Robust Control of an Active Suspension System Using H_2 & H_∞ Control Methods

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Abstract: In this paper, H2 & H∞ contro1 for an active suspension system are presented. These Controllers are designed for the order reduced model of the plant that makes the design problem so easy, But preserves the performances and stability of the nominal closed loop system. Some constraints on the Input and output sensitivity functions are considered. The results show control specifications are met to large extent with both methods.
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1. Introduction

An active suspension system is used for disturbance attenuation in a large frequency band and in the presence of the load variation. Active suspension systems are currently of great interest in both academia and industry. A literature survey on suspension system shows that several models and controllers have been developed in attempts to enhance and improve the ride and handling qualities in today's vehicle Amirifar (2003), Ray (1991). Linear controllers are the main group of these controllers. In the linear control philosophy, it is assumed that the system's state exhibit only small variations around the equilibrium point, so that a linear approximation model can be used. Existing linear controllers range from PID to robust multivariable controllers Thompson (1989), Landau (1995), Landau (1995), Amirifar (2006), Thompson (1976), Kuo (1999), Aghaie (2007)1]. In Landau (1995) robustness analysis and synthesis methods based on stochastic stability robustness for a quarter-car model was presented that can be applied to higher order active suspension systems. However, such approach requires large feedback gain and reasonable phase must be selected. In Landau (1995) the centralized/ local optimal output feedback controller (CLOFC) was developed for active suspension systems. In Amirifar (2006) and Thompson (1976) optimal control theory was applied to the design of an active suspension system. The used performance index is based on ride quality, suspension deflection, and tire deflection. In Thompson (1989) a combination of the H∞ and LQR methods was used to improve the system performance when it is subject to external disturbances, e.g., road irregularities, and parameter uncertainties, e.g., vehicle weight as payload varies. Even though this method provided better performance, its application to a vehicle suspension

System is difficult, since the H_∞ method often results in complex high- order controllers even the design model is of reasonable size Karnopp (1983). In order to reduce the order of high-order controllers, controller order reduction techniques can be used. In Sunwoo (1991) a new controller order reduction technique with stability and performance preservation via LMI optimization was presented and implemented on an active suspension system. The controller order reduction problem was reduced to an LMI problem, so it can be solved efficiently. The fixed parts of the high-order controller which should be preserved in the reduced-order controller, and many other specifications on the reduced-order controller that can be expressed as LMIs constraints, can easily be treated.

The novelty of this paper is the presentation of the classical H_2 & H_∞ control schemes for the active suspension system. The control specifications are enforced in the H_2 & H_∞ problems as the constraints on the input and output sensitivities. The desired performances are obtained by precise selection of the weighting functions that may affect the stability of the closed loop system in turn. In this paper, the reduced order model of the system is used to design the controllers, but performance indexes and stability are examined for the main system. The simulation results for both of H_2 & H_∞ controllers are given and compared.

The reminder of this paper is organized as follows. In section 2, an active suspension system is introduced. Section 3 is devoted to the H_2 & H_∞ controller design for robustifying, achieving the wanted performances and the stability margins. Finally, section 4 presents the concluding remarks.

2. Active suspension system

The schematic diagram of the active suspension system is shown in Figure 1.

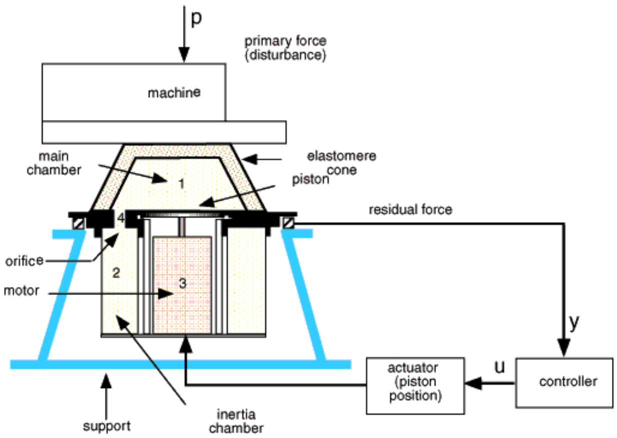

Figure 1. Schematic diagram of the active suspension system

Two models for the system will be identified, corresponding to the primary and secondary path. The input of the primary path (excitation of the shaker), u is the input of the secondary path (proportional to the piston position) and y is the system output (residual force).

The active hydro-suspension system reduces the machine vibration at the resonance frequency. The principal parts of the suspension system are given below:

1. an elastomere cone that encloses the main chamber filled with silicon oil (1);
2. an inertia chamber enclosed with a flexible membrane (2);
3. A piston (3) that is fixed on a DC motor. When the position of the piston is fixed, the suspension system is passive;
4. An orifice (4) that allows oil flow between two chambers.

The principal idea of the active suspension is to change the elasticity of the system in order to absorb the vibrations generated by the machine that we want to isolate. For the experimental purposes the machine is replaced by a shaker which is driven by a computer generated control signal.

The output of the system is the measured voltage corresponding to the residual force. The control input drives the position of the piston via an actuator. The transfer function, F(s) between the excitation of the shaker and the residual force is called the primary path. The secondary path is defined as the transfer function between the control input and the residual force.

The magnitude analysis of the frequency response of the primary path obtained by spectral analysis is shown in Figure 2, this analysis shows that there are several vibrational modes, with the first mode at 31.47 Hz and the second mode around 160Hz are the most important ones.

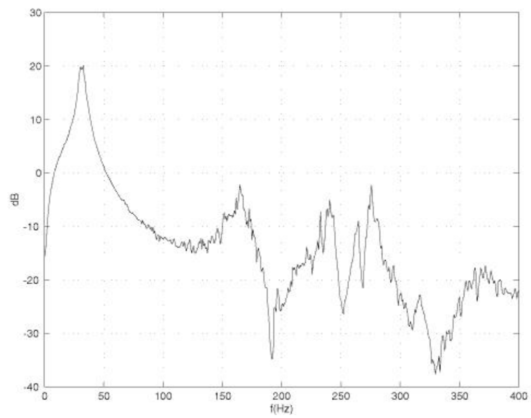

Figure 2. Magnitude of the frequency response of the primary path

The structure of the controlled active suspension system is given in the Figure 3.

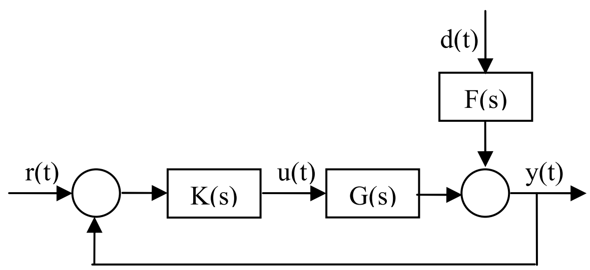

Figure. 3. Block diagram of the controlled active suspension system

In this section, the design of a controller for the active suspension system is considered. A 15-th order continuous-time model of the secondary path G(s) is used for scheme. G(s) consists of several vibration modes, whereas the first mode around 30 Hz, and the second mode around 160 Hz are the most important ones. In order to consider the effects of the most important modes during the controller design procedure, a low-order model, namely, $G_1(s)$ is obtained by truncating the high frequency modes. Figure 4 shows the magnitude Bode plot of G(s) and $G_1(s)$.

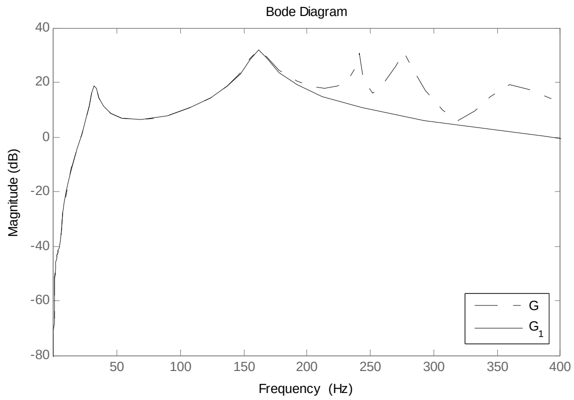

Figure 4. The magnitude Bode plot of G(s) and G_1(s).

3. Controller design and simulation results

The control goal of this system is to compute a linear controller which minimizes the residual force around the main vibration modes of the primary path model and to try to distribute the amplification of the disturbances over the higher frequencies.

The control objective can be presented in terms of the constraints for the closed-loop sensitivity functions. The output sensitivity function of the nominal closed loop system, $S_{yp}(s)$ is defined as

$$S_{yp}(s)=\frac{1}{1+G(s)K(s)} \tag{1}$$

The input sensitivity function of the nominal closed loop system, Sup(s) is defined as

$$S_{up}(s)=-\frac{K(s)}{1+G(s)K(s)} \tag{2}$$

The modulus of the output and input sensitivity functions must be bounded above in the frequency domain by the special constraints. Figure 5 and Figure 6 show the magnitude Bode plot of the constraints on the input and output sensitivity functions used in the controller designs.

The resulting 12-th order H∞ Controller achieves the control specifications approximately. The nominator and denominator of K(s) are given below. Figure 7 and 8 show the magnitude bode plot of the sensitivity functions obtained by controller implementation.

$$d_{K_\infty}(s)=s^{12}+1996.8s^{11}+4.2813\times10^{6}s^{10}+4.8207\times10^{9}s^{9}$$
$$+5.5322\times10^{12}s^{8}+3.4406\times10^{15}s^{7}+2.4111\times10^{18}s^{6}$$
$$+6.4411\times10^{20}s^{5}+1.8\times10^{23}s^{4}+2.5085\times10^{25}s^{3}$$
$$+3.4302\times10^{27}s^{2}+1.9448\times10^{29}s+3.2362\times10^{30}$$

$$n_{K_\infty}(s)=2.302\times s^{11}+9801\times s^{10}+1.8232\times10^{7}\times s^{9}$$
$$+2.8513\times10^{10}s^{8}+2.9356\times10^{13}s^{7}+2.7491\times10^{16}s^{6}$$
$$+1.4999\times10^{19}s^{5}+9.3233\times10^{21}s^{4}+1.6425\times10^{24}s^{3}$$
$$+3.6921\times10^{26}s^{2}+3.5976\times10^{28}s+1.0494\times10^{30}$$

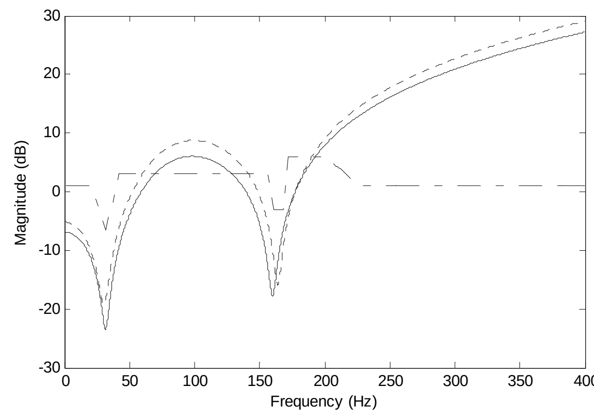

Figure 5: Output sensitivity constraint (dashed-dot) and the inverse of the Output sensitivity weighting function of H_∞ Controller (dotted) and the inverse of the Output sensitivity weighting function of H_2 Controller (solid)

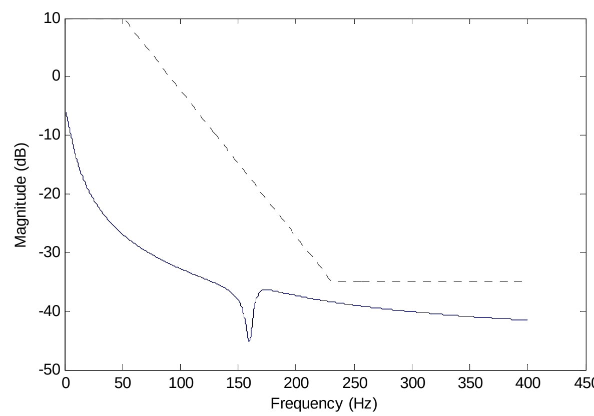

Figure 6. Input sensitivity constraint (dotted) and the inverse of the Output sensitivity weighting function of H_∞ Controller and H_2 Controller (solid)

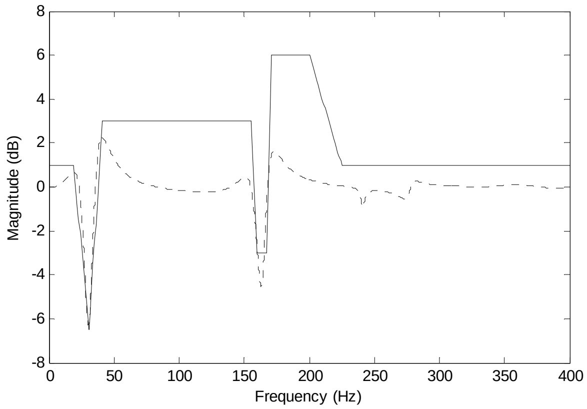

Figure 7. Output sensitivity function constraint (solid) and output sensitivity function (dotted), with H_∞ controller

The resulting 10-th order H2 Controller achieves the control specifications approximately. The nominator and denominator of K(s) are given below. Fig. 9 and 10 show the magnitude bode plot of the sensitivity functions obtained by controller implementation.

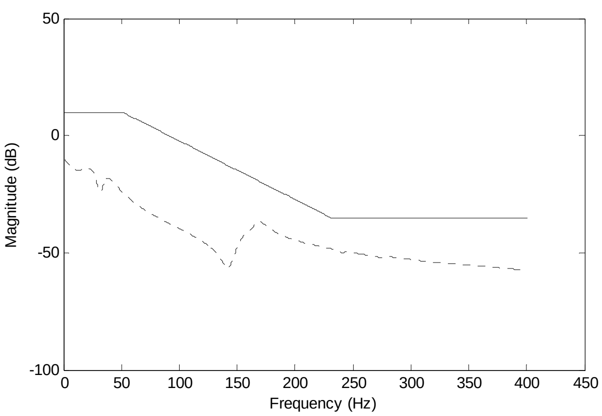

Figure 8. Input sensitivity function constraint (solid) and input sensitivity function (dotted), with H∞ controller

$$n_{K_2}(s) = 2.1505s^9 + 9519.1s^8 + 1.832\times10^7 s^7 + 2.7996\times10^{10} s^6 + 2.8233\times10^{13} s^5 + 2.4727\times10^{16} s^4 + 1.2337\times10^{19} s^3 + 6.4232\times10^{21} s^2 + 5.0442\times10^{23} s + 1.0294\times10^{25}$$

$$d_{K_2}(s) = s^{10} + 1942.3s^9 + 4.14\times10^6 s^8 + 4.5216\times10^9 s^7 + 5.126\times10^{12} s^6 + 2.9771\times10^{15} s^5 + 2.0559\times10^{18} s^4 + 4.081\times10^{20} s^3 + 8.1062\times10^{22} s^2 + 5.1225\times10^{24} s + 8.7689\times10^{25}$$

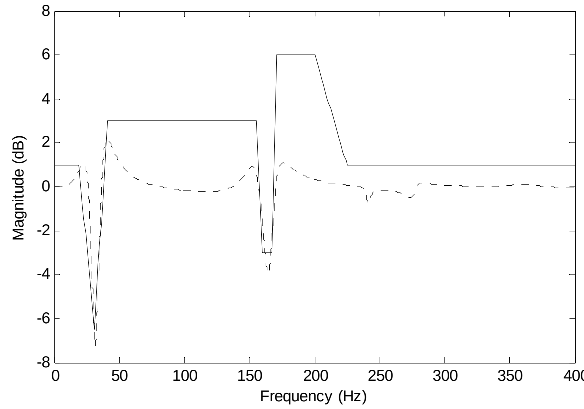

Figure 9. Output sensitivity function constraint (solid) and output sensitivity function (dotted), with H_2 controller

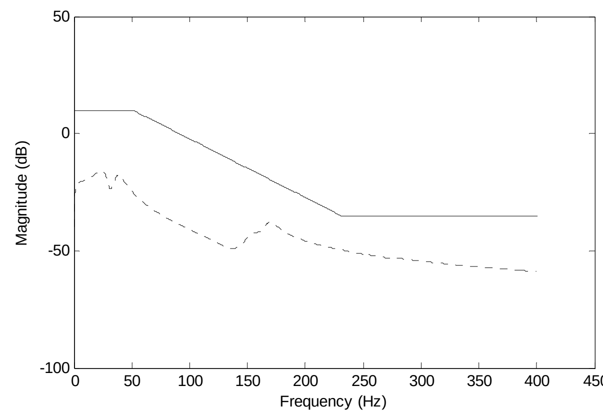

Figure 10. Input sensitivity function constraint (solid) and input sensitivity function (dotted), with H_2 controller

The gain margin and phase margin of the controlled system are presented in table. 1. Results show that the controlled system is stable and is sufficiently far from stability margins.

Table. 1 stability margins

	H_2 Controller	H_∞ Controller
Gain Margin	5.0897	6.5464
Phase Margin	109.9230	130.5152

3. Conclusion

An application of the H_2 & H_∞ controls to an active suspension system has been presented. By translating the control goals on the input and output sensitivity functions, a mixed sensitivity problem has been obtained. The reduced order model of the system is used to design the controllers, but performance indexes and stability are examined for the main system. The final solutions are the 10th-order H_2 controller and the 12th-order H_∞ controller that achieve all design specifications and have good stability margins. It is clear that the H_∞ controller has higher order than the H_2 controller but give better performances.

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