Classification of dynamical geometric figures

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Abstract: In this paper, we discuss the effect of the time on geometric figures. Also, we deduce the algebraic transformations and geometric transformations which occur on geometric figures by the time. We give some important results.
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1. Introduction

In this article, the transformations of the geometric figures discussed in many papers. These papers explained how geometric figures are deformed by folding, retraction, desperation…etc. More studies of the folding of many types of manifolds were studied in [1-6]. Here, we shall discuss the deformations which will occur on geometric figures by the change of the time. Also, we will discuss the changing in the dimension, the volume, the position ….etc.

2. Definitions

(1) An n-dimensional manifold M is a Hausdorff topological space such that each point has an open neighborhood to open n-dimensional disc $U^n = \{x \in R^n : |x| < 1\}$.

(2) Let M and N be two manifolds of dimension m and n respectively. A map $f : M \to N$ is said to be an isometric folding of M into N if for every piecewise geodesic path $\gamma : I \to M$, the induced path $f \circ \gamma : I \to N$ is piecewise geodesic and of the same length as γ [9]. If f does not preserve length, it is called a topological folding [6].

(3) A subset A of topological space X is called a retract of X if there exist a continuous map $r : X \to A$ (called a retraction) such that $r(a) = a,\ a \in A$ [7].

(4) A dynamical system in the space X is a function $q = f(p,t)$ which assigns to each point p of the space X and to each real number t $(-\infty < t < +\infty)$ a definite point $q \in X$ and possesses the following properties [8]:

1. Initial condition: $f(p,0) = p$ for any point $p \in X$.
2. Property of continuity in both arguments:
$$\lim_{\substack{p \to p_0 \\ t \to t_0}} f(p,t) = f(p_0,t_0).$$

3. The Main Results

Let A be a set of Geometric figures defined on topological space X. Define dynamical set A by A_{dyn} such that any element in A change with time. Now, we will discuss different cases of dynamical sets in dynamical spaces.

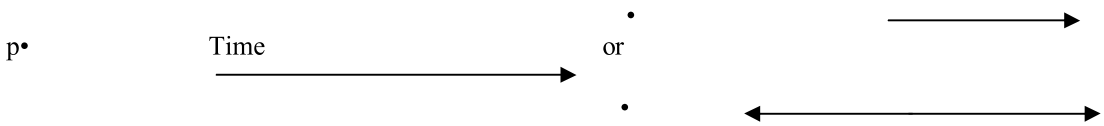
CASE (A):

Let A be a set defined by A={a, b, c, d}. The types of change by the time which occurs on a set A are:

$A_{dyn} = \{a, a_1, \ldots, a_n, b, c, d\}$ or $A_{dyn} = \{a, b, b_1, \ldots, b_n, c, d\}$ or $A_{dyn} = \{a, b, c, c_1, \ldots, c_n, d\}$
or $A_{dyn} = \{a, b, c, d, d_1, \ldots, d_n\}$
or $A_{dyn} = \{a, a_1, \ldots, a_n, b, b_1, \ldots, b_n, c, c_1, \ldots, c_n, d, d_1, \ldots, d_n\}$
or $A_{dyn} = \{a_1, \ldots, a_n, b, c, d\}$ or $A_{dyn} = \{a_1, \ldots, a_n, b_1, \ldots, b_n, c_1, \ldots, c_n, d_1, \ldots, d_n\}$

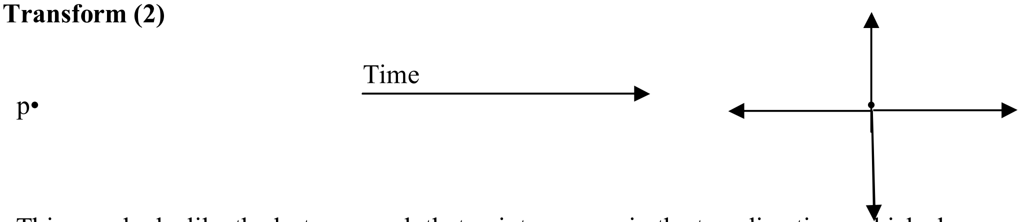
or $A_{dyn}=\{b, c, d\}$ or $A_{dyn}=\{a, c, d\}$ or ….or $A_{dyn}=\{a\}$ or $A_{dyn}=\{\ \} = \phi$.
Let A be subset of R^1- space. Define A = {set of points}. Under the change of time, we have the following transformations:

Transform (1):



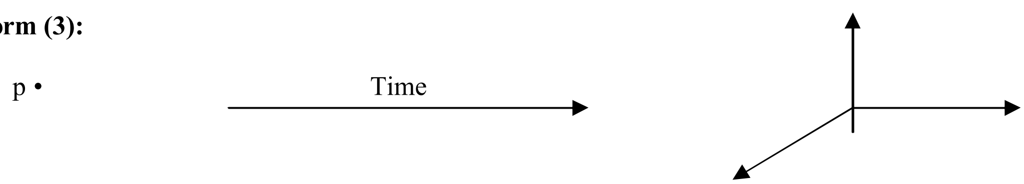
In this case, under change of the time the point p moves in one direction (along x-axis say) or in two direction. Notice that point changes its position and its dimension (from dimension zero into dimension one). Also, we deduce x-axis.

Transform (2)

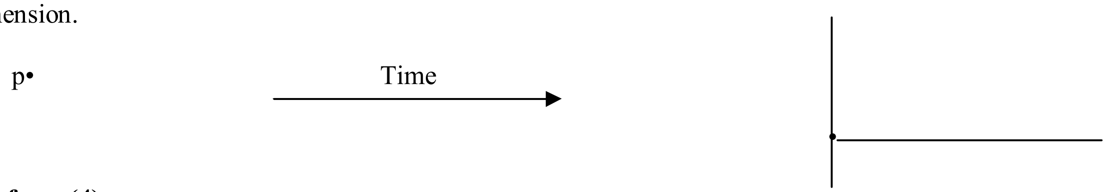


This case looks like the last case such that point p moves in the two directions which changes the dimension.

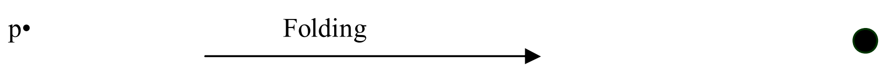
Transform (3):



In this transformation, the point p moves under the action of the time into three dimensions. We notice that the point p changes its dimension (from zero dimensions into three dimension) and its position.
In general case, the point p moves under the action of the time from zero dimensions into n-dimension.

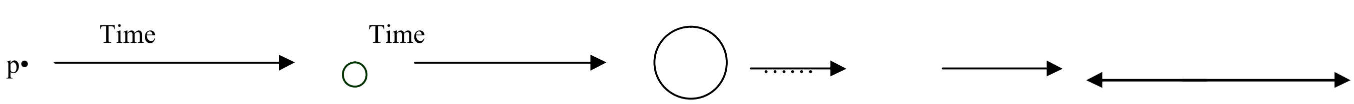


Transform (4):



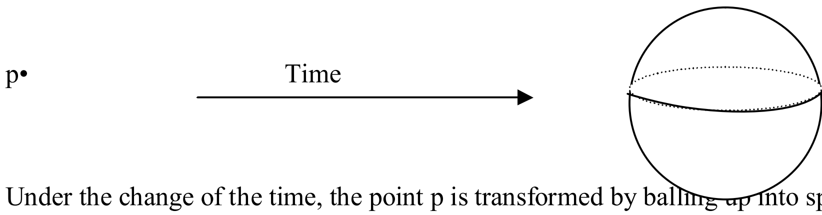
This transformation occurs for the point under the change of the time. The point p transform into small area with length and wide (*i.e.*)
Point p changes its dimension and its volume.

Transform (5):



The point p transforms by the time (under the action of the time) into circle of small radius, by increasing the time the radius of the circle will increase and we get on circle of infinite radius (line expanded into infinity). We notice that the volume is changed, the dimension also is changed (from zero dimension into one dimension) and the curvature of the circle changes.

Transform (6):



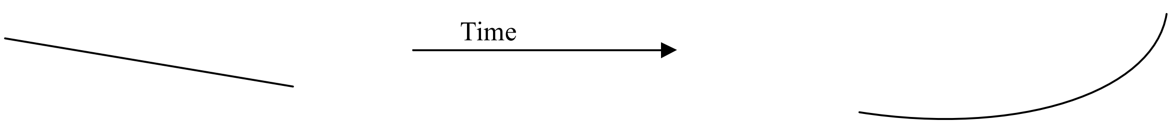
Under the change of the time, the point p is transformed by balling up into sphere (*i.e.*)
The transformations occur in dimension (from zero dimensions into two dimensions),
in the volume and the curvature changes.

Remark: We notice that the last transforms represented dynamical space (is the space which transforms into another space by the time). Here, we deduce a new dynamical set contained in dynamical space.

<u>CASE (B):</u>

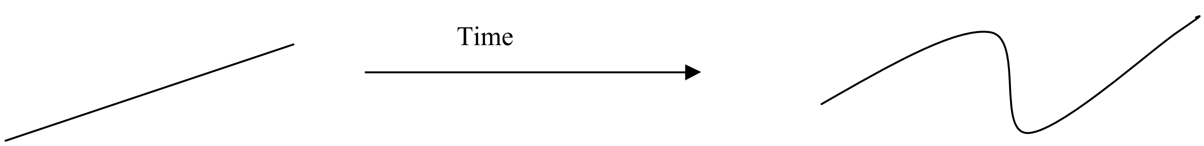
Consider a dynamical set A = {lines, circles of different radius a} defined on R^2-space. Under the change of time, we discuss transformations which will occur on this set.

Transform (1):



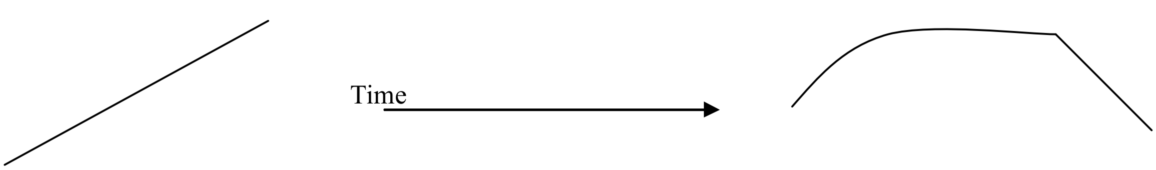
Take one element of this set, we observes transformation occur on this line to convert into curve which different about the line in curvature such that curvature of line K=0 but curvature of curve K > 0. Also, we notice that the curve is convex.

Transform (2):



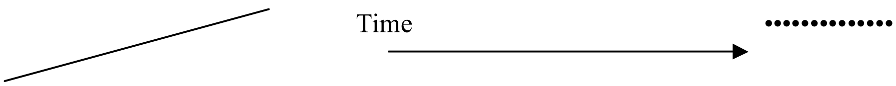
In this case, line transform-under the change of the time- into a curve. We notice that the curvature changes (from K=0 into K>0) and curve in this case is not convex.

Transform (3):



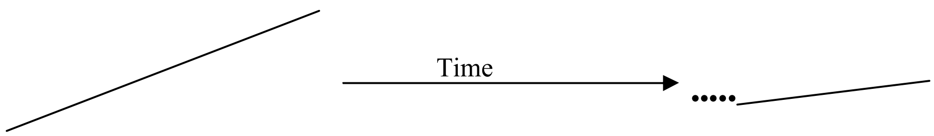
In this case, we get transformations in curvature, torsion not in dimension. We notice that the change by the time is equivalent to the folding.

Transform (4):



In this transform, we find that the line under the action of the time changed into set of points which different in dimensions such that dim (line) =1 and dim (point) =zero. This transformation is a type of desperation.

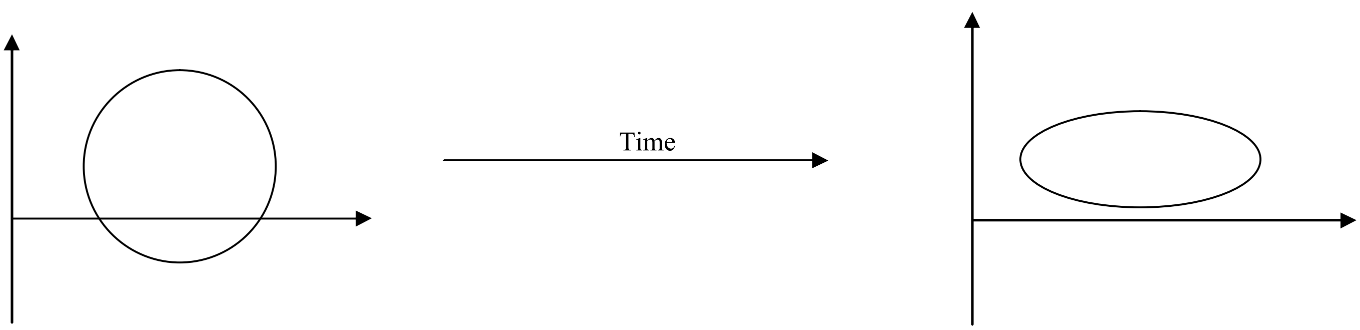
Transform (5):



This transform convert the line into smallest line and set of points such that the initial line changed under the action of the time into line different about the initial line in length and set of points different in dimension. We can remark that the result system not formed a manifold.

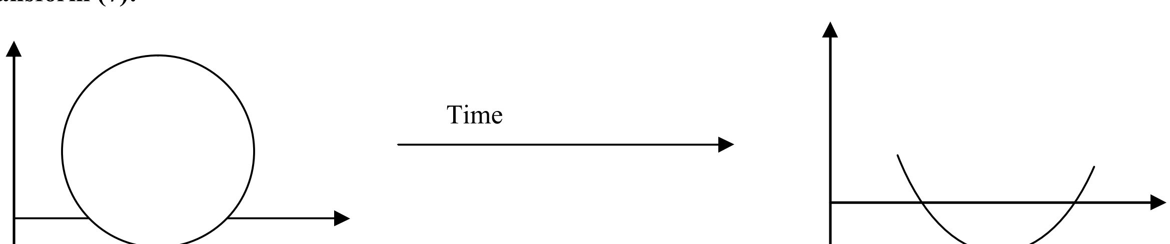
Now, we will discuss the transformations which occur on circles of different radius:

Transform (6):



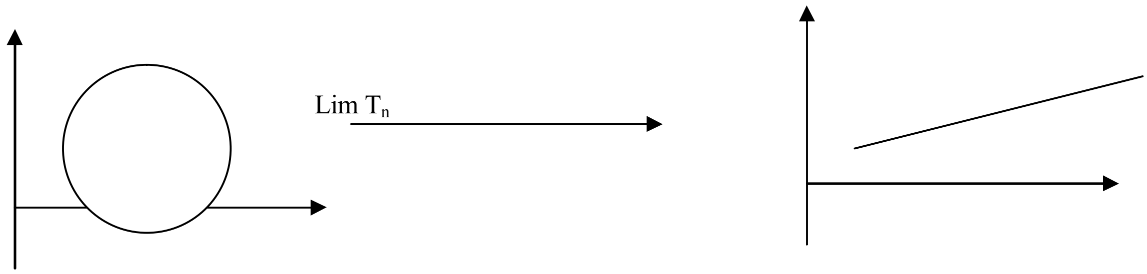
Circle from set A, under the change of the time, transform into ellipse which different in algebraic properties like (curvature, algebraic equation).

Transform (7):



In this case, we deduce that circle transforms from closed curve in R^2-space into open curve (half circle) which lies also in R^2-space.

Transform (8):



Under the action of balling up of the circle, the radius of the circle increase at infinity which converts the circle into line. We notice some different in the curvature, algebraic equation and convexity.

At the same moment, all circles which lie in A_{dyn} may be transformed into half circles and formed another dynamical set B which contained curves and half circles.
We discus transformations which occur on this new dynamical set:

Transform (1):

This curve in R^2-space belongs to B_{dyn} transformed under the action of time into loop. This transformation is a type of folding.

Transform (2):

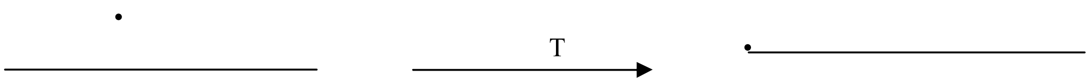
At the same moment of the time, we deduce all half circles of different radius transformed into loop. This loops different in curvature and convexity.

Now, let us consider a new set of geometric figures which are not manifolds and discussed the transformations which will be manifolds.

Case (C):

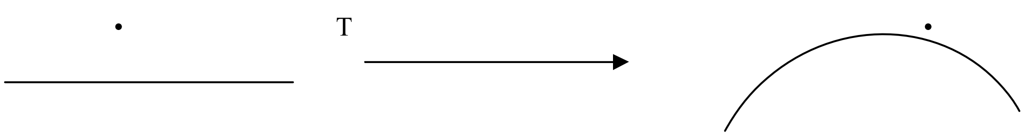
Let A = {Lines and points} in R^2-space, we have the following transformations:

Transform (1):

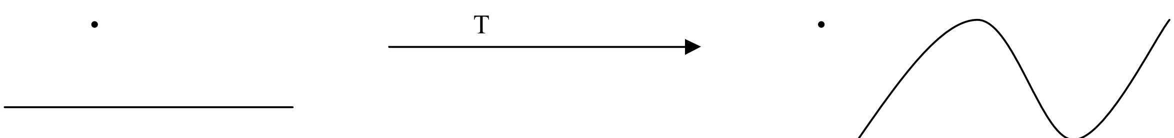


After the change of the time, the point moves to coincide with the line. We notice that the point changes its position and the result is a manifold. But A is not a manifold where T a map from A into B (T: A $\to$ B , B $\subset$ A).

Transform (2):



Under the action of the time, the line transformed into a curve and coincided with the point. In this case, we notice that the result is manifold with different curvature or change in convexity like



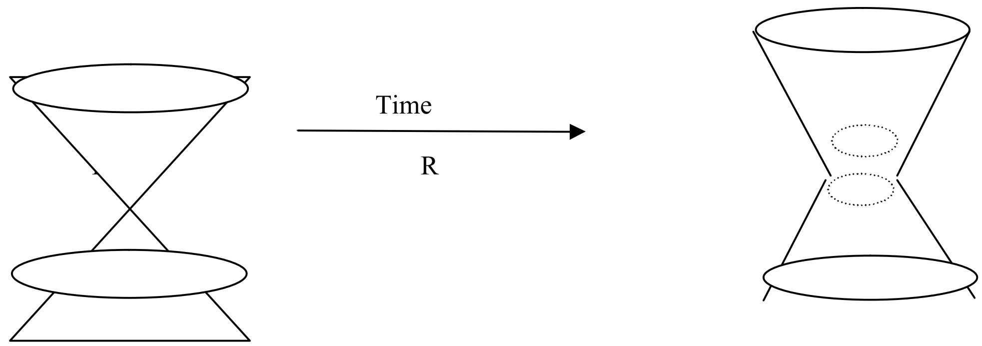
Transform (4):

This desperation on the line transforms the system of line and point into set of points. Every one of this set of points formed a manifold of dimension zero.

Case (D):

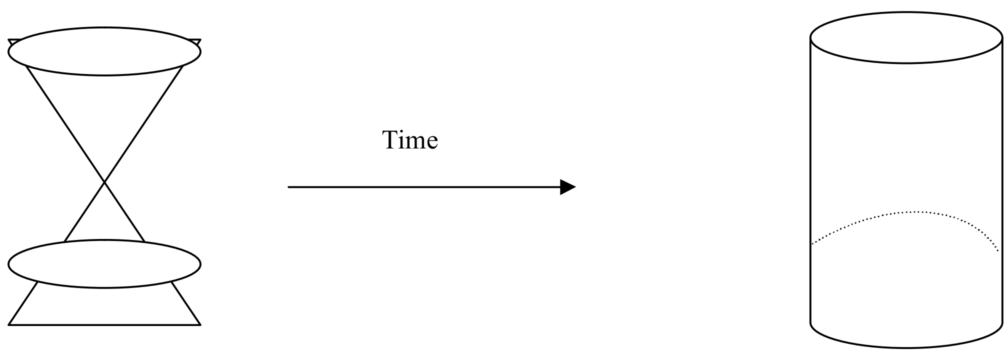
Consider a dynamical set A = {set of geometric figure which are not manifolds} for example A = {double cone, two disjoint spheres} defined on R^3-space.
Under the change of time, we discuss transformations on this set into set of manifolds by the following transformations:

Transform (1):

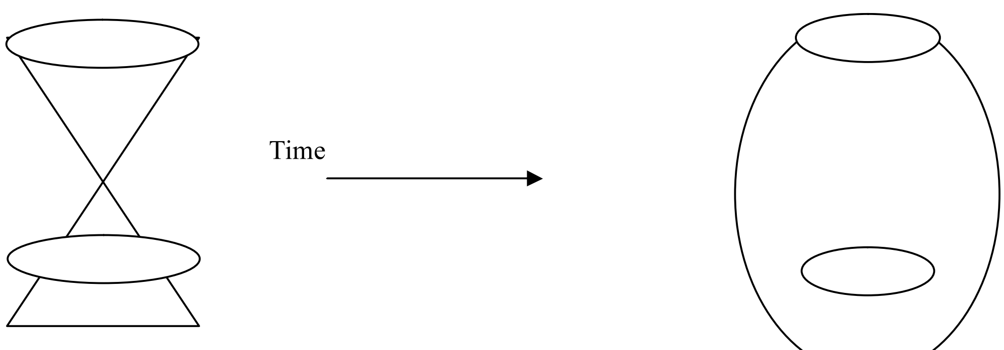


Under the change of the time, the double cone which are not manifold transformed by retraction into manifold without the vertex where R represent map from cone into cone without point p (R: C $\longrightarrow$ (C-{P}))

Transform (2):

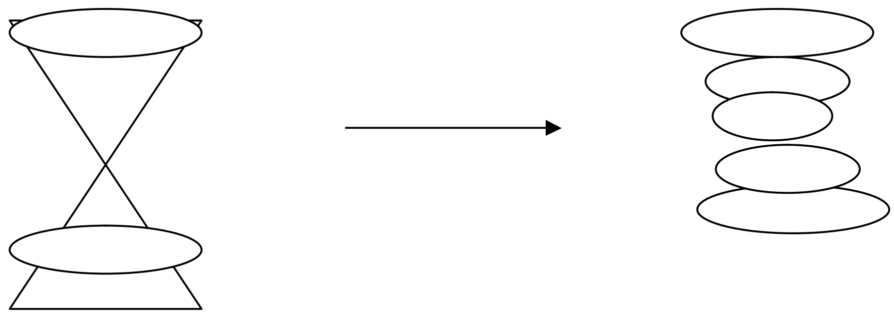


In this case, with the change of the time we notice change in double cone under the action of balling up such that the double cone transformed into cylinder which is a manifold or may be transformed into ellipsoid (cutting from two sides) like



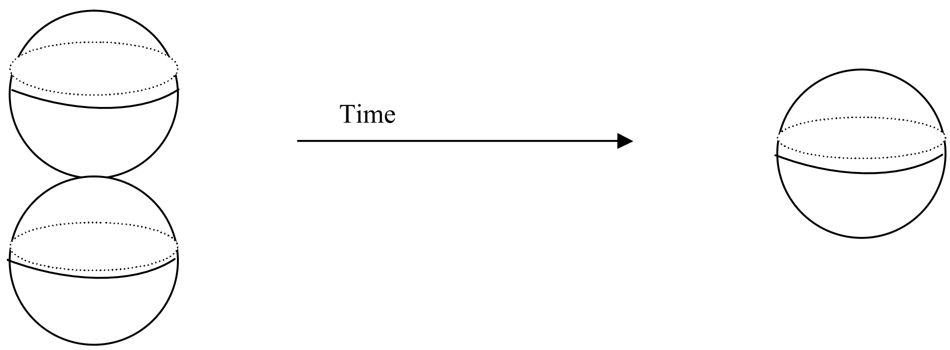
We notice that the double cone under the increasing of balling up transformed into cutting ellipsoid from edges which not represent a manifold with changes in curvature, algebraic properties (equation), volume (extension).

Transform (3):



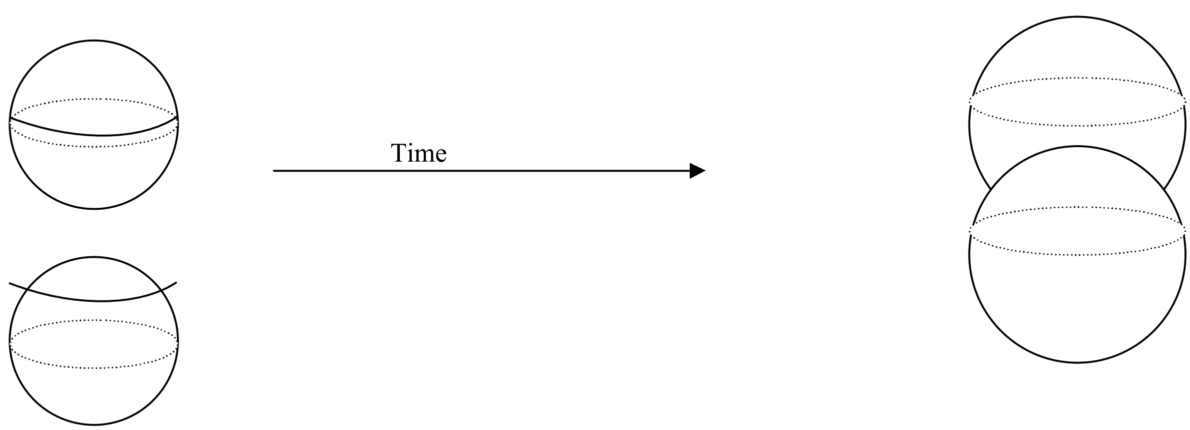
This transformation under the action of the time, the double cone transformed into set of sectors (represents circles of different radius). We notice that we deduce a dynamical space (transform from R^3-space into R^2-space). Every one of this circles represented a manifold.

Transform (4):



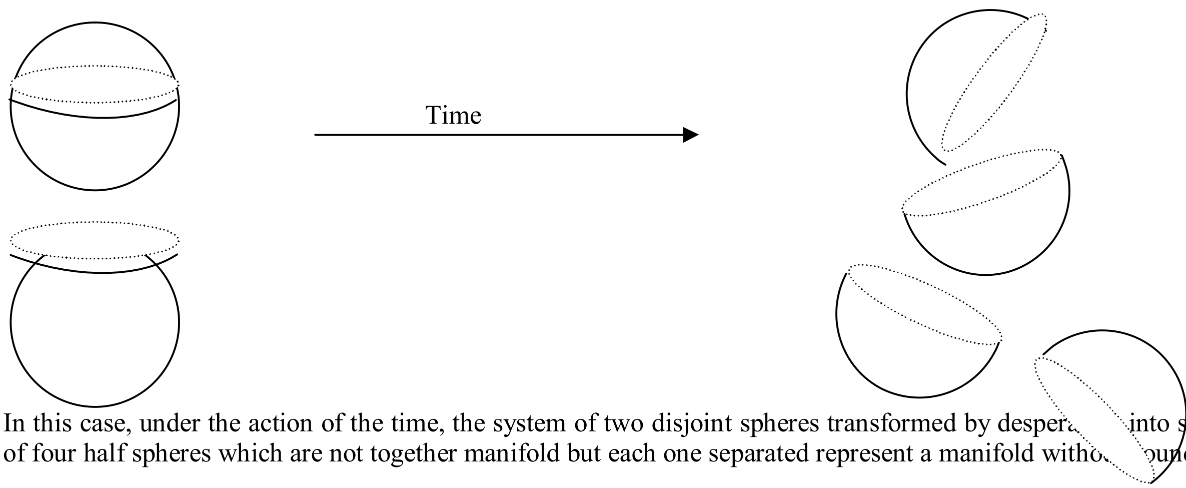
Under the change of the time, the two disjoint spheres which are not manifold together transformed by folding into one sphere which is a manifold.

Transform (4):



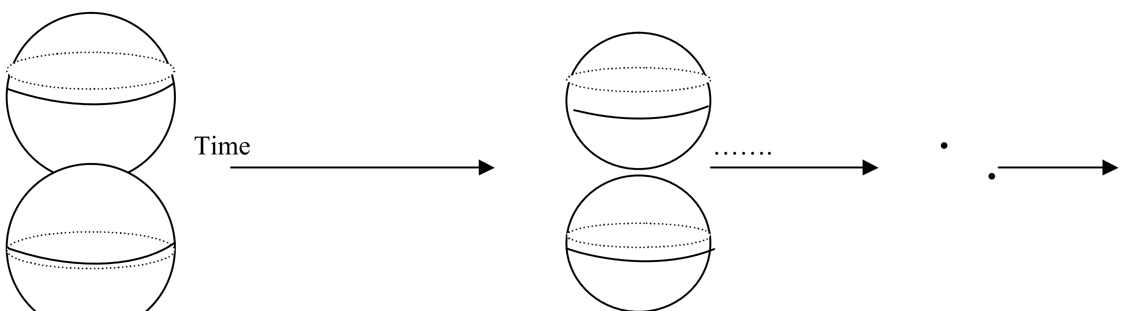
Under the action of balling up two disjoint spheres have the same radius; we deduce two joined spheres with larger radius which represent together manifold.
We notice that the dimension is not changed but the change in volume.

Transform (5):



In this case, under the action of the time, the system of two disjoint spheres transformed by desperation into system of four half spheres which are not together manifold but each one separated represent a manifold without boundary.

Transform (6):



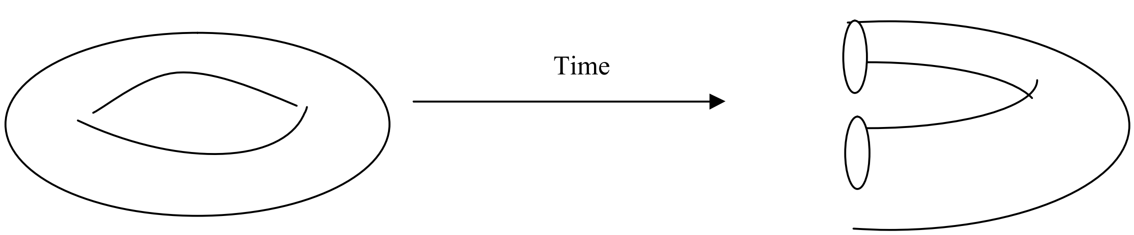
Under the action of the time, the two disjoint spheres transformed into two points which are different in dimension (from manifolds of dimension two into manifold of dimension zero).

In general, consider another dynamical set defined on R^n-space consists of set of spheres with varies radius and disjoint. We notice that the system of this spheres transformed under the action of the time into only one sphere (manifold) or under cartesian product into T^n (torus of dimension n).

<u>Case (E):</u>

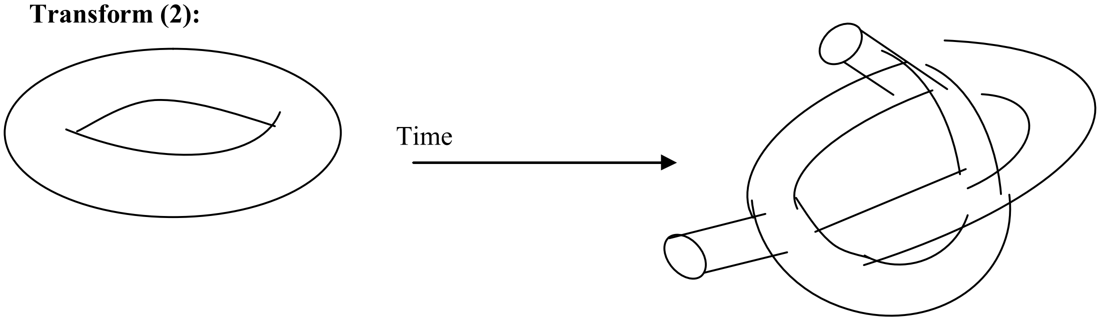
Let A = {set of all tori} $\subset$ R^3-space. Now, we discuss the transformation which occurs on A by changing of time. There are the following transformations:

Transform (1):



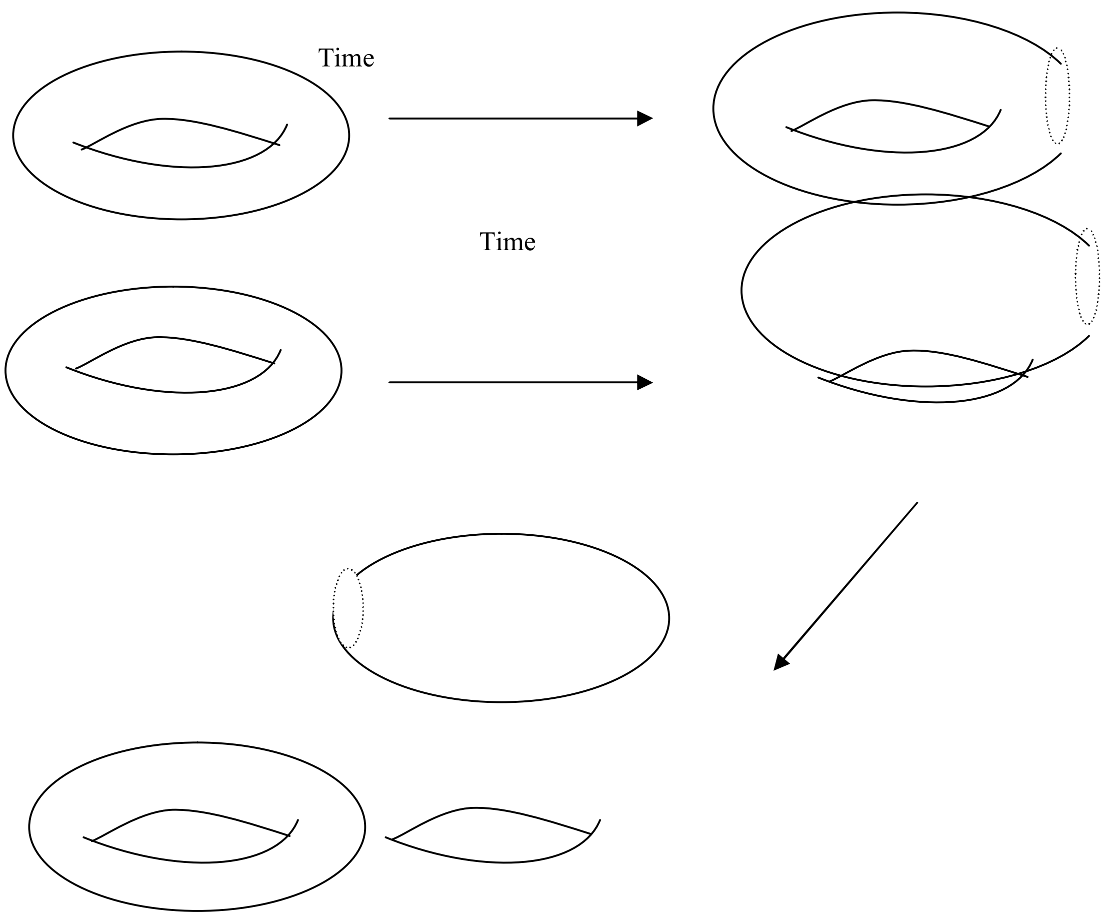
Under the action of the time, the torus transformed into half torus which has the same dimension.

Transform (2):



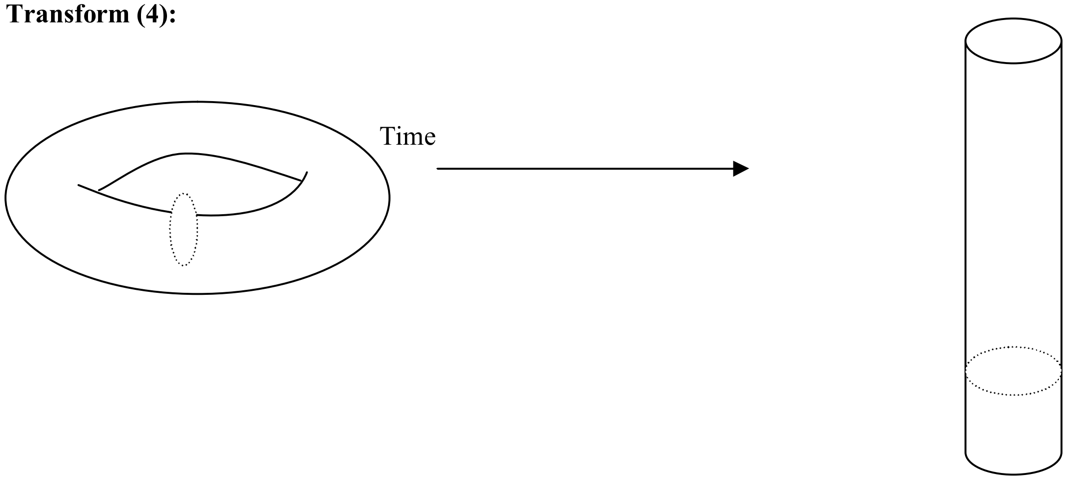
Under the action of the time, the torus transformed into knot. This transformation is a type of folding.

Transform (3):



Under the changing of the time, we deduce that sum of any two torus of this set of geometric figures gives geometric figure which represented a connected sum of two torus. We notice that the space of these geometric figures may be converted into dynamical space.

Transform (4):



Under the action of the time, the torus is transformed by cutting and straightens it out to form the cylinder. Also, under the action of the time, the result cylinder is cutting and transforming into rectangular which is different in dimension and algebraic properties.

4-References

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