Theory of Optimal Tax Rate "A theory and a mathematical equation using calculus to double the size of collected taxes by applying a tax rate below 6%"

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Abstract: There is a common misconception that to increase the size of collected taxes, governments should increase the annual tax rate, but this in fact leads to the degression of total size of collected taxes on the long run. The optimal tax rate is determined by the initial capital, the rate of profit of the project, project costs and the number of years during which the tax shall be collected i.e. the virtual lifetime of the project.
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1- Theory of inverse proportionality of tax size to tax rate:

There is a common misconception that to increase the size of collected taxes, governments should increase the annual tax rate, but this in fact leads to the degression of total size of collected taxes on the long run. This is because of the degression of the profit remaining after the deduction of tax, which is supposed to be added to the initial capital to start a new capital in the following year after withholding the tax. That is, by increasing the annual tax rate, the amount of money remaining after the deduction of the tax will be lesser than the case if we did not increase the tax rate because of slow growth of the capital.

On the other hand, the reduction of the annual tax rate leads to a rapid growth of capital and we can formulate the previous reasoning in the following statement.

The statement of the theory:

"When you increase the tax rate on annual income of commercial or industrial activity after deduction of costs (taxable amount), the size of the final collected taxes decreases year after year more than if you did not increase the tax rate, assuming the cost is fixed"

Definition of the size of the collected taxes:

It is the total final size of calculated taxes year after year on the project.

- **Theoretical assumptions:**

1. The remainder of the money after the tax deduction is used all as new capital for the project in the year following the tax-withholding year.
2. Fixed costs (but the theory remains valid at variable costs, but the optimal tax rate will be calculated for each cost separately)

- **Proof of the theory:**

It is well known that the tax is withheld from the net profit after deduction of the project costs.

And net profit depends on the project's capital and is directly proportional to it. Whenever you increase the capital, the net profit increases and vice versa.

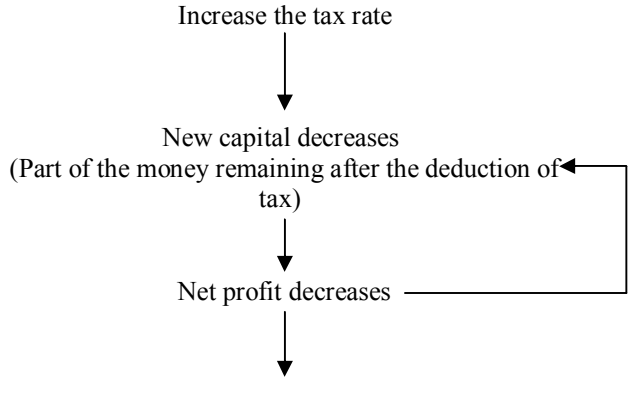
When you deduct a bigger tax from the net profit of the project, (by the application of a bigger tax rate), the remaining part of the money after deduction of tax is less than if you did not increase the tax rate.

Assuming that, the remaining part of the money after deduction of tax is re-used all as new capital for the project in the following year.

Year after year, the capital increases to a lesser degree than if you did not increase the tax rate.

Since the net profit decreases more than if you did not increase the tax rate, the final size of the taxes collected year after year decreases as well despite the fact that the tax rate is greater because the capital, with a bigger rate of tax, increases to a lesser extent as can be seen in the below numerical example.

2. Illustrative chart for the theory



3. Theory of optimal tax rate degression

- Definition of the optimal tax rate:

It is the tax rate that gives the maximum possible size of the final collected taxes after a certain number of years.

- **The statement of the theory**:

"Optimal tax rate for the project decreases when you increase the number of years during which to collect the tax"

It is clear by substitution in the equation of optimal tax rate with a different number of years, we find that the optimal tax rate decreases as the value of the number of years increases as in the below numerical example.

4 - Mathematical equation to derive the optimal tax rate

The optimal tax rate is determined by the initial capital, the rate of profit of the project, project costs and the number of years during which the tax shall be collected i.e. the virtual lifetime of the project.

We can proceed as follows:

For the first year:

Let M_1 be the initial project capital, where M_1 is the cost of goods produced or sold in that first year.

Now let D be the profit percentage of the project.

Thus, Gross profit in the first year = (D) (M_1)

Assuming the project costs is C and the net profit (taxable amount) is F, then

Net profit in the first year of the project is equal to gross profit minus costs as follows:

$F_1 = (M_1)\ (D)-C$

Now let V be the size of the tax and let X be the tax rate then Size of the tax for the first year is equal to $V_1 = (X)\ (F_1)$

For the second year:

The capital of the second year is equal to $M_2 = M_1 + F_1-V_1$

And the net profit in the second year is equal to

$F_2 = (M_2)\ (D)-C$

Thus, Size of the tax in the second year is equal to $V_2 = (X)(F_2)$

- **For any year Y**

Capital at the beginning of the year is $M_y = M_{y-1} + F_{y-1}-V_{y-1}$

And net profit is equal to $F_y = M_y D - C$

The size of the tax is equal to $V_y = X F_y$

Now let the size of the collected taxes year after year be $\sum V$

Where

$$\sum V=V_1+V_2+ \dots +V_y$$

Then $\sum V=(X)(\sum F)$

Now we have $\sum V$ as a function in the variable X of the Y-degree. From calculus, we know that this function has its maximum value at the value of X that satisfies the equation

$\{d/dx\ (\sum V)=0\}$

5- Guidance for the Ministry of Finance in determining the tax rate

A - The optimal tax rate, with respect to the government and at the same time just for the project, varies according to project profit and costs of capital must be calculated separately for each project.

B - The optimal tax rate should decrease as you increase the target number of years during which to collect taxes.

6 – Verification of the theory through a numerical example

Assuming initial capital in the first year of the project is thousand pounds for ease of calculation, then

$M_1 = 1000$

Assuming a 20% profit rate of the project

& assuming the costs of the project is fifty pounds

For the first year:

$M_1 = 1000$

$F_1 = M_1 D - C = 1000\ (20/100) -50 = 150$

$V_1 = XF_1 = 150X$

For the second year

$M_2 = M_1 + F_1-V_1 = 1000 +150-150\ X = 1150-150X$

$F_2 = M_2D-C = (1150-150X)(\ 20/100) -50 = 180-30X$

$V_2 = XF_2 = X\ (180-30X) = 180X-30X^2$

$\sum F=F_1+F_2=150+180-30X=330-30X$

$\sum V=X\sum F=X(330-30X)=330X-30X^2$

For the third year

$M_3=M_2+F_2-V_2=(1150-150X)+(180-30X)-(180X-30X^2)=$

$=1330-360X+30X^2$

$F_3=M_3D-C=(1330-360X+30X^2)(20/100)-50=216-72X+6X^2$

$V_3=XF_3=X(216-72X+6X^2)=216X-72X^2+6X^3$

$\sum F=F_1+F_2+F_3=546-102X+6X^2$

$\sum V=546X-102X^2+6X^3$

For any year y

$M_y=M_{y-1}+F_{y-1}-V_{y-1}$

$F_y=M_yD-C$

$V_y=XF_y$

$\sum F=F_1+F_2+F_3+...+F_y$

$\sum V=X\sum F$

∑V is a function in the variable X of degree Y and the root of its differentiation is the optimal tax rate

Now substituting for the number of years Y = 60, then We get the optimal tax rate = 0.11 or 11%

To prove that it is truly the optimal tax rate, we can substitute for X with different smaller values & larger values than optimal tax rate, we shall find that the size of collected tax is maximum only at the optimal tax rate. For example if we take X as 15% , we shall find that size of taxes after 60 years equals 1632500 pounds which is smaller that the value obtained at optimal tax rate which is 1720900 pounds. We also note that the difference is 88400 pounds which is a considerable figure.

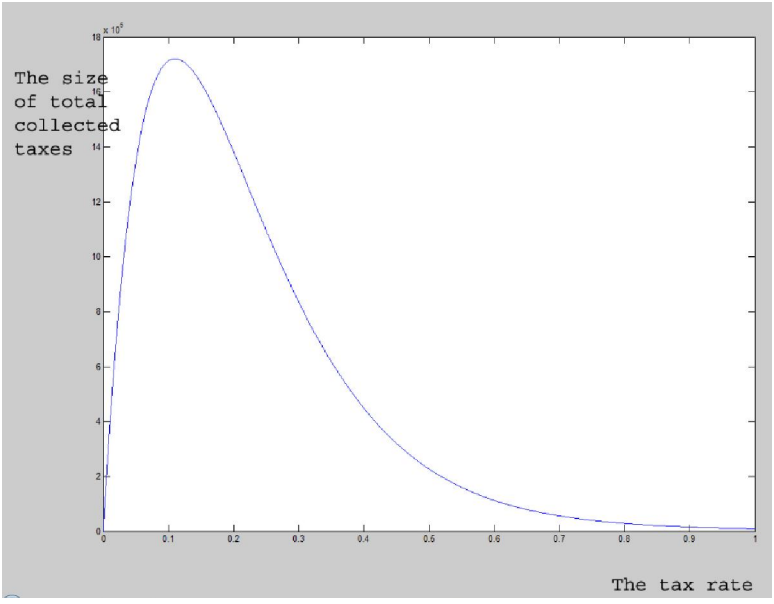
On substituting for the number of years Y= 100 , we shall find that the optimal tax rate becomes 6.34 % .

The previous example can be summarized by a comparison between the size of the tax collected at the optimal rate and corresponding size at a greater tax rate in the following table (Table 1):

Table 1. The previous example

Item	Value	Relationship	Value
The tax rate	11%	<	15%
The initial capital for the project	1000 pounds	=	1000 pounds
Profit rate of the project	20%	=	20%
Net profit for the first year	150 pounds	=	150 pounds
Size of the tax in the first year	16.5 pounds	<	22.5 pounds
Capital in the second year	1133.5 pounds	>	1127.5 pounds
Net profit in the second year	176.7 pounds	>	175.5 pounds
Size of the tax in the second year	19.437 pounds	<	26.325 pounds
The capital of the third year	1290.8 pounds	>	1276.7 pounds
Net profit in the third year	208.1526 pounds	>	205.335 pounds
Size of the tax in the third year	22.8968 pounds	<	30.8002 pounds
The size of the collected taxes after 3 years	58.8338 pounds	<	79.6253 pounds
The size of the collected taxes after 60 years	1720900 pounds	>	1632500 pounds

7- Graph showing the relationship between tax rate and the size of total collected taxes and the validity of the theory



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