Novel Switching H_2/H_∞ Control: Combination of Dwell Time Switching Signal and Multiple Lyapunov Function

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Abstract: In this paper, a switching strategy is employed to solve the H_2/H_∞ multi objective controller design. Two controllers are designed to meet the H_2 and H_∞ performance specifications. Linear matrix inequalities are used in the controller design process. New switching signal is defined which is the combination of dwell time switching signal and multiple Lyapunov function such that stability of closed loop system is guaranteed as well as desired performance. Simulation results show that proposed switching strategy improves the performance of the controller and reduces the conservation in comparison with the common H_2/H_∞ controller.

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1. Introduction

There has been increasing interest in hybrid control in recent years, due to its potential to overcome limitations of adaptive control and benefits in controlling of systems that cannot achieve the desired performance by a single controller. Indeed, hybrid control scheme provides an effective mechanism when facing large modelling uncertainty and highly complex systems. Even for simple linear time invariant systems, controllers switching can be utilized in improving the performance (Sun, 2005, Feuer, 1997, and McClamroch, 2000). To date, Morse, Hespanha and Liberzon have established a theoretical backbone for hybrid controllers (Morse, 1997, Hespanha, 1999, and Liberzon, 2003). By now, stabilizing a continuous system via hybrid output feedback has attracted a number of authors, such as (Santarelli, 2008) where a comparison between the responses of the switching controller and two other forms of LTI control have been made. An experimental assessment of controller switching with state and control magnitude constraints is carried out in Kogiso, 2004. In Zheng, 2006, the multi-objective robust control of an induction motor with tracking and disturbance rejection specifications is proposed via switching. In Essounbouli, 2006, DeCarlo, 1988, and Jamshidi, 2010 controller switching has been proposed to improve the trade-offs in design multi objectives.

Supervisory control employs logic-based switching for adaptation, instead of continuous tuning of parameters as in conventional adaptive control. This type of switching-based supervisory control scheme consists of the following subsystems: a plant to be controlled, a bank of controllers, and a switching logic. Dwell-time method is representative of the trajectory independent switching logic for supervisory control (see Yoon, 2007 and its references). On the other hand, Lyapunov functions are employed in such trajectory dependent switching methods as in Yoon, 2007.

In an actual engineering control problem, different contradictory requirements must be satisfied such as attenuation of various types of disturbances, set point tracking, bounds on the signal peaks, and robustness to changing conditions and plant uncertainties. The synthesis problems with a combination of performances are known as multi objective control. General multi objective control problems are difficult and remain mostly open up to date. The usual approach for the general multi objective control problem is to find a controller transfer matrix for all objective designs and to use the same Lyapunov matrix for the separate design specifications. Though meeting all the objectives of a control application is desirable, the design of a single multi-objective controller is a trade-off among competitive problems such as disturbance rejection, tracking, regulation, constraints of the signals…. So a single controller may be restrictive (Boyd, 1991, Scherer, 1997, and Khargonekar 1991).

The mixed H_2/H_∞ control is an important robust control method and has been studied by many researchers. The mixed H_2/H_∞ control is concerned with the design of a controller that minimizes the H2 performance of the system with respect to some inputs while guarantees certain worst case H_∞ performance with respect to other inputs. In engineering applications, the mixed H_2/H_∞ control is more attractive than the sole H_∞ control since the H_∞ control is a worst-case design which tends to be conservative whereas the mixed H_2/H_∞ minimizes the average performance with a guaranteed worst-case performance.

Here, the H_2/H_∞ control problem of complex systems is treated using switching controller. That is, the desired plant behavior is achieved by switching between pre-designed controllers, each to meet a set of relevant specifications. Our aim is using switching controller to reduce the conservatism of the controller synthesis and the resultant performance degradation; therefore, we apply the concept of multiple controllers and utilize the switching signal to orchestrate the switching among pre-designed controllers to improve performance of closed-loop system.

In this paper, we present a new switching logic. At every time instant, we search for a controller corresponding to the best performance. We then decide whether to switch to that controller or not by comparing the value of Lyapunov function at the previous switching instant to this controller with its prospective value that would result from the switching; if a certain inequality condition is satisfied, switching is allowed. We then further employ a dwell-time algorithm together. We show that asymptotic convergence is ensured by the proposed switching control scheme resulting from the combination of the Lyapunov-function-based switching and the dwell-time switching.

This paper is organized as follows: Section 2 presents the system definition, and the controllers used in this paper. The problem of synthesis switching signal is described in Section 3. A simple illustrative example is presented in Section 4. Section 5 contains some concluding remarks.

2. Problem Statement

This paper presents a controller design strategy for fulfilling H_2/H_∞ control objectives resulting in no compromise between distinctive specifications. Instead of considering all of the objectives in a single controller, switching between these controllers in the timely manner is proposed to meet the desired performance. In this case, we can obtain our multi-criterion goal without introducing conservatism to the problem, as each specification or a set of relevant objectives are assumed to be accomplished by a single controller without considering any contradictory objective in the design procedure. As switching can cause instability, a new switching strategy is introduced that lets switching based on either dwell time switching or multiple Lyapunov function.

The state-space realization of the LTI plant, P, for which H_2 and H_∞ output-feedback controller is designed is as follows:

$$P:\begin{cases}\dot{x}=Ax+B_u u_C+B_w w\\ z=C_z x+D_{zu}u_C+D_{zw}w\\ y=C_y x+D_{yw}w\end{cases} \qquad (1)$$

$x\in R^{n_x}$ is state, $w\in R^{n_w}$ is the exogenous input signal (noise, disturbance and reference input), $u_C\in R^{n_u}$ is the control input, $y\in R^{n_y}$ is the measured output and $z\in R^{n_z}$ is the output to be regulated and defines the performance objectives of the closed loop system. The assumptions of H_2 and H_∞ controls are true. The diagram of the closed loop system is depicted in Figure 1.

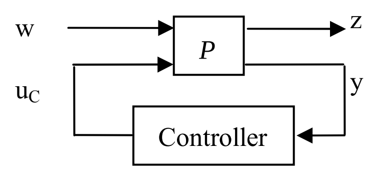

Figure 1. The Standard Diagram for H_2 and H_∞ Controls

The standard H_2/H_∞ performance criterion is considered: The objective is synthesis of the switching controller that internally stabilizes the closed loop system and minimize $\left\|T_{w_2\to z_2}\right\|_2$ while $\left\|T_{w_\infty\to z_\infty}\right\|_\infty<\gamma$. $w_i=R_i^{-1}w$, $i\in I=\{H_2,H_\infty\}$ is the exogenous input and $z_i=L_i z$ is the controlled output (Scherer, 1997). The available design methods in addition to convex optimization problem defined by different Linear Matrix Inequalities (LMI) for H_2 and H_∞ closed-loop specifications given in Scherer, 1997 is used to find transfer matrices of the controllers.

Figure 2. illustrates the closed loop configuration used in this context, where u denotes the control input, y_P the process output, r a bounded reference signal (set point), d unknown but bounded input disturbance, and n unknown but bounded measurement noise. The process is a LTI system with strictly proper transfer matrix $H_P(s)$. $\{A_P,B_P,C_P\}$ is a minimal realization for $H_P(s)$. For the sake of conformity of the closed loop configuration in Figure 2. and the closed loop block diagram in Figure 1., the dashed line in Figure 2. is

considered as P in Figure 1., external inputs $\begin{bmatrix} r \\ d \\ n \end{bmatrix}$ in Figure 2. forms w in Figure 1.

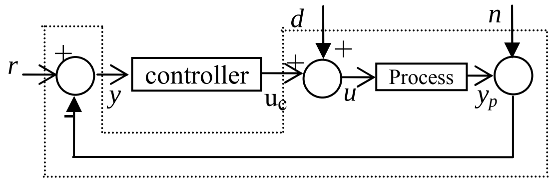

Figure 2. The Closed Loop Configuration

3. Purposed Switching Signal

We present a switching logic using the Multiple Lyapunov functions in Liberzon, 2003. The proposed algorithm is referred to as S_{MLF} and is given as follows: (Algorithm S_{MLF})

1- Initialize $\sigma(t)$ and Δt

2- Find the best controller: i := arg Best Controller

3. If $\sigma(t-\Delta t) \neq i$, and

$$V_i(t_i) \geq V_i(t) \qquad (2)$$

$V_i(.)$ is the Lyapunov function of the closed loop system with i th controller, t_i is the last time that switching to i th controller occurred, then let $\sigma(t^+) = i$ and $t_i = t$.

4. $t = t + \Delta t$ and go to step 2.

Switching takes place in S_{MLF} when two conditions are met: firstly there should be a better controller leading to the better performance, and secondly the inequality in (2) should hold to grantee the stability according to the following Lemma.

Lemma. 1. (Liberzon, 2003): Let $\dot{x} = f_i(x), i \in I$ be a finite family of globally asymptotically stable systems, and let $V_i, i \in I$ be a family of corresponding radially unbounded Lyapunov functions. Suppose that there exists a family of positive definite continuous functions $W_i, i \in I$ with the property that for every pair of switching times (t_p, t_q), $p < q$ such that $\sigma(t_p) = \sigma(t_q) = i \in I$ and ($\sigma(t_k) \neq$ i for $t_p < t_k < t_q$, we have $V_i(x(t_q)) - V_i(x(t_p)) \leq -W_i(x(t_p))$. Then the switched system is globally asymptotically stable.

In other words, switching is not allowed even when there is a better controller, if use of this new controller violates the condition given in (2). Checking the two conditions implies that both stability and performance is considered in S_{MLF}. Using this algorithm, switching system is asymptotically stable, the state variables are bounded and converge to zero, and all the signals remain bounded.

A dwell time, τ_D, is a lower bound for the difference between two consecutive switching instants; switching is allowed after waiting for the dwell time. Here we combine the dwell-time algorithm with switching logic proposed above. A switching logic using the dwell time switching is referred to as S_D which is time dependent and trajectory independent.

Lemma. 2. (Liberzon, 2003): The switched system $\dot{x} = A_i x$ is asymptotically stable if the time interval between consecutive switching instant between their asymptotical stable subsystems is not smaller than $\tau_D \geq \sup_{i \in I}\left(\frac{a_i}{\lambda_i}\right)$. Where $\left\|e^{A_i t}\right\| \leq e^{a_i - \lambda_i t}$ and $a_i = \log\left(\frac{\sigma_{\max}(M_i)}{\sigma_{\min}(M_i)}\right)$, M_i is the modal matrix (i.e. the matrix with eigenvectors as its columns) of the stable matrix A_i and $\sigma_{\max}(M_i)$ and $\sigma_{\min}(M_i)$ are the maximum and minimum singular values of M_i, respectively. The positive scalar λ_i is simply the absolute value of the real part of the eigen values A_i nearest to the imaginary axis (stability degree of stable matrix A_i).

As there are two switching logics involved, we use two subscripts for switching times to clarify which logic causes the switching; let $t_{p,q}$ denote the switching instant which is due to the q th switching by S_D in a row after the p th switching by the S_{MLF} . $t_{p,0}$ denotes the p th switching instant due to switching by S_{MLF} .

If there exists other controller with better performance but the condition (2) for S_{MLF} does not hold for $\bar{t} \in (t_{p,q} \quad t_{p,q} + \tau_D]$, then S_D forces switching to take place, leading to $t_{p,q+1} = t_{p,q} + \tau_D$. If the condition in (2) holds for some $\bar{t} \in (t_{p,q} \quad t_{p,q} + \tau_D)$ then switching results from S_{MLF} , leading to $t_{p+1,0} = \bar{t}$.

The two switching logics S_{MLF} and S_D are employed together in the proposed switching control scheme; as a result, switching takes place whichever logic allows without destroying stability as is shown below. The proposed switching logic is referred to as $S_{MLF} \cup S_D$, and is depicted in Figure 3.

Plant with each controller forms a linear subsystem of switching signal. Suppose that at time instant $t_{p,0}$ switching to i th subsystem has occurred by S_{MLF} logic. The subsystems that switchings to them have occurred in the time interval $\left[t_{p,0} \quad t_{p+1,0}\right)$ by S_D logic can be considered as a subsystem of switching system with zero input nonlinear dynamic $\dot{x} = f_i(x)$ which is asymptotically stable according to Lemma 2.

As a result, the asymptotically stable linear subsystems, A_i, under switching logic $S_{MLF} \cup S_D$ is a combination of the asymptotically stable linear subsystems, A_i, and the asymptotically stable non linear subsystems, f_i, under S_{MLF} switching logic.
If all the linear subsystems that switch together consecutively by S_D switching logic, and their switching start from i th subsystem are considered as one nonlinear subsystem, the number of subsystems is bounded. And according to Lemma 1 are asymptotically stable.

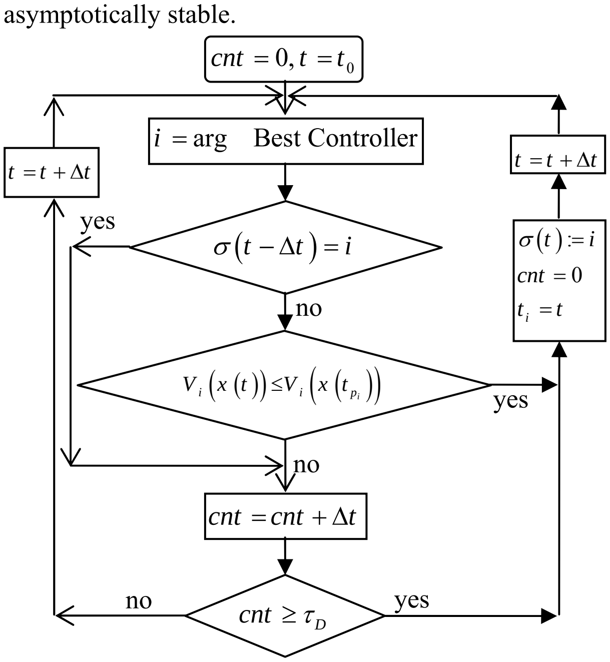

Figure 3. The Proposed Switching Logic $S_{MLF} \cup S_D$

4. Example

In this section, we briefly illustrate the enhancement of multi objective control performance via switching. The proposed approach is applied to the dynamic model of the roll angle of an aircraft taken from Vegte, 1994, and Hespanha, 2002:

$$H_P(s) = \frac{-1000}{s(s+0.875)(s+50)} \qquad (3)$$

In this example, it is considered that white measurement noise with a large variance is injected in the time intervals $t \in [18 \quad 40]$, $t \in [73 \quad 93]$ and $t \in [128 \quad 148]$. In the presence of the measurement noise, noise rejection and the slower response are the objectives, while in the absence of the measurement noise a fast response and good tracking should be considered in the design procedure. This design problem is a multi-objective problem with conflicting criteria, because if a controller has low closed-loop bandwidth and is therefore not very sensitive to noise, it will exhibit a slow response and if a controller has high bandwidth and is therefore fast, it will be very sensitive to noise.

Following realization is considered for $H_P(s)$:

$$\dot{x} = \underbrace{\begin{bmatrix} -50.875 & -43.75 & 0 \\ 1 & 0 & 0 \\ 0 & -1000 & 0 \end{bmatrix}}_{A_P} x + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{B_P} u$$

$$y_P = \underbrace{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}_{C_P} x$$

It is easy to find that $\dot{y}_P = \begin{bmatrix} 0 & -1000 & 0 \end{bmatrix} x$.

Since $u = u_C + d$, $w = \begin{bmatrix} r \\ d \\ n \end{bmatrix}$ and $y = r - n - y_P$:

$$\dot{x} = \underbrace{\begin{bmatrix} -50.875 & -43.75 & 0 \\ 1 & 0 & 0 \\ 0 & -1000 & 0 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{B_u} u_C + \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{B_w} w$$

$$y = \underbrace{\begin{bmatrix} 0 & 0 & -1 \end{bmatrix}}_{C_y} x + \underbrace{\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}}_{D_{yw}} w$$

To have a good measurement noise rejection the controller, K_{H_2}, with more robust performance regarding measurement noise is designed using LQG/LQR. The regulator gains are computed by minimizing the cost $J = E\left\{ \lim_{t \to \infty} \frac{1}{T} \int_0^T \left(y_P^2(t) + \dot{y}_P^2(t) + 100 u_C^2(t) \right) dt \right\}$.
The design of the optimal LQG gain was done assuming that the load disturbance, d, and the measurement noise, n, were uncorrelated white noise processes with $E\left(d(t)d(\tau)\right) = \delta(t-\tau)$ and $E\left(n(t)n(\tau)\right) = \mu\delta(t-\tau)$, where $\mu = 10^{-1}$. It is easy to show that

$$J = E\left\{\lim_{t\to\infty}\frac{1}{T}\int_0^T \left(z_2(t)^T z_2(t)\right)dt\right\} = \left\|T_{w_2\to z_2}\right\|_2^2,$$

where $z_2 = [y \quad \dot{y} \quad \rho u_C]$ and $w_2 = \begin{bmatrix} d \\ n \\ \sqrt{\mu} \end{bmatrix}$, in other words

$$z_2 = \begin{bmatrix} y_P \\ \dot{y}_P \\ \rho u_C \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1000 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{C_{z_2}} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \rho \end{bmatrix}}_{D_{z_2 u}} u_C$$

. It is clear that, $B_{w_2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $D_{z_2 w_2} = \begin{bmatrix} 0 & -\sqrt{\mu} \end{bmatrix}$.

To get a relatively good tracking and to prevent the extra increasing of the value of input signal, the controller, K_{H_∞}, is designed by minimizing $\left\|T_{w_\infty \to z_\infty}\right\|_\infty$ where $w_\infty = r$ and:

$$z_\infty = \begin{bmatrix} y \\ u_C \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}}_{C_{z_\infty}} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{D_{z_\infty u}} u_C + \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}}_{D_{z_\infty w}} w,$$

$$z_\infty = \begin{bmatrix} y \\ u_C \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}}_{C_{z_\infty}} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{D_{z_\infty u}} u_C + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{D_{z_\infty w_\infty}} w_\infty$$

In Figure 4, the left plots show the closed-loop response of controller K_{H_∞} and the right plots show the closed-loop response of controller K_{H_2} to a square set point. It can be seen from this Figure that the controller K_{H_∞} exhibits a faster response but is more sensitive to measurement noise. The top plots show the output, y_P, and the bottom plots the tracking error, y.

The left plots in Figure 5 show the closed-loop response of the switching controller, $K_{switching}$, to square reference. This structure inherits the fast performance of K_{H_∞} in normal cases, and a good noise rejection of K_{H_2} in the presence of the white measurement noise.

Using Lemma 2 the minimum time interval between consecutive switching between controllers K_{H_2} and K_{H_∞} is equal to $\tau_D = 22.9083s$. $V_{K_2}(t)$ and $V_{K_\infty}(t)$ denote the Lyapunov functions of closed loop system with controller K_{H_2} and K_{H_∞} at time instant t, respectively.

In time interval $[0 \quad 18]$ controller K_{H_∞} and in time interval $[18 \quad 38]$ controller K_{H_2} are in the loop. Since $V_{K_\infty}(0) > V_{K_\infty}(38)$, according to algorithm we can switch to controller K_{H_∞}. At $t = 73$, because of the time interval between consecutive switching is $73 - 38 > 22.9083$, according to Figure 3. without checking inequality (2) we switch to controller K_{H_2}. Since $V_{K_\infty}(38) < V_{K_\infty}(93)$, according to Figure 3, we check the constraint (2) until $\tau_D = 22.9083s$ after previous switching. Any time that the constraint is satisfied, we can switch to controller K_{H_∞}. Since during this time interval the constraint is not satisfied switching to controller is occurred at $t = 73 + \tau_D$. At $t = 128$ similar to $t = 73$ we switch to controller K_{H_2}. Since $V_{K_\infty}(93) < V_{K_\infty}(148)$, according to Figure 3, we check the constraint (2) until $\tau_D = 22.9083s$ after previous switching. Any time that the constraint is satisfied, we can switch to controller K_{H_∞}. At $t = 148.03$ the constraint is satisfied and switching to controller K_{H_∞} is occurred.

The right plots show the closed-loop response of the common multi objective controller K_{H_2/H_∞} that minimize $\left\|T_{w_2\to z_2}\right\|_2$, subject to $\left\|T_{r\to z_\infty}\right\|_\infty < \gamma$ (a single controller that considers both design objectives concurrently). The top plots show the output, y_P, and the bottom plots the tracking error, y.

The comparison illustrates the conservatism reduction by means of the switching controller in comparison with the performance of the common multi objective controller. As depicted in this Figure, it is apparent that we have approached meeting both design objectives better using the switching controller. It can be seen, from Table 1. that the switching controller minimizes the 2- norm and ∞-norm of tracking error, y, in comparison with other controllers.

Table 1. The Performance of the Controllers

	K_{H_2}	K_{H_∞}	K_{H_2/H_∞}	$K_{switching}$
$\sup_n \frac{\|y\|_\infty}{\|n\|_2}$	3.658	3.8836	4.7811	3.6575
$\sup_n \frac{\|y\|_2}{\|n\|_2}$	84.162	83.2744	100.334	82.0997

5. Conclusion

In this paper, a switching H_2/H_∞ controller is developed. This structure allows us to take the benefits of both H_2 and H_∞ controllers and to efficiently eliminate their disadvantages. The transients caused by switching may result in instability which is avoided by appropriate choice of switching signal. Purposed new switching signal is the combination of dwell time switching and multiple Lyapunov function. The simulations illustrate that a switching controller scheme inherits characteristics of all the controllers in the time intervals they have been in the loop, and diminish the performance degradation caused by considering all the control objectives in the design of a unique controller.

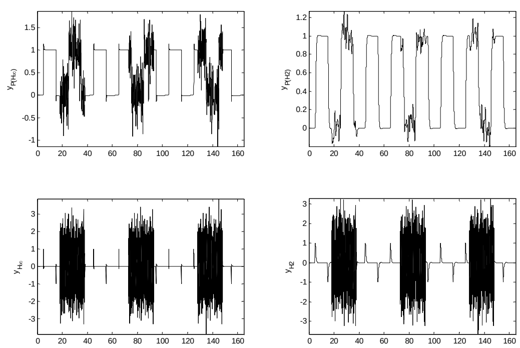

Figure 4. The Closed Loop Response of K_{H_∞} and K_{H_2}.

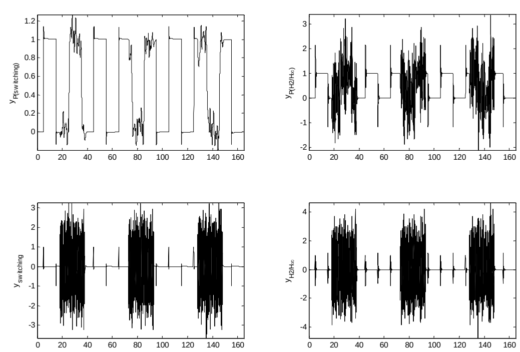

Figure 5. The Closed Loop Response of Supervisory Based Switching Controller, $K_{switching}$, and Common Multi Objective Controller, K_{H_2/H_∞}.

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