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Analysis of difference equations via Laplace Transform Technique

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Abstract: In this paper we will analyze difference equations via Laplace transform technique. These equations appear in most of the science and engineering disciplines like Biology, Economics, circuit analysis, numerical analysis, electrical and electronics engineering etc. The paper inquires the difference equations by Laplace transform technique. The purpose of paper is to prove the applicability of Laplace transform technique to analyze the difference equations.

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I. Introduction

A difference equation is a relation between the differences of unknown function at one or more general values of the independent variable [1, 2, 3, 4, 5, 6, 7]. Laplace transformation is very useful tool in various disciplines of science and engineering [8, 9, 10, 11, 12, 13, 4, 15]. It also comes out to be very effective tool to analyze the difference equations which appear mostly in science and engineering disciplines like Biology, Economics, circuit analysis, numerical analysis, electrical and electronics engineering etc [16, 17, 18, 19, 20, 21, 22]. These are generally solved by adopting Numerical method or by Z- transform, Matlab etc. In this paper, we present a Laplace transform technique to solve the difference equations.

II. Basic Definition

The Laplace transformation [6, 7] of f (t), t ≥ 0, denoted by f (p) or L {F (t)}, is defined as L {F (t)} $=\int_0^\infty e^{-pt} F(t)dt$, provided that the integral exists, i.e. convergent.

III. Methodology

Difference Equation

Let $f(t) = b^{[t]}$, where $[t]$ is the greatest integer less than or equal to t and $b > 0$ then $f(t)$ is a exponential order [1, 2, 3] and by definition

L {f (t)} $=\int_0^\infty e^{-pt} F(t)\,dt$

L {f (t)} $=\int_0^\infty e^{-pt} b^{[t]} dt$

$= \int_0^1 e^{-pt} b^0 dt + \int_1^2 e^{-pt} b^1 dt + \int_2^3 e^{-pt} b^2 dt$

$= \frac{1 - e^{-p}}{p} + b\frac{(e^{-p} - e^{-2p})}{p} + b^2\frac{(e^{-2p} - e^{-3p})}{p} + \cdots,$

$= \frac{1 - e^{-p}}{p}[1 + be^{-p} + b^2 e^{-2p} + \cdots]$

$= \frac{1 - e^{-p}}{p} \cdot \frac{1}{1 - be^{-p}}$

Hence $L\{F(t)\} = \frac{1 - e^{-p}}{p(1 - be^{-p})}$

We know that,

$L\{F(T)\} = \bar{f}(p)$

Then by Heaviside expansion [1, 2]

$L^{-1}\{\bar{f}(p)\} = f(x) = \begin{cases} f(t-1), & t > 0 \\ 0, & t < 0 \end{cases}$

Also,

$\left\{\frac{1 - e^{-p}}{p(1 - be^{-p})}\right\} = b^n, for\ n = 0,1,2, \ldots$

And

$\left\{\frac{1 - e^{-p}}{p(1 - be^{-p})^2}\right\} = nb^{n-1}, for\ n = 0,1,2, \ldots$

(A) Solve the difference equation

$$x_{n+2} - 42x_{n+1} + 441\, x_n = 0$$

Or

$$x(t+2) - 42x(t+1) - 441x(t) = 0 \quad \ldots. (1)$$

We know that $L\{F(t)\} = \bar{f}$ (p) then

$$L^{-1}\{e^{-p}\bar{f}(p)\} = \begin{cases} f(t-1), t > 1 \\ 0, t < 1 \end{cases}$$

Also,

$$L^{-1}\left\{\frac{1-e^{-p}}{p(1-be^{-p})}\right\} = b^n \ for\ n = 0,1,2 \ldots.$$

$$And\ L^{-1}\left\{\frac{e^{-p}(1-e^{-p})}{p(1-be^{-p})}\right\} = nb^{n-1}$$

$for\ n = 0,1,2 \ldots.$

Now,

$$Lx(t+2) = \int_0^\infty e^{-pt}x(t+2)\,dt$$

Let $(t+2) = u, dt = du$, then

$$Lx(t+2) = \int_0^\infty e^{-pt}x(t+2) = \int_2^\infty e^{-p(-2+u)}x(u)\,du$$

$$= e^{2p}\int_0^\infty e^{-pu}x(u)\,du - e^{2p}\int_0^2 e^{-pu}x(u)\,du$$

On solving, we get,

$$Lx(t+2) = e^{2p}L\{x(t)\} - \frac{e^p}{p}(1-e^{-p})$$

And,

$$Lx(t+1) = e^p L\{x(t)\}$$

From (1),

$$x(t+2) - 42x(t+1) - 441x(t) = 0$$

Taking Laplace Transform [1, 2] on both sides

$$L\{x(t+2)\} - 42L\{x(t+1)\} - 441L\{x(t)\} = 0 \ \ldots..(2)$$

Putting the value of

$L\{x(t+2)\}, L\{x(t+1)\}$ in (2)

We get,

$$(e^{2p} - 42e^p + 441)L\{x(t)\} = \frac{e^{2p}}{p}(1-e^{-p})$$

$$L\{x(t)\} = \frac{e^{2p}(1-e^{-p})}{p(e^{2p} - 42e^p + 441)} \ \ldots..(3)$$

Or

$$x(t) = L^{-1}\left\{\frac{e^{2p}(1-e^{-p})}{p(e^{2p} - 42e^p + 441)}\right\}$$

Or

$$x(t) = L^{-1}\left\{\frac{e^{-p}(1-e^{-p})}{p(1-21e^{-p})^2}\right\}$$

Or

$$x(t) = n(21)^{n-1},$$

Here n = 0, 1, 2……

Since

$$L^{-1}\left\{\frac{1-e^{-p}}{p(1-be^{-p})^2}\right\} = nb^{n-1}, for\ n = 0,1,2, \ldots$$

(B) Solve the Differential difference equation

$$x'(t) - 2x(t-1) = t, x(t) = 0, t \le 0$$

Taking Laplace Transform [1, 8] on both sides, we get

$$L\{x'(t)\} - 2\{x(t-1)\} = L\{t\}$$

Or

$$\bar{x}(p) - x(0) - 2e^{-p}\bar{x}(p) = \frac{1}{p^2}$$

Or

$$(p - 2e^{-p})\bar{x}(p) = \frac{1}{p^2}$$

Or

$$\bar{x}(p) = \frac{1}{p^2(p-2e^{-p})}$$

Or

$$\bar{x}(p) = \frac{1}{p^3\left(1 - \frac{2e^{-p}}{p}\right)}$$

Or

$$\bar{x}(p) = \frac{1}{p^3}\left[1 + \frac{2}{p}e^{-p} + \frac{4}{p^2}e^{-4p} + \frac{8}{p^3}e^{-6p} \ldots\ldots\ldots\right]$$

Or

$$\bar{x}(p) = \sum_{n=0}^{\infty}\frac{2^n e^{-2np}}{p^{n+3}}$$

Or

$$x = L^{-1}\sum_{n=0}^{\infty}\frac{2^n e^{-2np}}{p^{n+3}}$$

But, $L^{-1}\left\{\frac{e^{-2np}}{p^{n+2}}\right\} = \frac{(t-2n)^{n+1}}{n+1!}$, t> $2n$

Hence,

$$L^{-1}\sum_{n=0}^{\infty}\frac{2^n e^{-2np}}{p^{n+3}} = \frac{2^n(t-2n)^{n+1}}{n+1!}$$

Therefore, if *f*[t] denotes the greatest integer less than is equal to t, then

$$x(t) = \sum_{n=0}^{[t]}\frac{2^n(t-2n)^{n+1}}{n+1!}$$

(C) Solve the Differential difference equation

$$x''(t) - x(t-1) = f(t), x(t) = 0,\ x'(t), t \le 0$$

Where, $f(t) = \begin{cases} 0,\ t \le 0 \\ 11t,\ t > 0 \end{cases}$

Taking Laplace Transform [2, 7] on both sides, we get

$$L\{x''(t)\} - L\{x(t-1)\} = L\{f(t)\}$$

Or

$$p^2\bar{x}(p) - px(0) - e^{-p}\bar{x}(p) = \frac{11}{p^2}$$

$$(p^2 - e^{-p})\bar{x}(p) = \frac{11}{p^2}$$

Or

$$\bar{x}(p) = \frac{11}{p^2(p^2 - e^{-p})}$$

Or

$$\bar{x}(p) = \frac{11}{p^4}\left[1 - \frac{e^{-p}}{p^2}\right]^{-1}$$

Or

$$\bar{x}(p) = 11\left[\frac{1}{p^4} + \frac{e^{-p}}{p^6} + \frac{e^{-p}}{p^8} + \cdots \ldots\ldots\right]$$

Or

$$\bar{x}(p) = 11\sum_{n=0}^{\infty}\frac{e^{-np}}{p^{2n+4}}$$

But, $L^{-1}\left\{\frac{e^{-np}}{p^{2n+4}}\right\} = \frac{(t-n)^{2n+3}}{2n+3!}$

Hence,

$$x(t) = 11\sum_{n=0}^{[t]}\frac{(t-n)^{2n+3}}{2n+3!}$$

IV. Conclusion

This paper presents the use of Laplace transform technique to analyze the difference equations. It may be finished that the technique is very foremost and accomplished in analyzing the difference equations appearing in science and engineering disciplines like Biology, Economics, circuit analysis, electrical and electronics engineering etc. It may be finish that this technique is very foremost and accomplished in solving the difference equations.

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