Flexural-torsional Interaction about R.C. Interfacial Beams Neighboring Slabs of Different Depths

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Abstract: Structural analysis of reinforced concrete elements is the first step in the design process, where straining actions, of different types, are extracted to figure out the stress condition within structural elements. Interfaces between various structural elements or elements of same type, but different configuration (dimensions, characteristic strength, or reinforcement) present a typical analysis argument. This paper deals with interfaces between slabs of different depths on the two sides of an interfacial beam. This case usually presents in case of slab depression or cantilever slabs, where the cantilever is deeper than the neighboring slab. The most severe case presents when a cantilever is neighbored by a hollow block slab, in this case the maximum difference between slabs depths takes place. The problem has been mathematically formulated, based on a non-linear concrete stress strain hypothesis, as presented in the Egyptian code of Practice. A finite element model has been prepared to study the effect of beam torsional rigidity on moment transfer. Moreover a parametric study has been established to clarify the significance of various structural parameters on the behavior of interfacial beams.

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Keywords: Flexure, torsion, concrete slab.

1. Introduction

Torsional analysis of reinforced concrete beams has been presented in the literature in two main forms. The first is the truss model which has first been introduced in the beginning of the 20th century[1]. Later on, so many models have been presented in the literature. One of the most comprehensive three dimensional truss models was that proposed by Hsu[2], where his proposed truss model could evaluate both shear and torsion, as shown in figure (1).

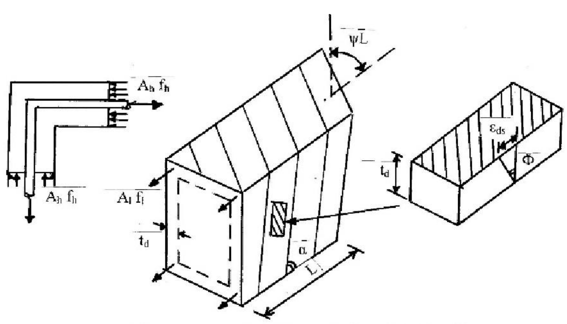


Figure 1. 3 D truss model [Hsu[1]]

The second method for analysis of torsion in reinforced concrete beams was via equilibrium equations, regarding forces in both concrete and reinforcement. Mitchell and Collins[2]presented a simple model based on equilibrium equations and boundary conditions, concrete stress strain geometry, and deformed shape. Figure (2) and (3) present both strain and stresses diagrams, as proposed by Collin's[2]. Khaldoun[3,4,5] has evaluated the modified compression field theory to take into consideration effects of variations in longitudinal strains, as a result of flexure, as shown in figure (4).

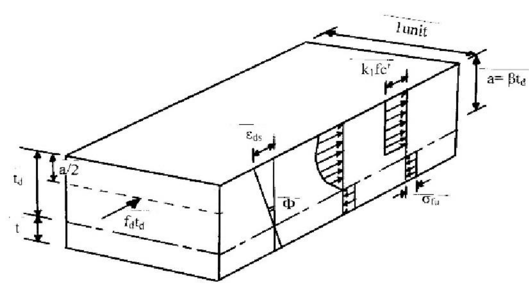


Figure 2. Torsional equilibrium model longitudinal Direction.

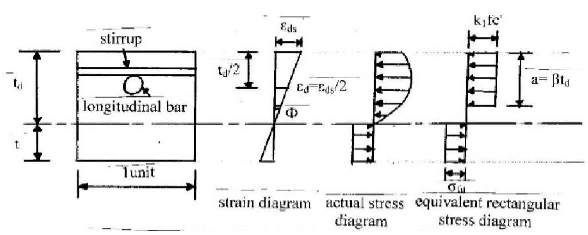


Figure 3. Torsional equilibrium model transversal Direction.

This paper has been prepared to evaluate the generated torsion within interface beam, between slabs of different depths; resulting from difference in flexural capacities and location in resultant concrete compression. Figure (5) presents that typical case for a deep cantilever one way slab neighboring a slimmer internal one way slab, where an interfacial beam exists in-between. The main research target is determining the percentage of slab flexure to be transferred to torsion in the interfacial beam, as a result of the difference in slab depths on both sides of the interfacial beam. Two approaches have been adopted to tackle the problem.

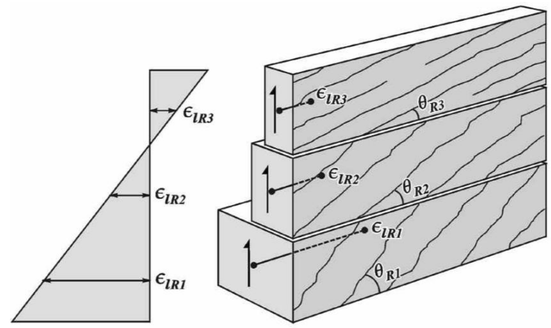


Figure 4. Variation in longitudinal strain across a wall in a beam subjected to torsion

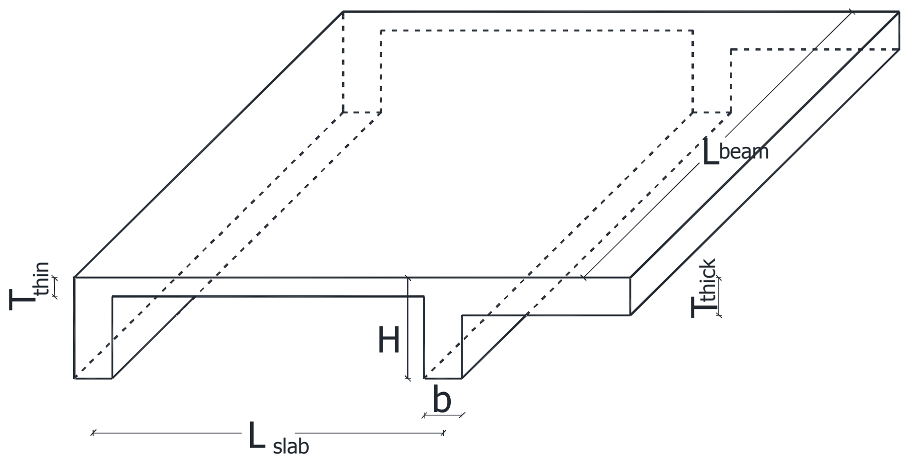


Figure (5) Interfacial beam neighbored by slabs of different depths

The first wasapproach was direct mathematical formulation for stresses in both reinforcement and concrete. Both parabolic and parabolic-rectangular stress-strain relations presented in ECP 203[16] have been considered. Hogging reinforcement was assumed to be extending from side to another by the same configuration. The first approach considered equilibrium equations across the two neighboring slabs have been considered, then evaluated the difference between flexural capacities of both slabs, and considered that difference is to transfer to the beam in the form of torsional moment.

The second approach was finite element modeling. Where a non-linear solid elements model has been presented to study effects of relative beam torsional stiffness to slab flexural stiffness on moment transfer from slab to neighboring beam. After formulating the mathematical approach and preparing the finite element model a verification procedure has been implemented between the both techniques, then a parametric study has been conducted to figure out the significance of various structural parameters on flexural torsional interaction across the interfacial beam.

2. Mathematical Formulation

The mathematical formulation has been prepared based on the equilibrium about the beam slabs node. Figure (6) shows a free body diagram of the considered beam-slab node.

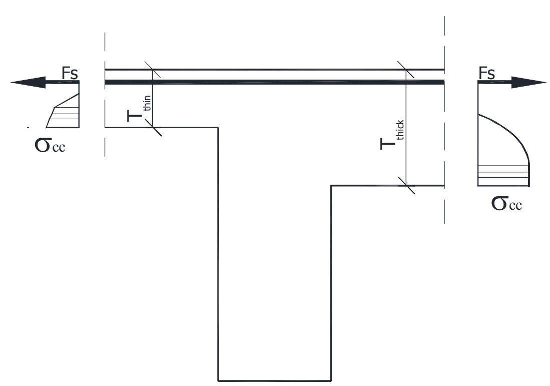


Figure 6. Free body diagram of the beam-slab node

Analysis of the node shown in figure (6) requires evaluation of bending moments across the beam from both sides. Figure (7) represents the parabolic distribution of concrete stress across the section subjected to flexure, while figure (8) represents the parabolic-rectangular distribution. Evaluation of bending moment could be done using equations (1), (2), (3), and (4); for any level of reinforcement strain considering the hypothesis of plane section rotation to represent the flexural behavior. Equations (1) and (2) could be used for evaluation both force and location of Cg for concrete parabolic stress distribution, while equations (3) and (4) could be used for evaluation both force and location of Cg for concrete parabolic-rectangular stress distribution. The four equations have been formulated by the integration of the concrete stress-strain relations to extract the force and cg location. It should be noted that the analysis of using the mathematical formulation does not take into consideration effects of both flexural rigidities of slabs and torsional rigidity of interfacial beam.

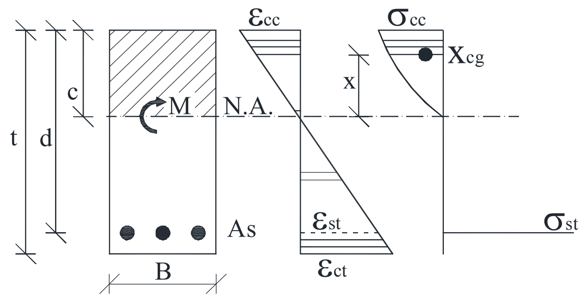


Figure 7 Concrete parabolic stress distribution

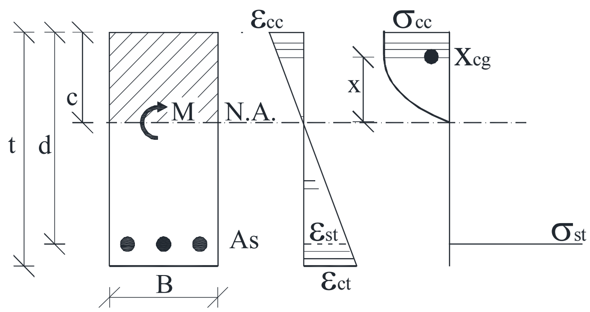


Figure 8 Concrete parabolic-rectangular stress distribution

$$F=\left(\frac{-167500 f_{cu}\varepsilon_c^2}{c^2}\frac{x^3}{3}+\frac{670 f_{cu}\varepsilon}{c}\frac{x^2}{2}\right)*\mathrm{B} \quad \_\_\_\_\_\_\_\_(1)$$

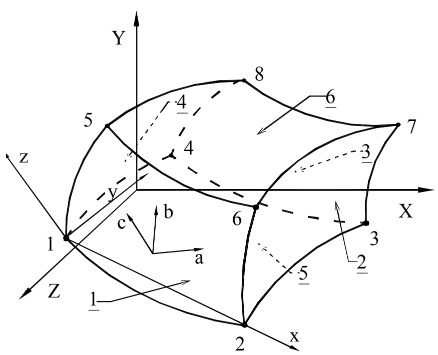
$$x_{cg}=\frac{\dfrac{-167500 f_{cu}\varepsilon_c^2}{c^2}\dfrac{x^2}{4}+\dfrac{670 f_{cu}\varepsilon}{c}\dfrac{x}{3}}{\dfrac{-167500 f_{cu}\varepsilon_c^2}{c^2}\dfrac{x}{3}+\dfrac{670 f_{cu}\varepsilon_c}{2c}} \quad \_\_\_\_\_\_\_\_\_\_\_(2)$$

$$F=\left[\frac{-167500 f_{cu}\varepsilon_c^2}{\left(\frac{0.002c}{\varepsilon_c}\right)^2}\frac{x^3}{3}+\frac{670 f_{cu}\varepsilon_c}{\frac{0.002c}{\varepsilon_c}}\frac{x^2}{2}+\left(c-\frac{0.002}{\varepsilon_c}c\right)\left(\frac{-167500 f_{cu}\varepsilon_c^2}{\left(\frac{0.002c}{\varepsilon_c}\right)^2}x^2+\frac{670 f_{cu}\varepsilon}{\frac{0.002c}{\varepsilon_c}}x\right)\right]*\mathrm{B} \quad \_\_\_(3)$$

$$\mathrm{X}_{cg}=\frac{\left(\frac{-167500 f_{cu}\varepsilon_c^2}{\left(\frac{0.002c}{\varepsilon_c}\right)^2}\frac{x^3}{3}+\frac{670 f_{cu}\varepsilon_c}{\frac{0.002c}{\varepsilon_c}}\frac{x^2}{2}\right)\left[\dfrac{\frac{-167500 f_{cu}\varepsilon_c^2}{\left(\frac{0.002c}{\varepsilon_c}\right)^2}\frac{x^2}{4}+\frac{670 f_{cu}\varepsilon}{\frac{0.002c}{\varepsilon_c}}\frac{x}{3}}{\frac{-167500 f_{cu}\varepsilon_c^2}{\left(\frac{0.002c}{\varepsilon_c}\right)^2}\frac{x}{3}+\frac{670 f_{cu}\varepsilon}{\frac{0.002c}{\varepsilon_c}}\frac{1}{2}}\right]+c\left(1-.\frac{0.002}{\varepsilon_c}\right)\left(\frac{-167500 f_{cu}\varepsilon_c^2}{\left(\frac{0.002c}{\varepsilon_c}\right)^2}x^2+\frac{670 f_{cu}\varepsilon}{\frac{0.002c}{\varepsilon_c}}x\right)\left[\frac{c(0.002+\varepsilon)}{2\varepsilon}\right]}{\left(\frac{-167500 f_{cu}\varepsilon_c^2}{\left(\frac{0.002c}{\varepsilon_c}\right)^2}\frac{x^3}{3}+\frac{670 f_{cu}\varepsilon}{\frac{0.002c}{\varepsilon_c}}\frac{x^2}{2}\right)+c\left(1-.\frac{0.002}{\varepsilon_c}\right)\left(\frac{-167500 f_{cu}\varepsilon_c^2}{\left(\frac{0.002c}{\varepsilon_c}\right)^2}x^2+\frac{670 f_{cu}\varepsilon}{\frac{0.002c}{\varepsilon_c}}x\right)} \quad \_\_\_\_\_(4)$$

3. Finite Element Modelling

A finite element model has been prepared using Ansys. The model used solid elements to represent the concrete mass of both slabs and beam. Solid 65 has been chosen as a basic modeling element. Figure (9) shows solid 65 configuration. The finite element model has been prepared to represent slab depth ratios ranging from one up to five. End conditions have been defined for both beam and slabs to represent the required variation in relative stiffness. Figure (10) shows a cross section in the finite element model.



XYZ	Global Cartesian Coordinate System
xyz	Element Coordinate System
abc	Material Coordinate System
1,2,3,4,5,6	Face Numbers for Applying Loads

Figure 9 Solid element configuration

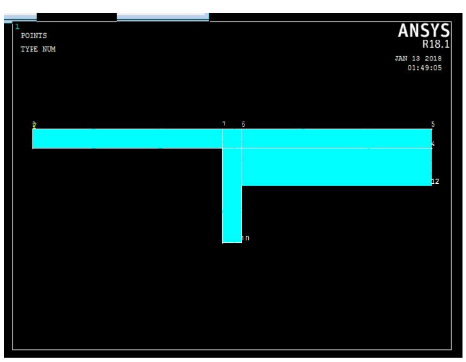


Figure 10 Finite element model

A nonlinear analysis using a time linearly variable load curve, as shown in figure (11) was adopted.

4. Models verification

There was a lack in the literature, regarding testing torsional behavior of reinforced concrete beams, resulting from being surrounded by concrete slabs of different depths. That is why the verification process has been performed between the mathematical formulation and the finite element analysis. Five models, representing the most practical slab depth ratios, ranging from (1) up to (5), have been prepared. A beam and an interior neighboring slab of the same stiffness have been modeled using both finite element and mathematical approaches. Figure (12) represents outputs of beam torsional to slab flexural moments for both finite element and mathematical approaches. It could be noticed that both models coincide when the exterior slab thickness is double the interior one (slab ratio = 2). While for slab ratios below two, the mathematical model results in torsional to flexural ratios below the finite element. On the other hand for slab ratios exceeding two the mathematical modeling results in ratios above the finite element. Average tolerance in $M_{torsion}/ M_{flexure}$ between both models is in the order of 10%

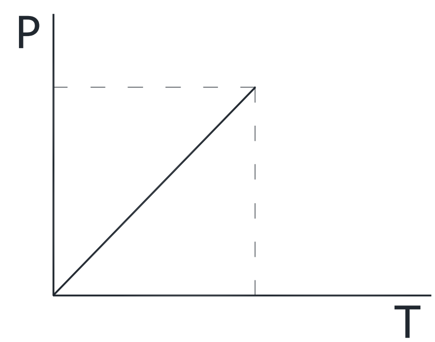


Figure 11 F.E. load curve

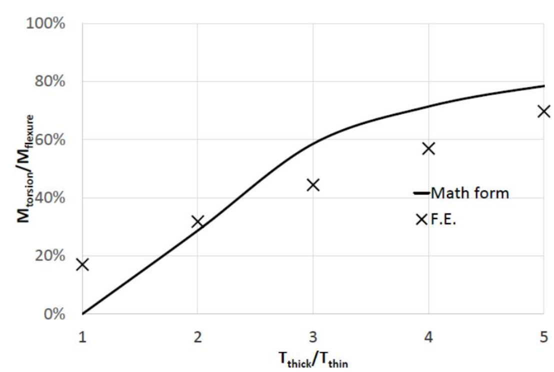


Figure 12 Models verification

5. parametric study

After validation of analysis tools (mathematical formulation and finite element models) a parametric study has been conducted to figure out the significance of various parameters on the flexural-torsional interaction between slabs and interfacial beam. The main output under consideration was the percentage of moment transferred by torsion to the interfacial beam to the moment crossing the beam across the neighboring slabs by flexure. The parametric study adopted the level of loading, reinforcement ratio, slab flexural failure load, thickness ratio between neighboring slabs, and slab flexural to beam torsional rigidities; as studied parameters.

Reinforcement ratio has been studied using the mathematical model, where models with maximum allowed reinforcement ratios (2/3 balanced reinforcement) and minimum reinforcement ratio (ensuring ductile failure) have been studied. Figure (13) shows percentage of moment transfer to beam torsion for various depths ratios. It was found that the full load has been transferred between slabs across interfacial beam, by flexure; in case of depth ratio equals to one. While in case of slabs depth ratios exceeding 20 all flexure at the thicker side has been transferred to torsion about the interfacial beam. The rate of increase in flexure transfer to torsion is high for depth ratios from one to three and low for depth ratios exceeding six, while intermediate between three and six.

Moreover it could be noticed that the more the slab reinforcement ratio the more the percentage of flexure transfer to torsion. This could be attributed to that in case of low reinforcement ratio concrete stress distribution follows the parabolic distribution, for both thick and thin slabs; while for high reinforcement ratios concrete stress distribution follows the parabolic-rectangular distribution for the thin slab and parabolic distribution for thick slab. Since the distance between tensiled steel reinforcement and compressed concrete C.G. increases in case of concrete parabolic stress distribution than parabolic-rectangular one; then difference between thick and thin slabs flexural capacities increase in case of high reinforcement ratios.

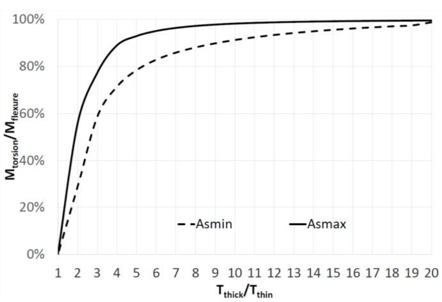


Figure 13. Flexural-torsional interaction as a function of slabs depths ratio for high and low reinforcement ratios

The case of moment transfer from the thinner slab to the thicker one has also been studied. According to the mathematical formulation; this case does not transfer slab flexure to beam torsion, if reinforcement is overlaid above the beam, between the two neighboring slabs. In this case the full slimmer slab flexure is transferred to the thicker one, representing only a portion of thicker slab flexural capacity. This could be presented as a decay in tensile reinforcement strain within the thicker slab. Tensile reinforcement strain at failure has been traced by the mathematical formulation approach, where the failure has been defined, based on the thinner slab flexural capacity. An upper limit of elastic reinforcement stress-strain relation has been defined by 0.2% (yield of reinforcement steel at stress of 400MPa). Reinforcement ratio, based on thin slab configurations, was taken into consideration as a co-parameter. Figure (14) represents reinforcement tensile strain release at failure for different slab thickness ratios. Relation between thicker slab reinforcement strain at which slimmer slab reaches its flexural capacity and relative slab thickness appeared non-linear, where the more the slab depth ratio the less the failure steel reinforcement strain. Moreover the more the reinforcement ratio the less the reinforcement strain at slab failure.

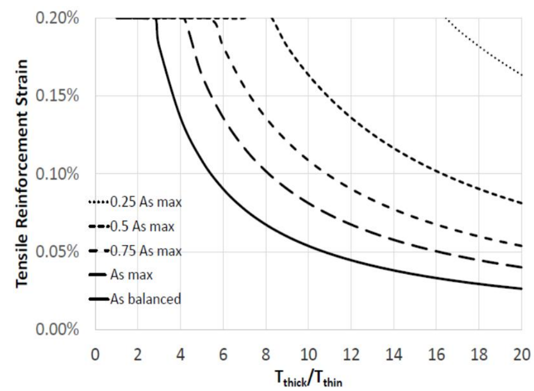


Figure 14. Thick slab reinforcement tensile strain at slim slab failure

Since the studied parameter is the ratio between moments transferred to torsion to that crossing the interfacial beam; parameters with identical values in both slabs are insignificant. That is why concrete characteristic strength is insignificant for all depth ratios, as shown in figure (15).

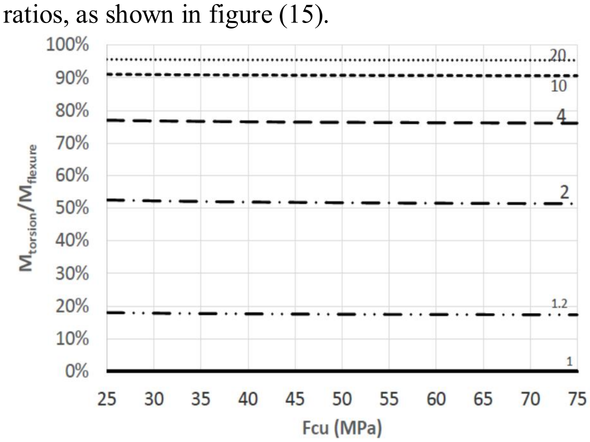


Figure 15. Effect of Fcu on torsional-flexural interaction for various depth ratios.

Slab flexural to beam torsional rigidities is a highly significant parameter, where it judges the distribution of moments between slabs (flexure) and beam (torsional), for consistent problem configurations, based on end conditions. This parameter has been tackled using the finite element analysis. Models including slab ratios ranging from two up to five (most practical range) have been prepared. Slab flexural stiffness relative to beam torsional stiffness ranging from (0.001) to (1000) have been considered. Values of moment transferred to beam have been extracted at the point, once the thinner slab snaps into failure. It was found that beam torsional to slab flexural moment ratio is dependent on neighboring slabs relative depths for, for the same relative stiffness, where the more the relative slab depth the more the percentage of flexure transfer into torsion. In addition the more the beam to slab stiffness the more the increase of moment transferred by torsion to that by flexure. Figure (16) represents torsional to flexural moment ratio as a function of slab-beam relative stiffness, for different slab depth ratios. It could be noticed that relative slab depths is a significant parameter for percentage of slab flexure transfer into beam torsion, in case of slab-beam relative stiffness ranging from 0.1 up to 100. Moreover it is obvious that in case of slab to beam stiffness exceeding 100 all the thicker slab flexure should transfer to the slimmer one, while in case of ratios below 0.001 all the thicker slab flexure will transfer to the form of beam torsion.

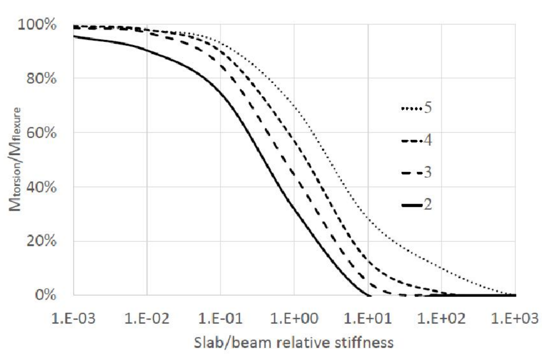


Figure 16. Torsional-flexural interaction for different depth and stiffness ratios

6. Conclusions

Studying outputs of the parametric study, resulted in the following conclusions.

1- The proposed mathematical model could be used to evaluate torsional moments in interfacial beams, neighboring slabs of different depths when both slimmer slab and interfacial beam have the same stiffness.

2- The more the reinforcement ratio within the neighboring slabs the more the torsional moment transfer.

3- No flexure interaction takes place between neighboring slabs of depth ratio exceeding 20.

4- Torsion-flexure interaction is independent of concrete characteristic strength.

5- The more the neighboring slab depth ratio the less the failure steel reinforcement strain, for moments transferred from slimmer to thicker slabs.

6- Slab-beam relative stiffness is a highly significant parameter, regarding beam-slab torsional flexural interaction.

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