Excluding Decorrelation) | (Including Decorrelation) |
| $x_{6-12}$            | <b>6.7902</b>             | <b>0.5269</b>             |
| $x_{12-13}$           | <b>6.7667</b>             | <b>4.4250</b>             |
| $x_{6-13}$            | 4.6628                    |                           |
| $p_{12-13}$           | 4.6505                    |                           |
| $q_{6-12}$            | 4.3490                    |                           |

**Table IV: Test Case 1(b): Error Identification Results**

| Measurement/Parameter | $r^N/\lambda^N$           | $r^{Nd}/\lambda^{Nd}$     |
|-----------------------|---------------------------|---------------------------|
|                       | (Excluding Decorrelation) | (Including Decorrelation) |
| $x_{10-11}$           | 27.7765                   | 6.7735                    |
| $x_{13-14}$           | 27.7706                   | 46.8020                   |
| $x_{6-11}$            | 27.5534                   | 6.9430                    |
| $x_{9-14}$            | 26.8986                   | 2.9573                    |
| $r_{9-10}$            | 20.5636                   | 5.1666                    |

*Test Case 2:* Errors are individually introduced to different parameters of IEEE 30-bus test network as shown by Table (V).

**Table V: Test Case 2: Simulated Errors**

| Sub-Case | Parameter   | True Value | Erroneous Value |
|----------|-------------|------------|-----------------|
| a        | $x_{18-19}$ | 0.1292     | 0.5             |
| b        | $x_{27-25}$ | 0.2087     | 0.6             |

Tables VI and VII show the values of the highest normalized residuals for different parameters and measurements, resulting from errors introduced by test cases 2(a) and 1(b), before and after applying data decorrelation.

**Table VI: Test Case 2(a): Error Identification Results**

| Measurement/Parameter | $r^N/\lambda^N$           | $r^{Nd}/\lambda^{Nd}$     |
|-----------------------|---------------------------|---------------------------|
|                       | (Excluding Decorrelation) | (Including Decorrelation) |
| $x_{15-18}$           | 15.4741                   | 0.0484                    |
| $x_{18-19}$           | 15.4470                   | 4.2831                    |
| $x_{20-10}$           | 15.3713                   | 0.0205                    |
| $x_{19-20}$           | 15.2465                   | 0.7348                    |
| $p_{18-19}$           | 9.0775                    |                           |

**Table VII: Test Case 2(b): Error Identification Results**

| Measurement/Parameter | $r^N/\lambda^N$           | $r^{Nd}/\lambda^{Nd}$     |
|-----------------------|---------------------------|---------------------------|
|                       | (Excluding Decorrelation) | (Including Decorrelation) |
| $r_{24-25}$           | 24.836                    | 4.235                     |
| $x_{27-25}$           | 27.7589                   | 24.967                    |
| $x_{28-27}$           | 21.4546                   | 12.652                    |
| $p_{25-27}$           | 13.257                    |                           |
| $p_{24-25}$           | 12.6138                   |                           |

### Optimum Placement of PMUs using ILP

In what follows we will explain the basic problem formulation as explained in [5]. We shall then present the modifications and extensions to the existing procedures to allow for network errors identification.

#### A. Problem Formulation

The objective function of the Optimum Placement Problem (OPP) of PMUs is to render the full network observable using a minimum number of PMUs. For an n-bus system, the PMU placement problem can be formulated as follows:

$$\min \sum_i^n w_i \cdot x_i \quad (50)$$

$$\text{s.t. } f(X) \geq \hat{1}$$

where X is a binary decision variable vector, whose entries are defined as:

$$x_i = \begin{cases} 1 & \text{if a PMU is installed at bus } i \\ 0 & \text{otherwise} \end{cases} \quad (51)$$

$w_i$  is the cost of the PMU installed at bus i

$f(X)$  is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measurement set and zero otherwise.

$\hat{1}$  is a vector whose entries are all ones.

The procedure for building the constraint equations follows the below series of steps;

- 1) Step 1: No flow or injection measurements are present.
- 2) Step 2: Power Flow measurements are available.
- 3) Step 3: Power injection measurements are available.

Description for the procedure for building the constraint equations will be provided using a 7-bus sample system that is shown by Figure (1).

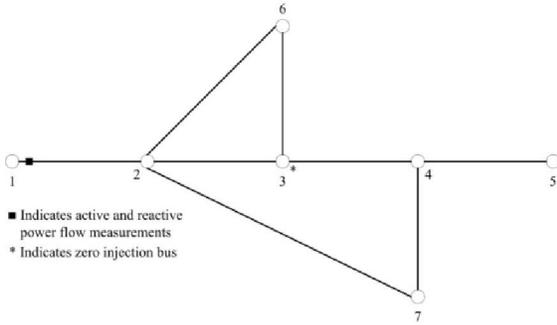


Figure 1 7-Bus Sample System

The binary connectivity matrix  $A$  of the sample network is defined as follows:

$$A_{k,m} = \begin{cases} 1 & \text{if } k = m \\ 1 & \text{if } k \text{ and } m \text{ are connected} \\ 0 & \text{otherwise} \end{cases} \quad (52)$$

The constraints function  $f(X)$  is, then, given as:

$$f(X) = A \cdot X \quad (53)$$

where  $X$  is a vector representing the solvability of each bus and it has a non-zero value if the bus voltage is solvable and zero otherwise.

*Step 1: No Flow or Injection Measurements are Present*

This step assumes that no flow or injection measurements are present in the network. This will help building up the full constraints function.

The binary decision vector  $X$  for the sample network is in the form;

$$X = [x_1 x_2 x_3 x_4 x_5 x_6 x_7]^t \quad (54)$$

Matrix  $A$  for the sample network is obtained as;

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \quad (55)$$

The general constraint, for this case, is that; at least one PMU should be placed at each bus in order to render all links, directly connected to this bus, observable. Therefore, for each bus, the constraint

function  $f(X)$ , as obtained by Equation (53), can be written as:

$$f(X) = \begin{cases} f_1 = x_1 + x_2 & \geq 1 \\ f_2 = x_1 + x_2 + x_3 + x_6 + x_7 & \geq 1 \\ f_3 = x_2 + x_3 + x_4 + x_6 & \geq 1 \\ f_4 = x_3 + x_4 + x_5 + x_6 & \geq 1 \\ f_5 = x_4 + x_5 & \geq 1 \\ f_6 = x_2 + x_3 + x_6 & \geq 1 \\ f_7 = x_2 + x_4 + x_7 & \geq 1 \end{cases} \quad (56)$$

The ‘+’ operator serves as a logical ‘OR’. For example, the constraint of bus 1 can be interpreted as; At least one PMU can be placed at either bus 1 or 2 to render bus 1 observable.

Since, at this stage, no flow or injection measurements are considered, it is assumed that each group of buses forms an observable island on its own. Therefore, a minimum of one PMU shall be placed at any of the buses in the group.

*Step 2: Considering Flow Measurements*

When a flow measurement is introduced at any branch, this implies that the two observable island connected by this branch can be grouped into one observable island.

The mathematical interpretation of the above is that the two constraint functions of the two buses connected by the measured link can be grouped into one new constraint function which is the ‘sum’ of the two original functions. Again, the ‘+’ operator is used to indicate that the PMU can be placed at any of the buses forming the new observable island.

In the sample network shown by Figure (1), active and reactive measurements are introduced in the link between buses 1 and 2. Therefore, these two buses along with their connecting link will be considered as a new observable island that has a constraint function given as;

$$f_{1-new} = f_1 + f_2 = x_1 + x_2 + x_3 + x_6 + x_7 \geq 1 \quad (57)$$

Hence, the new constraint function for network observability will be given as;

$$f(X) = \begin{cases} f_{1-new} = x_1 + x_2 + x_3 + x_6 + x_7 & \geq 1 \\ f_3 = x_2 + x_3 + x_4 + x_6 & \geq 1 \\ f_4 = x_3 + x_4 + x_5 + x_6 & \geq 1 \\ f_5 = x_4 + x_5 & \geq 1 \\ f_6 = x_2 + x_3 + x_6 & \geq 1 \\ f_7 = x_2 + x_4 + x_7 & \geq 1 \end{cases} \quad (58)$$

*Step 3: Considering Power Injection Measurements*

In this step, injection measurements, including pseudo-measurements for zero injection buses, will be considered in addition to the flow measurements introduced in Step 2.

The analysis of this step is based on the fact that; if the voltage phasors at the buses directly connected

to the bus with injection measurements are known, then the voltage phasor at the bus of injection measurements can be calculated using Kirchhoff's Current Law.

There are two methods to proceed with this step;

- Non-linear constraint formation, and
- Topology transformation.

Topology transformation method is proposed by [5] to maintain the linearity of the problem.

Considering the sample network of Figure (1), bus 3 is shown to be a zero injection bus. Therefore, if the voltage phasors at buses 2, 4 and 6 are known, the same for bus 3 can be calculated using KCL. Hence, bus 3 can be merged with any of the adjacent buses. For the given illustrative example, bus 3 will be merged with bus 6 to form a new bus 6' and the network will be as shown by Figure (2).

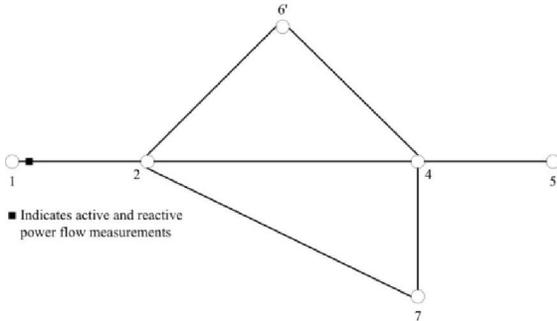


Figure 2 7-Bus Sample Network with Injection Bus Eliminated

The constraint functions for the merged buses 3 and 6 will be eliminated and a new constraint function for the newly introduced bus 6' will be developed as below;

$$f_{6'} = x_2 + x_4 + x_{6'} \geq 1 \quad (59)$$

and the new constraint function for network observability will be given as;

$$f(X) = \begin{cases} f_{1-new} = x_1 + x_2 + x_3 + x_6 + x_7 \geq 1 \\ f_4 = x_3 + x_4 + x_5 + x_6 \geq 1 \\ f_5 = x_4 + x_5 \geq 1 \\ f_{6'} = x_2 + x_4 + x_{6'} \geq 1 \\ f_7 = x_2 + x_4 + x_7 \geq 1 \end{cases} \quad (60)$$

The solution of the objective function given in (50) that is subject to the constraints given by Equation (60) can be obtained using ILP.

The solution may yield the placement of a PMU on one of the fictitious buses (such as 6'). In this case, the PMU can be placed at either of the buses that were merged to create the fictitious bus (i.e. bus 3 or 6).

It is sometimes recommended to add a backup set of PMUs that is redundant to the primary set obtained from above method. The backup set shall be, on its own, capable of maintaining network observability.

The backup set is chosen in a similar manner as the primary set except that all  $x_i$  terms in the constraint function that correspond to buses which were previously selected to have PMUs will be excluded from the new constraint function for the placement of the backup set.

#### B. Limitations to the Existing Approach

The use of the above approach offers the following limitations;

1) The approach is instance-based, that, the solution for the strategic placement of the PMUs is based on certain measurement scheme for the given network topology. Therefore, a change in this scheme, caused by the elimination of a measurement from the network model due to being erroneous, might change the solution obtained. Considering that having erroneous measurements in a network is a dynamic process, the placement of PMUs will dynamically changed with the change of the measurement scheme.

2) Since different types of PMUs are manufactured by different companies with various measuring capabilities, the assumption that each PMU has multi-channel that is capable of measuring bus voltage phasors and line current phasors incident to that bus limits the application of this approach to PMUs having the same features.

The approach is mainly concerned with network observability. However, it does not cover providing critical measurements with redundancy should these measurements be erroneous and eliminated from the network model causing the network not to be fully observable.

#### C. Modifications to Existing PMU OPP Solution using ILP

The modification introduced in this article complements the OPP solution using ILP mentioned above. However, the approach followed by the modified method is intended to overcome the limitations offered by the exiting method and extend the implementation of this method to cover;

- 1) Measurements criticality assessment.
- 2) Identification of erroneous measurements and parameters.

The proposed approach considers that the network to be analyzed is already observable and that the placement of the additional PMUs is intended to maintain its observability even in case of loss of a single measurement. PMUs considered for our study are single-channel type that are capable of measuring only voltage phasors at network buses.

The problem of error identification in cases where there is a strong correlation between a measurement and a parameter which one of them is erroneous is also considered in the proposed approach.

The proposed method as per the description above can maintain the network observability and

error identification capability in case of single error in a parameter or conventional measurement. However, in case of a simultaneous error in a parameter or conventional measurement and their redundant PMU, the method will fail to maintain the function it is intended for. Therefore, the need to provide redundant set of PMUs that replace the original set in case of their total failure may be required. Providing this backup set of PMUs will involve major impact on the network costs. Therefore, it is important to study the network measurement scheme and assess the vulnerability of each measurement to errors. This assessment will be used as a measure to weight the necessity to increase the redundancy level of each measurement.

The use of single-channel type PMUs that are capable of measuring bus voltage phasors is considered in this thesis as it provides more practical implementation of the OPP problem solution using ILP. However, this limited capability of the utilized PMUs will cause each network bus to be initially considered as an observable island on itself.

Recalling the rules for network observability using PMUs [1], an error in PMUs used to connect observable islands can only be identified if at least three PMUs are installed in each observable island. Therefore, the OPP problem formulation explained above will be reconsidered as below;

*Step 1: No Flow or Injection Measurements are Present*

This step assumes that no flow or injection measurements are present in the network. This will help building up the full constraints function.

A new matrix D will be used to identify buses that are part of an observable island. When no flow or injection measurements are considered, each bus will be forming a single observable island on its own. Therefore, matrix D can be defined as;

$$D_{k,m} = \begin{cases} 1 & \text{if } k = m \\ 0 & \text{otherwise} \end{cases} \quad (61)$$

In other words, this matrix will be a diagonal unity matrix.

The constraint function will be given as;

$$f(X) = D \cdot X \geq 3 \quad (62)$$

The general constraint, for this step, is that; at least three PMUs should be placed at each bus in order to render the whole network observable and to enable error identification in any of the PMUs. For the sample network given by Figure (1), the constrain function will be given as:

The '+' operator serves as a logical "OR" which, along with the right-hand side of the inequality, can represent the general constraint mentioned above. For example, the constraint of bus 1 can be interpreted as; At least one PMU can be placed at either bus 1 or 2 to render bus 1 observable.

$$f(X) = \begin{cases} f_1 = x_1 \geq 3 \\ f_2 = x_2 \geq 3 \\ f_3 = x_3 \geq 3 \\ f_4 = x_4 \geq 3 \\ f_5 = x_5 \geq 3 \\ f_6 = x_6 \geq 3 \\ f_7 = x_7 \geq 3 \end{cases} \quad (63)$$

*Step 2: Considering Flow Measurements*

When a flow measurement is introduced at any branch, this implies that the two observable islands connected by this branch can be grouped into one observable island.

The mathematical interpretation of the above is that the two constraint functions of the two buses connected by the measured link can be grouped into one new constraint function which is the "sum" of the two original functions. The "+" operator is used to indicate that the PMU can be placed at any of the buses forming the new observable island.

In the sample network shown by Figure (1), active and reactive measurements are introduced in the link between buses 1 and 2. Therefore, these two buses along with their connecting link will be considered as a new observable island that has a constraint function given as;

$$f_{1-new} = f_1 + f_2 = x_1 + x_2 \geq 3 \quad (64)$$

Hence, the new constraint function will be given as;

$$f(X) = \begin{cases} f_{1-new} = x_1 + x_2 \geq 3 \\ f_3 = x_3 \geq 3 \\ f_4 = x_4 \geq 3 \\ f_5 = x_5 \geq 3 \\ f_6 = x_6 \geq 3 \\ f_7 = x_7 \geq 3 \end{cases} \quad (65)$$

*Step 3: Considering Power Injection Measurements*

This step will process the constraint function in a similar fashion as explained in [5].

Considering the sample network of Figure (1), bus 3 is shown to be a zero injection bus. Therefore, if the voltage phasors at buses 2, 4 and 6 are known, the same for bus 3 can be calculated using KCL. Hence, bus 3 can be merged with any of the adjacent buses. For the given illustrative example, bus 3 will be merged with bus 6 to form a new bus 6' and the network will be as shown by Figure (2).

The constraint functions for the merged buses 3 and 6 will be eliminated and a new constraint function for the newly introduced bus 6' will be developed as below;

$$f_{6'} = x_{6'} \geq 3 \quad (66)$$

and the new constraint function for network observability will be given as;

$$f(X) = \begin{cases} f_{1-\text{new}} = x_1 + x_2 \geq 3 \\ f_4 = x_4 \geq 3 \\ f_5 = x_5 \geq 3 \\ f_{6'} = x_{6'} \geq 3 \\ f_7 = x_7 \geq 3 \end{cases} \quad (67)$$

The solution of the objective function given in (50) that is subject to the constraints given by Equation (67) can be obtained using ILP.

The solution may yield the placement of a PMU on one of the fictitious buses (such as 6'). In this case, the PMU can be placed at either one of the buses 3 or 6.

Critical measurements are identified in earlier literatures [6] as being those measurements whose removal from the measurement set will render the network unobservable. Additionally, the method presented in [1] and improved in this article has identified another source of unidentifiable measurements and as being those measurements that are strongly correlated with network parameters.

Measurement defined to be critical for network observability can be identified as those measurements having null columns in the measurement error covariance matrix  $\Omega$  as defined by Equation (23) [5].

For measurements that are critical to network observability, an incidence matrix B can be constructed such that;

$$B_{k,m} = \begin{cases} 1 & \text{if } \Omega_{k,m} = 0 \\ 0 & \text{otherwise} \end{cases} \quad (68)$$

A constraint function  $g(X)$  can, then, be obtained to provide the constraint for the placement of PMUs to increase the redundancy of these measurements.

$$g(X) = B \cdot X \geq 1 \quad (69)$$

For measurements that are strongly correlated with parameters, these can be identified as the critical measurement-parameter pair that has a correlation coefficient equals to unity. The correlation coefficients between measurements and parameters can be obtained from the associated matrix  $R_{mp}$  that can be obtained from Equations (34) and (43). Therefore, an incidence matrix C can be constructed such that;

$$C_{k,m} = \begin{cases} 1 & \text{if } R_{mp_{k,m}} = 1 \\ 0 & \text{otherwise} \end{cases} \quad (70)$$

A constraint function  $j(X)$  can, then, be defined to provide the constraint for the placement of PMUs to enable the identification of errors occurring in either of the correlated quantities.

$$j(X) = C \cdot X \geq 1 \quad (71)$$

The overall constraint function  $F(X)$  can be formed by combining the constraint functions  $f(X)$ ,  $g(X)$  and  $j(X)$  given by Equations (62), (69) and (71) respectively. Thus,

$$F(X) = \begin{cases} f(X) \geq 3 \\ g(X) \geq 1 \\ j(X) \geq 1 \end{cases} \quad (72)$$

#### D. Solution Algorithm

- 1) Run the algorithm presented in Section III (D).
- 2) Build the binary decision vector X as explained in Equation (54).
- 3) Built the bus incidence matrix A as explained by Equation (52).
- 4) Construct the basic network observability constraint function  $f(X)$  as explained by Equation (53) considering the no power flow or injection measurements are available.
- 5) Refine  $f(X)$  to include the available power flow measurements.
- 6) Refine  $f(X)$  to include power injection measurements including zero (pseudo) injections.
- 7) Construct the incidence matrix for critical measurements B as explained by Equation (68) and calculate the relevant constraints function  $g(X)$  as explained by Equation (69).
- 8) Construct the incidence matrix for critical measurement-parameter pairs C as explained by Equation (70) and calculate the relevant constraint matrix  $j(X)$  as explained by Equation (71).
- 9) Construct the overall constraint function  $F(X)$  as explained by Equation (72) and use it along with the objective function given by Equation (50) to obtain the ILP solution.

In this algorithm, the impact of installation costs of each PMU is not considered. Therefore, all  $w_i$ s will be given equal value.

#### E. Simulation Results

##### Test Case 3: IEEE 14-Bus Test Network

This test is applied to IEEE 14-bus test network with a measurement scheme that offers a completely measured network. Therefore, no critical measurements exist for this test case.

Since the resulting requirement for PMUs are not related to network observability and measurement criticality, it should be expected that the proposed PMU is resulting from strongly correlated measurement-parameter pairs. Upon examining the measurements-parameters correlation coefficient matrix  $R_{mp}$ , the following table shows the measurement-parameter pairs that have a correlation coefficient of unity.

**Table VIII Test Case 3: Measurement-Parameter Pairs with Unity Correlation Coefficients**

| Measurements | Parameters     |
|--------------|----------------|
| $P_{7-8}$    | $r_{7-8}$      |
| $P_{8-7}$    | $r_{7-8}$      |
| $Q_{8-7}$    | $b_{sh_{7-8}}$ |
| $Q_{8-8}$    | $b_{sh_{7-8}}$ |

The resulting binary decision vector calculated

for the given network measurement scheme has indicated that only bus 8 is required to be provided with PMU. It can be seen from the above table that the placement of a PMU at bus 8 will help identifying an error that may occur at any of the strongly correlated measurements and parameters.

#### Test Case 4: IEEE 30-Bus Test Network

This test is applied to IEEE 30-bus test network with a measurement scheme that offers a completely measured network. Therefore, no critical measurements exist for this test case.

Since the resulting requirement for PMUs are not related to network observability and measurement criticality, it should be expected that the proposed PMU is resulting from strongly correlated measurement-parameter pairs. Upon examining the measurements-parameters correlation coefficient matrix  $R_{mp}$ , the following table shows the measurement-parameter pairs that have a correlation coefficient of unity.

**Table IX Test Case 4: Measurement-Parameter Pairs with Unity Correlation Coefficients**

| Measurements | Parameters       |
|--------------|------------------|
| $P_{9-11}$   | $r_{9-11}$       |
| $P_{11-9}$   | $r_{9-11}$       |
| $Q_{10-10}$  | $b_{sh_{10-10}}$ |
| $Q_{11-9}$   | $b_{sh_{9-11}}$  |
| $Q_{11-11}$  | $b_{sh_{9-11}}$  |
| $Q_{13-12}$  | $b_{sh_{12-13}}$ |
| $Q_{24-24}$  | $b_{sh_{24-24}}$ |
| $P_{26-25}$  | $r_{25-26}$      |
| $P_{26-26}$  | $r_{25-26}$      |
| $Q_{26-25}$  | $x_{25-26}$      |
| $Q_{26-26}$  | $x_{25-26}$      |

The resulting binary decision vector calculated for the given network measurement scheme has indicated that only buses 10, 11, 12, 24 and 26 are required to be provided with PMUs. It can be seen from the above table that the placement of a PMUs at buses 10, 11, 12, 24 and 26 will help identifying an error that may occur at any of the strongly correlated measurements and parameters.

#### Conclusion and Future Work

The approach presented by this article has successfully implemented an optimized PMU placement solution to accommodate the requirements

for network observability and errors identification. Although the article has provided a mean to calculate the cross-correlation between measurements and parameters, the method proposed was not useful to apply the same decorrelation technique used with measurement-parameter k-tuples. Therefore, it is recommended that future works may deduce a combined covariance matrix from which the measurements and parameters cross-correlation coefficients may be calculated and the deceleration technique may be applied.

The procedure for obtaining optimally placed backup set of PMUs was presented in [5]. Although this procedure will increase the redundancy of the measurements obtained from the PMUs, its application is only limited to where network observability is of concern. The need to provide redundancy for the existing measurement scheme depends mainly on the vulnerability of the existing measurements to be prone to error. This vulnerability varies from one measurement to another. Therefore, it would be useful if measurement vulnerability weighting factor is developed and introduced to the basic ILP problem formulation to control the redundancy level required for each measurement.

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