Live Load Distribution Factors Suitable For Concrete Bridges Under Ecp 201 And Euro Code 1991 Loading

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Abstract: Live Load Distribution Factors (LLDF) method is widely used to simplify bridge deck analysis. In this method, the superstructure is modeled as a single spine. The straining actions obtained are then distributed among the different girders using the LLDFs. Finite element modeling is used for the analysis of several concrete bridges of slab-on-girder and box-girder types. The modeling details are verified by comparing deflections with site measurements. Over 6000 cases were analyzed to calculate LLDF for truck loading specified by the Euro Code EN 1991 and Egyptian Code ECP 201. Non-linear regression analysis is applied on the obtained results to calibrate the parameters of LLDF equations. LLDF equations suitable for use with ECP 201 and EN 1991 truck loading for straight and skew concrete bridges of slab-on-girder and box-girder types are proposed.

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Key Words: Live Load distribution factors; LLDF; Bridge Design; Bridge Decks; Euro Code 1991, Egyptian Code ECP 201

1. Introduction

Live Load Distribution Factor (LLDF) method is a simple method used for preliminary design and fast checks of bridge superstructures. In this method, the bridge superstructure is analyzed as a single spine, Figure 1, where the determination of design quantities is simple and fast. The LLDF are then are used to distribute the obtained values among the main girders.

Because of the simplicity of the method, design codes and standards [e.g. 1, 2] provide equations that can be used to evaluate the LLDF for most common bridge systems. The applicability of such formulas is subject to specified limits. The LLDF are specific for certain live load values and patterns.

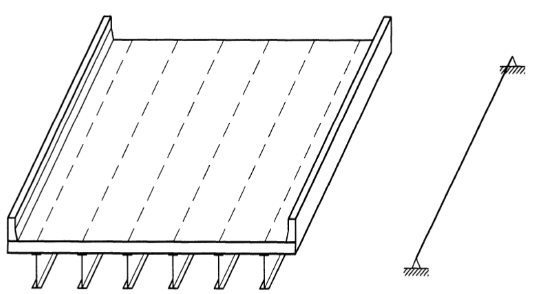


Figure 1: Slab-on-Girder system and its representation in a spine model.

The work presented in this paper aims at developing appropriate LLDF equations suitable for the live loads specified in the Euro Code 1991[6]. As the Egyptian Code ECP 201[5] adopts similar live loads, the proposed LLDFs are also applicable in that context.

2. Methodology

Grillage analysis [3] is utilized to build numerical models capable of accurately represent actual bridge superstructures. Verification of the modeling is made by comparing numerical results with field measurements. A parametric study is then carried out to evaluate LLDF for different parameters. Finally, non-linear regression analysis is used to obtain useful equations to evaluate the LLDF.

Modeling Technique

A bridge deck is modeled by longitudinal and transverse frame element as shown in Figure 2. Where longitudinal elements represent webs or girders and transverse elements represent cross diaphragm and/or slab.

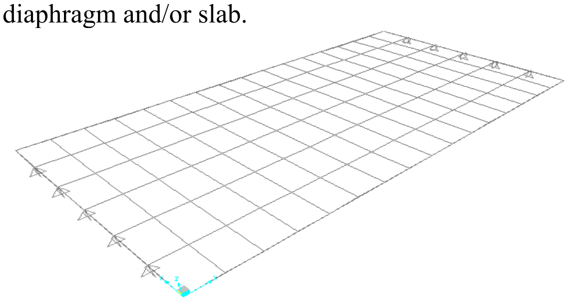


Figure 2: Grillage model representing a bridge deck.

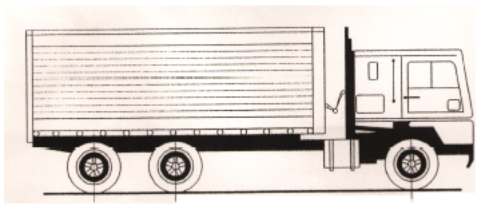
Grillage method for numerical modeling of bridge superstructures is a simple method that considers the interaction between bridge components as well as the difference between stiffness in the longitudinal and transverse directions [3]. Section moment of inertia is calculated at the centroid of each represented part. Transverse elements are spaced by 1.25 m. to generate a smooth mesh. Supports are modeled as roller at one end and hinged at the other end. SAP 2000 [4] is used as the numerical tool.

Model Verifications

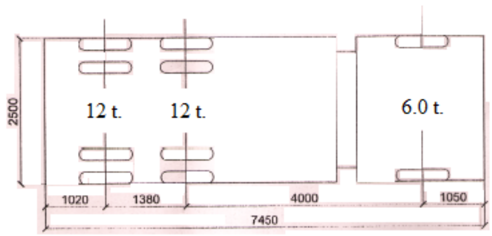
Data on five bridge load tests, conducted by the Concrete Research Laboratory at Cairo University, were used to verify the modeling technique described above. Data on bridge system, geometry, loading values and arrangements were used to build five models, one for each test. The tests were performed on the following structural systems:

Slab-on-Girder system (two tests).
Straight Box girder system (two tests).
Curved Box Girder system (one test).

One type of truck was used in all loading tests. The truck weighed 30 tons. Truck dimensions, axial spacing and axial loads are given in Figure 3. Numerically obtained deflections are compared to the corresponding field measurements.



Elevation



Truck 30.0 Ton

Figure 3: Truck dimensions and loads.

In load test 1 (extension of 6th of October bridge) a simply supported span of 25.10m between axes (R15) and (R16) was loaded by six typical trucks. The slab on girder superstructure has five main girders. Deflections were measured at quarter points, at mid span and at third quarter points. Comparison between measured and calculated deflections is shown in Figure 4. The difference between calculated and measured deflections varied between (- 4%) and (+17%).

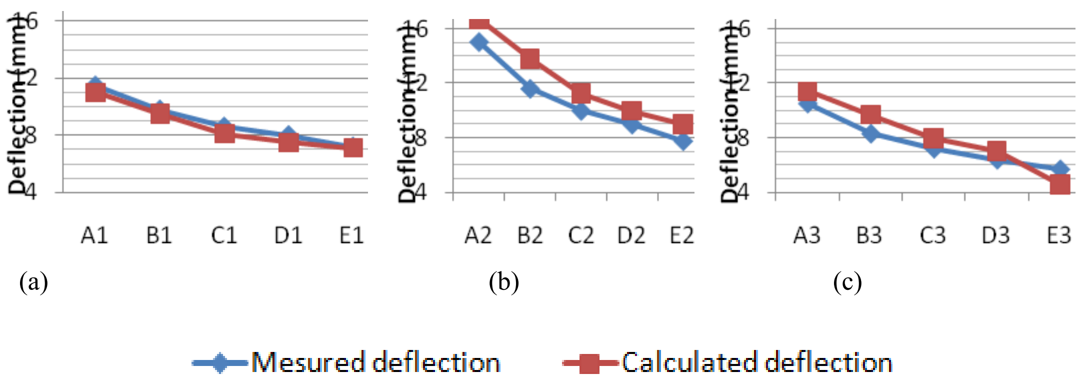


Figure 4: Load Test No 1 - Comparison between Model results and Load Test measurements (a) At quarter points; (b) At mid-span and (c) At third-quarter points

In load test 2 (Saft El-Laban bridge), a simply supported span of 24.10m between axes (6L) and (7L) was loaded by four typical trucks. The slab on girder superstructure has five main girders. Deflections were measured at quarter points, at mid span and at third quarter points of each girder. Comparison between measured and calculated deflections is shown in Figure 5. The difference between calculated and measured deflections varied between (-7%) and (+20%).

In load test 2 (Saft El-Laban bridge), a simply supported span of 24.10m between axes (6L) and (7L) was loaded by four typical trucks. The slab on girder superstructure has five main girders. Deflections were measured at quarter points, at mid span and at third quarter points of each girder. Comparison between measured and calculated deflections is shown in Figure 5. The difference between calculated and measured deflections varied between (-7%) and (+20%).

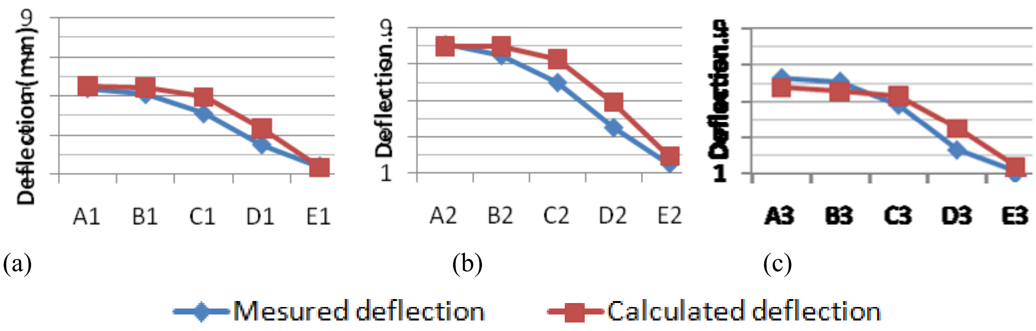


Figure 5: Load Test No 2 - Comparison between Model results and Load Test measurements (a) At quarter points; (b) At mid-span and (c) At third-quarter points

In load test 3 (Saft El-Laban bridge), a simply supported span of 30.15m between axes (15U) and (16U) was loaded by four typical trucks. The box girder superstructure has two cells. Deflections were measured at quarter points, at mid span and at third quarter points of each web. Comparison between measured and calculated deflections is shown in Figure 6. The difference between calculated and measured deflections varied between (+3%) and (+23%).

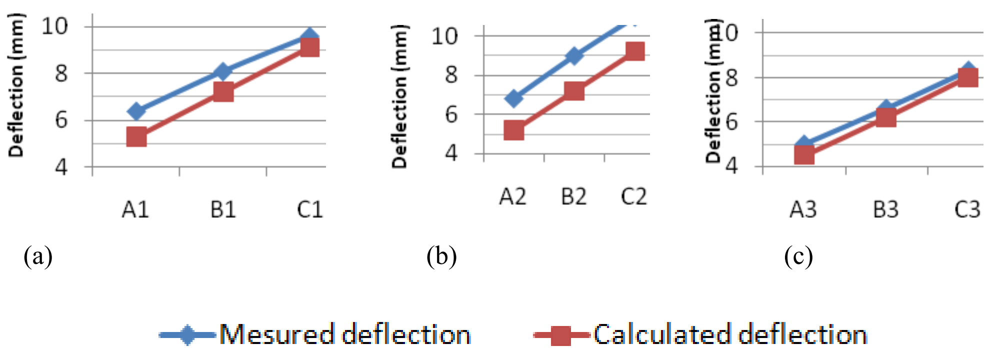


Figure 6: Load Test No 3 - Comparison Between Model Results And Load Test Measurements (A) At Quarter Points; (B) At Mid-Span And (C) At Third-Quarter Points

In load test 4 (Saft El-Laban bridge), two equal spans of 29.0m each between axes (50) and (52) were loaded by four typical trucks. The box girder superstructure has two cells. Deflections were measured at quarter points, at mid span and at third quarter points of the interior web. Comparison between measured and calculated deflections is shown in Figure 7. The difference between calculated and measured deflections varied between (0%) and (+3.8%).

In load test 5 (Saft El-Laban bridge), two equal spans of 34.0m each between axes (44) and (46) were loaded by four typical trucks. The box girder superstructure has two cells. The bridge has a horizontal curve of radius 125 m. Deflections were measured at quarter points, at mid span and at third quarter points of the interior web. Comparison between measured and calculated deflections is shown in Figure 8. The difference between calculated and measured deflections varied between (0%) and (+13%).

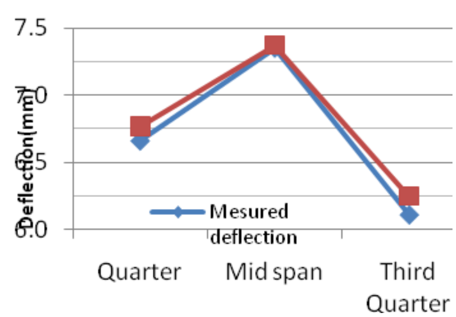


Figure 7: Load Test No 4 - Comparison between Model results and Load Test measurements along interior web

As seen from the details of the comparisons above, the modeling technique adopted is capable of capturing the measured trends of deformation and the values, in most of the cases considered.

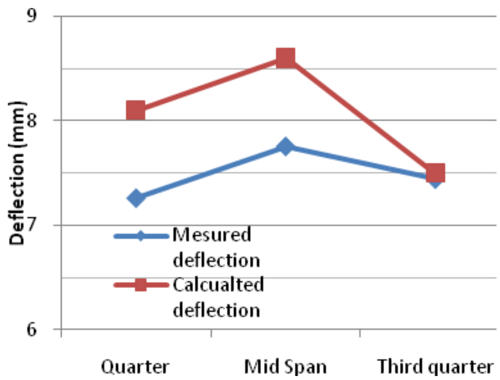


Figure 8: Load Test No 5 - Comparison between Model results and Load Test measurements along interior web

Parametric Study

The aim of this work is to evaluate LLDF for concrete bridges of the slab-on-Girder and box-girder systems of common articulations. Different parameters are studied including: span length, number of spans, girder spacing, skew angle and the existence of cross-girders. Cross-girders are only considered for slab-on-girder Bridges. Either one cross girder was located at mid span or two cross girders were located at third points. In both cases, the cross girders were of the same width as the main girders and were 200mm less in depth. Cross-girders are modeled by their full stiffness (un-cracked section). The range of the studied parameters is given in Table 1.

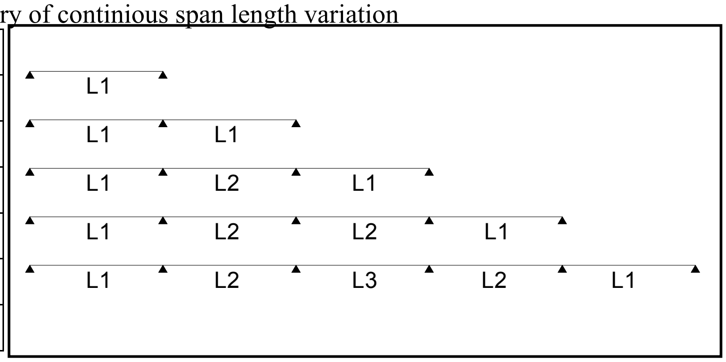
To consider the effect of continuity, different span lengths were used. Spans length variation in continuous spans is given in Table 2.

Table 1: Summry of Studied Parameters

Parameter	**Range**
Span Length (L)	20, 25, 35 and 45 m.
Number of spans	1, 2, 3, 4 and 5
Girder spacing (S)	1.80, 2.40, 3.20 m.
Skew Angle	0, 30, 40 and 50□
Cross girder	No cross girder, One or Two

Table 2: Summry of continious span length variation

Spans Length data		
L1	L2	L3
20	25	25
25	35	35
35	45	45
20	35	45
45	--	--



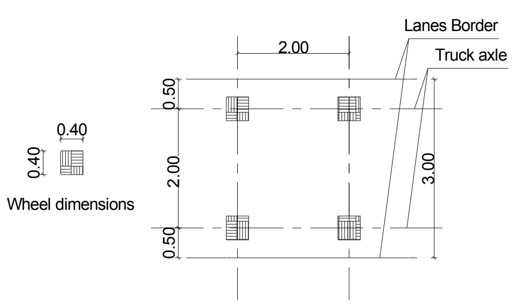


Figure 9: Truck configuration in Euro Code 1991 and Egyptian Code ECP 201

Euro Code 1991 [6] and Egyptian Code ECP 201 [5] specify three geometrically similar trucks of total weights 600kN, 400kN and 200kN equally divided on all wheels. Figure 9 shows the dimensions and configurations of the trucks. Loading, considered in the numerical models, is either for single truck or for multiple trucks. In the sequel, for brevity, the former is referred to as “Single Loading” while the later is referred to as “Multi Loading”. Also, “Interior” refers to loading on the middle interior girder while “Exterior” refers to loading on the outer most girder. In exterior girder loading, a sidewalk is taken 0.5m while notional lanes are aligned next to sidewalk. In single lane loading, only the main lane loading is applied, while in multi-lane loading three lanes are applied. For interior girders, an axle of main lane is placed at center line of the interior girder and the other lanes are aligned next to the main lane. In multi-lane interior loading only two lanes are applied as the calculated LLDFs is found to be greater than in the case where three lanes are loaded.

3. RESULTS

The results of the above described analyses are illustrated in Figures 10 to 17. In the figures, the calculated LLDFs are plotted against span length (*L*), girder spacing (*S*), (*S/L*) ratio, transverse stiffness, ratio of longitudinal stiffness (k_g) and the product of span (*L*) and the slab thickness (t_s) cubed.

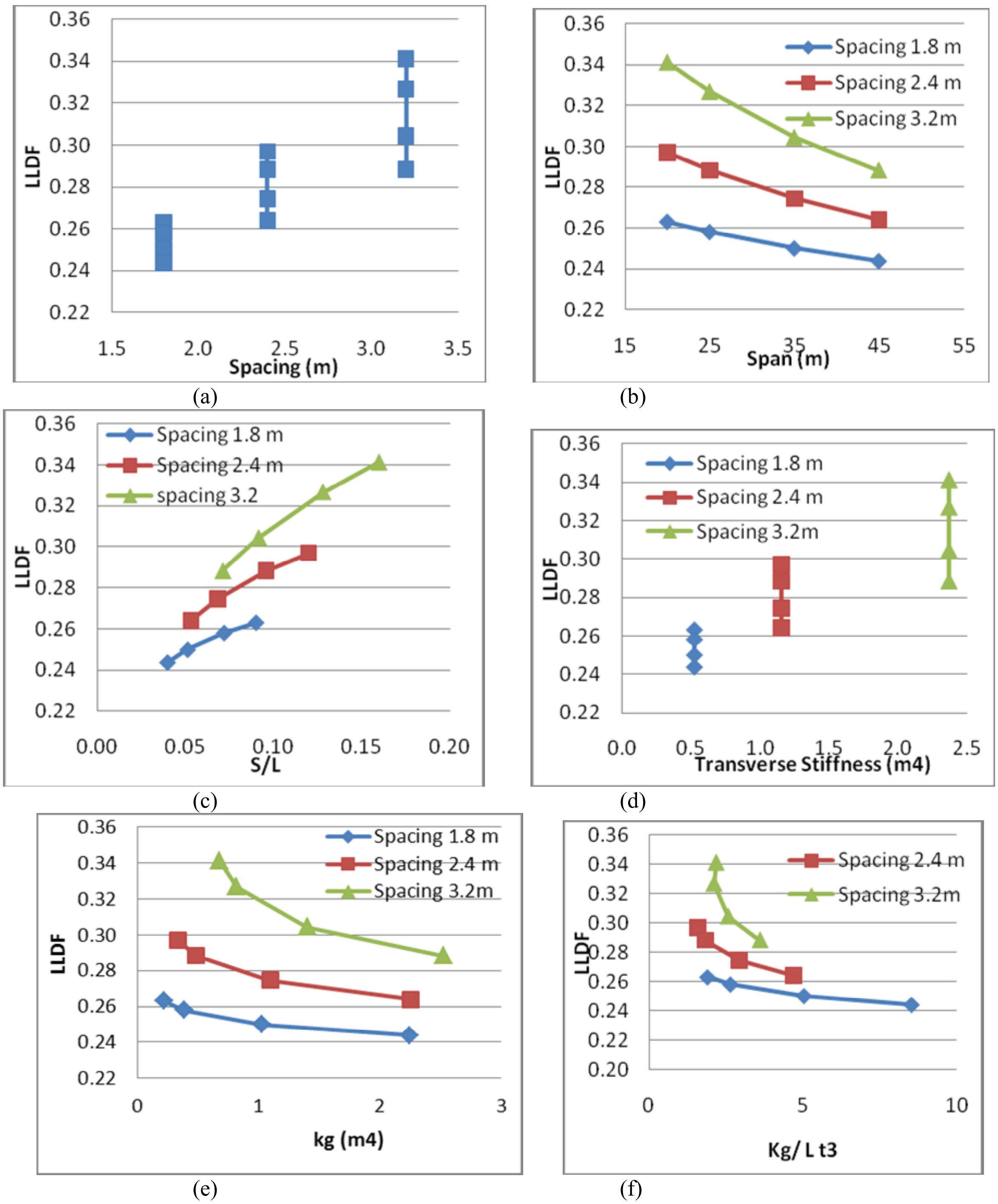


Figure 10: Relations between LLDF and studied parameters for Box Girder Bridge - multi interior lane loaded.

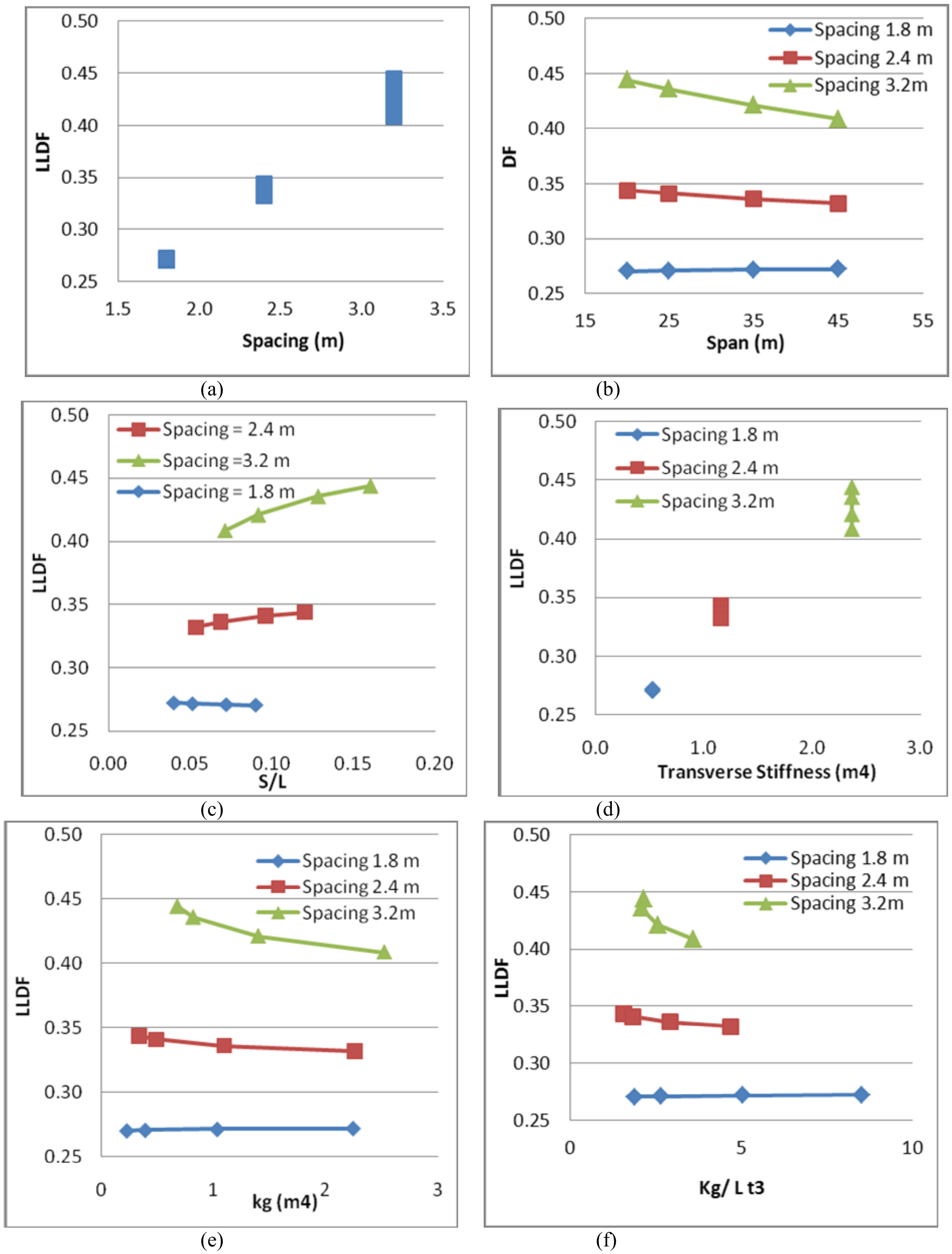


Figure 11: Relations between LLDF and studied parameters for Box Girder Bridge - multi exterior lane loaded.

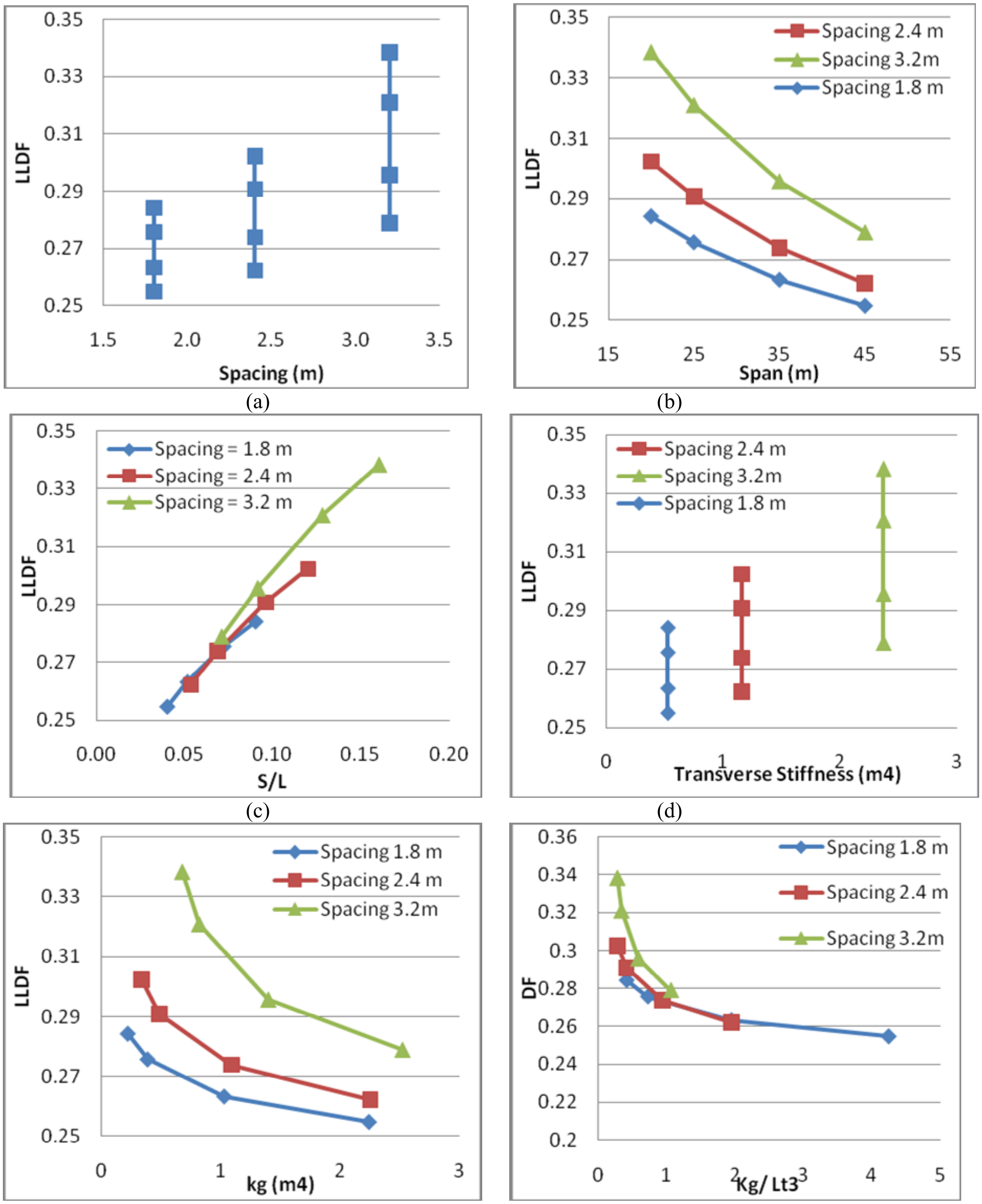


Figure 12: Relations between LLDF and studied parameters for Box Girder Bridge - single interior lane loaded

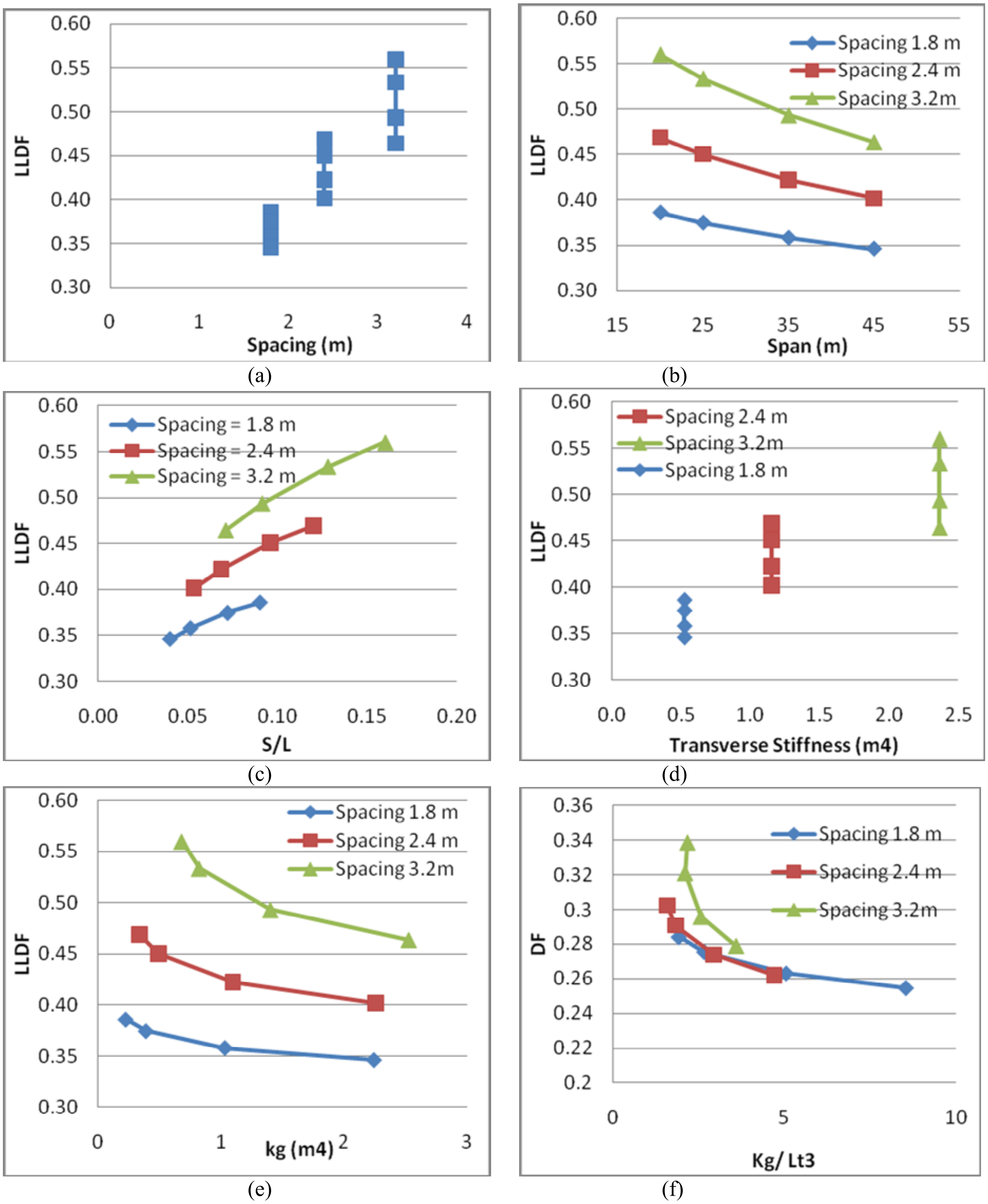
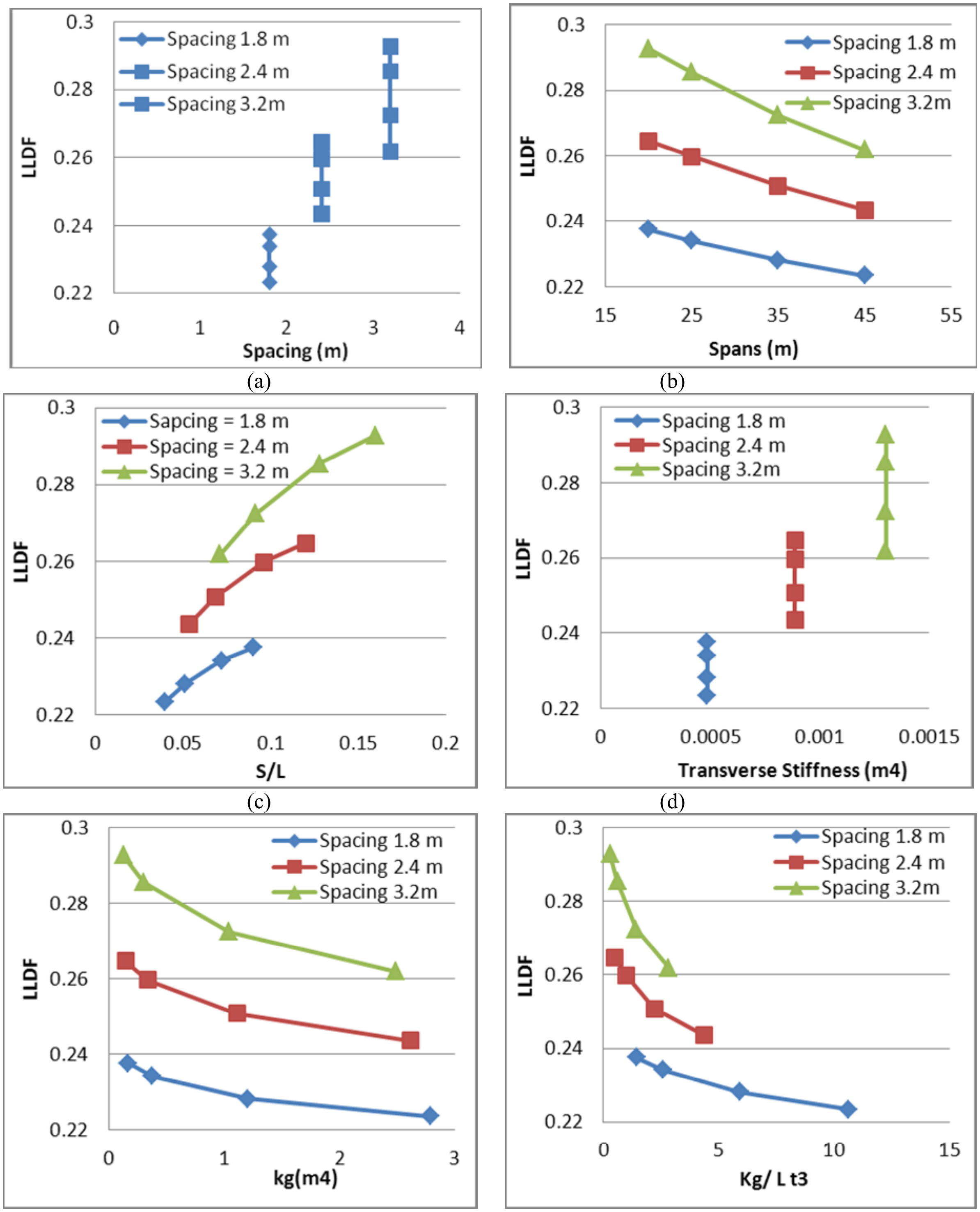


Figure 13: Relations between LLDF and studied parameters for Box-Girder Bridge - single exterior lane loaded



(a) (b) (c) (d) (e) (f)

Figure 14: Relations between LLDF and studied parameters for Slab-on-Girder Bridge - multi interior lane loaded

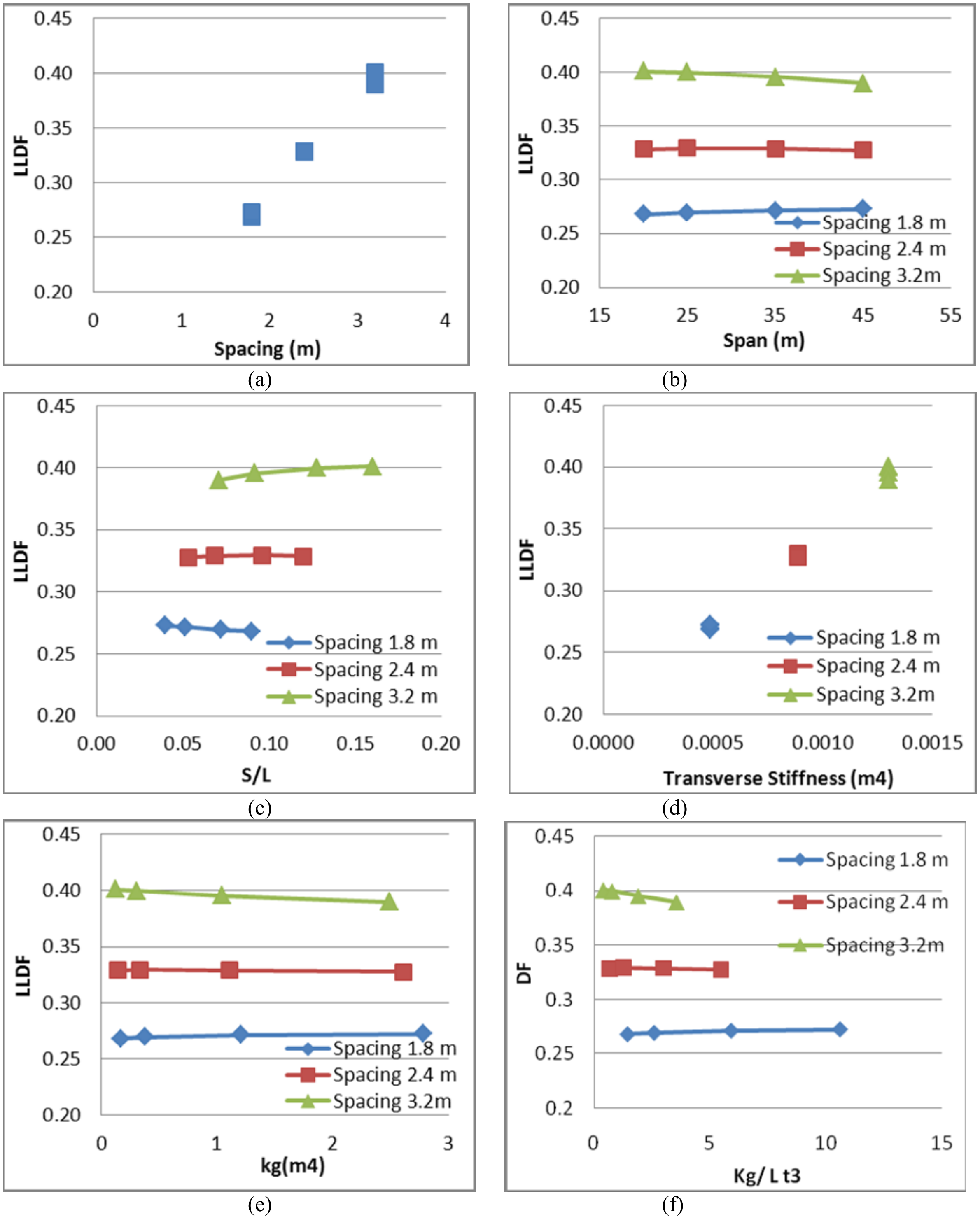
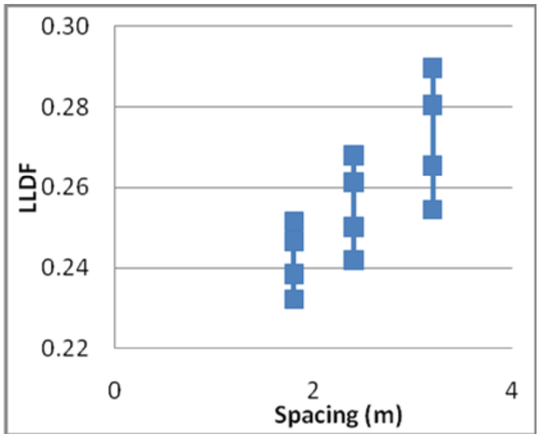
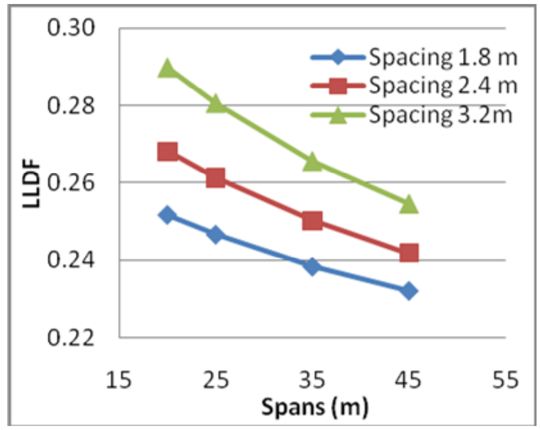
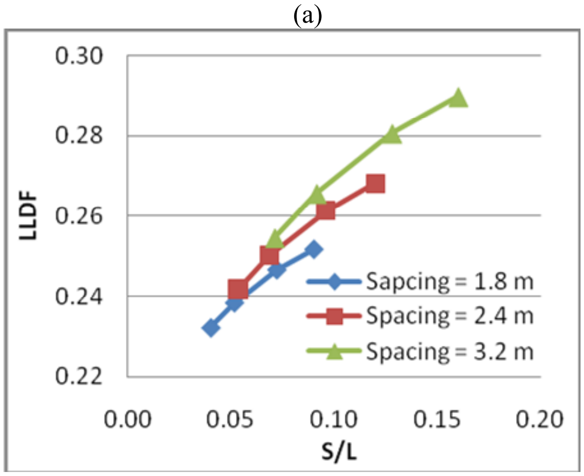


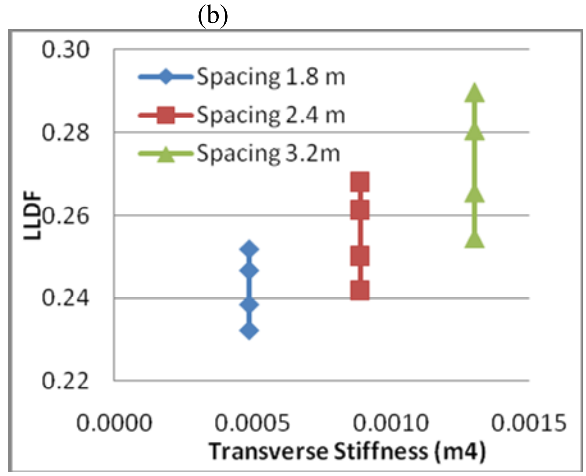
Figure 15: Relations between LLDF and studied parameters for Slab-on-Girder Bridge - multi exterior lane loaded



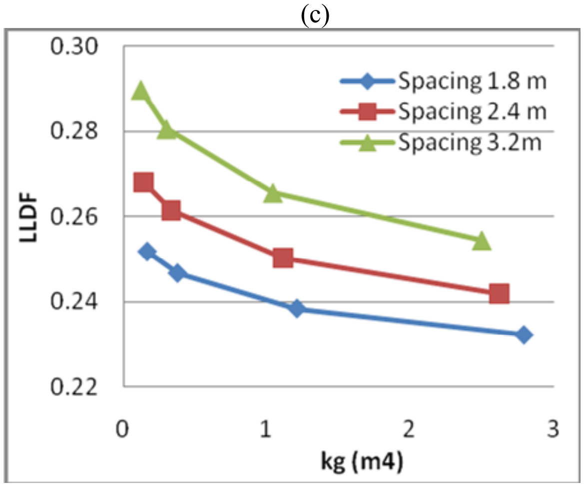


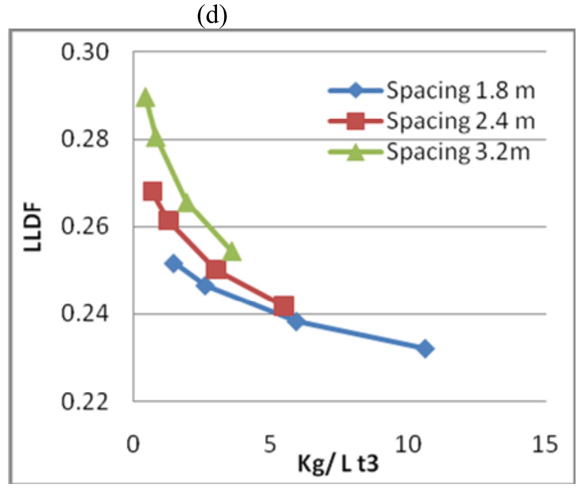
(a) (b)





(c) (d)





(e) (f)

Figure 16: Relations between LLDF and studied parameters for Slab-on-Girder Bridge - single interior lane loaded

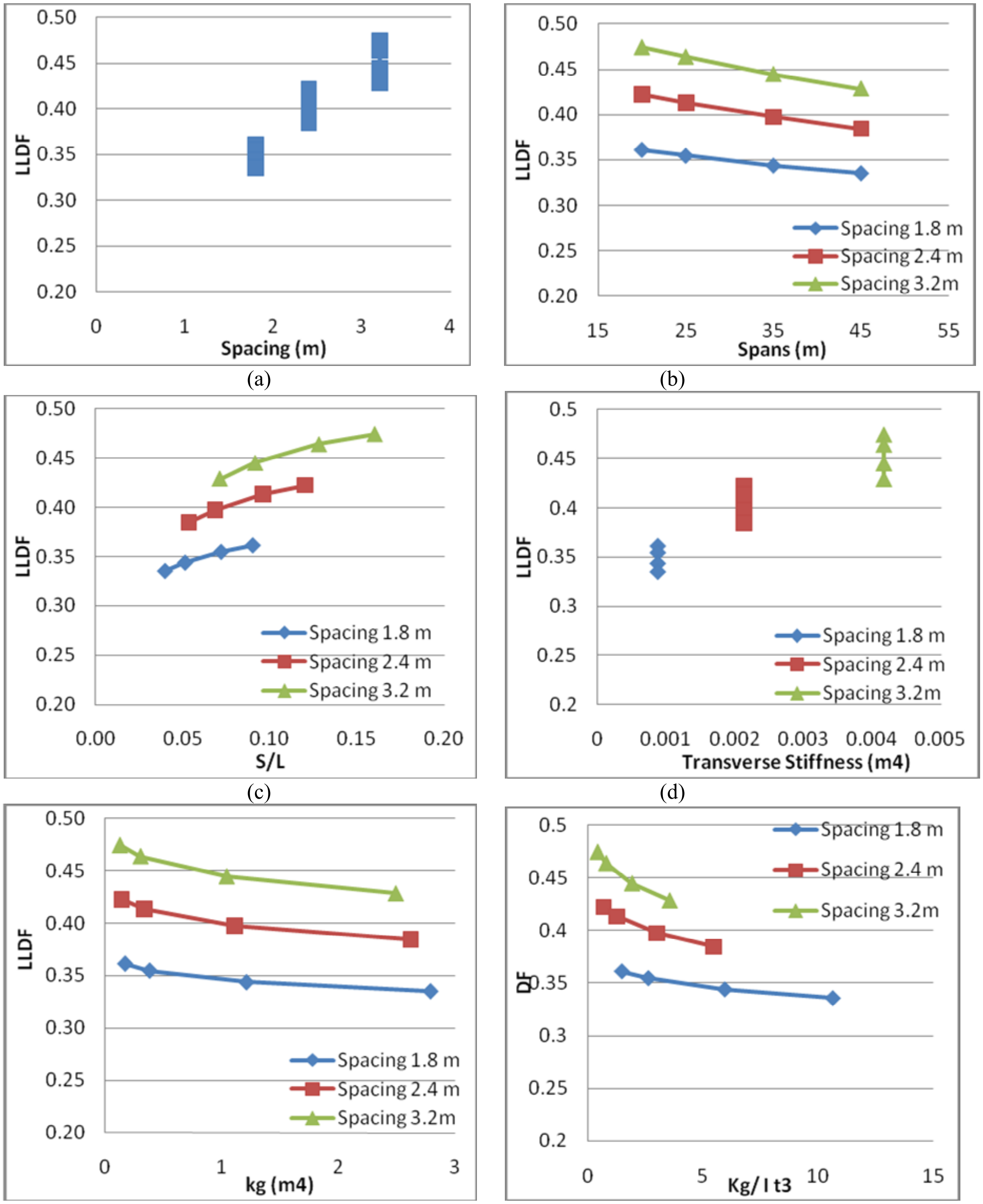


Figure 17: Relations between LLDF and studied parameters for Slab-on-Girder System - single exterior lane loaded

Regression Analysis

The previous analyses would only be useful to bridge designers in the form of formulas that can simply be used to evaluate the LLDF for common bridge articulations. Non-linear regression analysis is used to best fit the numerical results into usable formulas. Two formulas are proposed, the first (Equation 1 below) has the same general form as the AASHTO code formula, where the main parameters are (*S/a*), (*S/L*) and ($K_g/L\ t^3$) and S is girder spacing, L is span length; t is slab thickness and kg longitudinal stiffness ($I_g+A_g e^2$). All terms are defined as above. The second formula (Equation 2 below) is proposed by the authors.

$$LLDF = C_1 + \left(\frac{S}{a}\right)^b \left(\frac{S}{L}\right)^c \left(\frac{K_g}{Lt_s^3}\right)^d \quad (1)$$

The constants (C_1), (a), (b), (c) and (d) are to be evaluated via regression analyses. It is to be noted that the constant ($C1$) reflects the fact that the LLDF is non-zero even when the girder spacing (S) approaches zero. This is evidenced by many studies and is also reflected in the AASHTO LRFD 1998 [7]).

$$LLDF = C_1 + C_2\left(\frac{S}{L}\right)^a + C_3\left(\frac{K_g}{I_{trans}}\right)^b \quad (2)$$

The constants (C_1), (C_2), (C_3), (a) and (b) are to be evaluated via regression analyses.

IPM SPSS software [8] is used to carry out the regression analyses. The method of least squares is utilized to estimate the values of the unknown parameters. The coefficient of determination R^2 is used in the context of statistical models whose main purpose is the prediction of future outcomes on the basis of other related information. For the equations above, the coefficient of determination R^2 can explained as the ratio between the variance of the proposed equation prediction (modeled) to the total variance of the data set. It is a statistical measure of how well the regression line approximates the real data points. An R^2 of 1.0 indicates that the regression line perfectly fits the data.

R^2 obtained from regression analysis using Equation 2 varied from 0.7 to 0.99 while R^2 for Equation 1 varied from 0.95 to 0.99. Accordingly, Equation 1 is considered more suitable for the intent and is therefore presented in the sequel.

1. PROPOSED LIVE LOAD DISTRIBUTION FACTORS

The resulting formulas are presented in Tables 3 for Box-Girder system and in Table 4 for Slab-on-Girder system. Different formulas are presented for single spans, two equal spans and multiple spans. The regression analyses did not show significant difference between the analyzed cases of three, four and five spans. Accordingly, one set of formulas are presented for multiple spans. Single/multiple and interior/exterior are defined as above. For example, single-exterior refers to the distribution factor for exterior girder under loading of a single truck.

5.3 Effect of Skewness

It is known that skew supports change the load path. Load transferred through the shortest path would therefore have an effect on LLDFs. This effect depends on the amount of skewness. Multiplication factors that are to be used to correct the proposed formulas (given in Tables 3 and 4) are given in Table 5.

5.4 Effect of Intermediate Cross-Girders on the LLDFs for sagging moment

Cross-girders are typically used to allow larger load sharing between main girders. In the case of closed box girders, this is not very significant as the existence of top and bottom slabs provide sufficient lateral distribution in most cases. It is also noted that, especially in box girders, the existence of cross girders significantly increases the complexity of construction. In this section, the effect of cross girders on LLDFs in slab and beam system is investigated. Two arrangements are studied: one cross girder at mid-span and two cross girders at third points. Table 6 provides correction factors for the LLDFs for the two cases and for 0□, 30□, and 50□ skew angles.

Table 3: Proposed Equations for Box-Girder System

Case	Single Span	Two Equal Spans	Multiple Spans
Single Interior	$0.218+\left(\frac{S}{4000}\right)^{0.527}*\left(\frac{S}{L}\right)$	$0.24+\left(\frac{S}{6000}\right)^{0.315}*\left(\frac{S}{L}\right)$	$0.245+\left(\frac{S}{6478}\right)^{0.454}*\left(\frac{S}{L}\right)^{0.941}*\left(\frac{K_g}{Lt_s^3}\right)$
Single Exterior	$0.116+\left(\frac{S}{5645}\right)^{0.518}*\left(\frac{S}{L}\right)^{0.292}*\left(\frac{K_g}{Lt_s^3}\right)$	$0.0+\left(\frac{S}{4097}\right)^{0.40}*\left(\frac{S}{L}\right)^{0.22}$	$0.139+\left(\frac{S}{4675}\right)^{0.589}*\left(\frac{S}{L}\right)^{0.245}*\left(\frac{K_g}{Lt_s^3}\right)$
Multi Interior	$0.202+\left(\frac{S}{10000}\right)^{0.739}*\left(\frac{S}{L}\right)^{0.644}*\left(\frac{K_g}{Lt_s^3}\right)$	$0.197+\left(\frac{S}{10000}\right)^{0.691}*\left(\frac{S}{L}\right)^{0.589}*\left(\frac{K_g}{Lt_s^3}\right)$	$0.182+\left(\frac{S}{10000}\right)^{0.65}*\left(\frac{S}{L}\right)^{0.509}*\left(\frac{K_g}{Lt_s^3}\right)$
Multi Exterior	$0.0+\left(\frac{S}{7767}\right)^{0.704}*\left(\frac{S}{L}\right)^{0.125}*\left(\frac{K_g}{Lt_s^3}\right)^{0.058}$	$0.0+\left(\frac{S}{7116}\right)^{0.72}*\left(\frac{S}{L}\right)^{0.118}*\left(\frac{K_g}{Lt_s^3}\right)^{0.048}$	$0.054+\left(\frac{S}{6995}\right)^{0.921}*\left(\frac{S}{L}\right)^{0.081}*\left(\frac{K_g}{Lt_s^3}\right)$

*Range of Applicability:

$1.8\text{m} \le s \le 3.2\text{m}$; $180\text{mm} \le t_s \le 250\text{mm}$; $20\text{m} \le L \le 45\text{m}$; No. of spans ≤ 5; No intermediate diaphragms

Table 4: Proposed Equations for Slab-on-Girder System*

Case	Single Span	Two Equal Spans	Multiple Spans
Single Interior	$.165+\left(\frac{S}{5000}\right)^{0.085}*\left(\frac{S}{L}\right)^{1.0}*\left(\frac{K_g}{Lt_s^3}\right)^{0.241}$	$.19+\left(\frac{S}{5000}\right)^{0.199}*\left(\frac{S}{L}\right)^{1.0}*\left(\frac{K_g}{Lt_s^3}\right)^{0.185}$	$0.225+\left(\frac{S}{5000}\right)^{0.608}*\left(\frac{S}{L}\right)^{1.0}*\left(\frac{K_g}{Lt_s^3}\right)$

Single Exterior	$0.0 + \left(\frac{S}{6000}\right)^{0.301} * \left(\frac{S}{L}\right)^{0.272} * \left(\frac{K_g}{Lt_s^3}\right)^{0.06}$	$.0 + \left(\frac{S}{6000}\right)^{0.308} * \left(\frac{S}{L}\right)^{0.229} * \left(\frac{K_g}{Lt_s^3}\right)^{0.038}$	$0.0 + \left(\frac{S}{6000}\right)^{0.389} * \left(\frac{S}{L}\right)^{0.164} * \left(\frac{K_g}{Lt_s^3}\right)^{0.01}$
Multi Interior	$97 + \left(\frac{S}{7395}\right)^{0.736} * \left(\frac{S}{L}\right)^{0.864} * \left(\frac{K_g}{Lt_s^3}\right)^{0.151}$	$93 + \left(\frac{S}{8288}\right)^{0.713} * \left(\frac{S}{L}\right)^{0.722} * \left(\frac{K_g}{Lt_s^3}\right)^{0.11}$	$0.191 + \left(\frac{S}{9036}\right)^{0.846} * \left(\frac{S}{L}\right)^{0.544} * \left(\frac{K_g}{Lt_s^3}\right)$
Multi Exterior	$0.0 + \left(\frac{S}{6000}\right)^{0.596} * \left(\frac{S}{L}\right)^{0.253} * \left(\frac{K_g}{Lt_s^3}\right)^{0.091}$	$0.0 + \left(\frac{S}{6000}\right)^{0.653} * \left(\frac{S}{L}\right)^{0.205} * \left(\frac{K_g}{Lt_s^3}\right)^{0.07}$	$0.0 + \left(\frac{S}{6000}\right)^{0.75} * \left(\frac{S}{L}\right)^{0.141} * \left(\frac{K_g}{Lt_s^3}\right)^{0.035}$

*Range of Applicability: $1.8\text{m} \le s \le 3.2\text{m}$; $180\text{mm} \le t_s \le 250\text{mm}$; $20\text{m} \le L \le 45\text{m}$; No. of spans ≤ 5; No intermediate diaphragms

Table 5: Proposed LLDF correction factors for bridge skewness *

Case	Box-Girder	Slab-on-Girder
Single Interior	$e^{0.052\theta}$	$e^{0.05\theta}$
Single Exterior	1	1
Multi Interior	$e^{0.247\theta}$	$e^{0.25\theta}$
Multi Exterior	$e^{0.265\theta}$	$e^{0.222\theta}$

* ⍰ is the bridge skew angle in radians

Table 6: Ranges of LLDF reductions due to cross-girders for sagging moments in slab and beam bridges (%)

Loading Case	Continuity	No. of Cross-Girders	Skew Angle					
			0°		30°		50°	
			Min	Max	Min	Max	Min	Max
Single Exterior	Simple	One	4.8	20.7	4.0%	19.8	1.4%	18.0
		Two	4.8	10.9	5.1	11.1	5.6	11.5
	Two Equal	One	5.6	22.8	4.3	21.8	1.4	19.6
		Two	5.1	11.3	5.6	11.6	6.2	12.1
	Three Spans	One	6.8	26.3	2.3	25.0	0.8	22.3
		Two	5.8	14.8	6.3	14.6	5.4	13.7
	Four spans	One	6.8	26.2	5.0	28.7	0.8	22.2
		Two	5.8	14.8	6.3	17.4	6.0	13.7
	Five spans	One	6.8	28.1	5.0	25.7	0.8	22.7
		Two	5.8	17.0	6.3	17.2	6.0	16.4
Multi Exterior	Simple	One	0.3	13.2	0.1	14.0	3.0	15.2
		Two	2.6	7.7	2.8	8.6	4.6	10.5
	Two Equal	One	0.3	14.5	0.5	15.4	0.2	12.7
		Two	2.9	8.0	3.1	9.1	4.9	11.0
	Three Spans	One	0.5	16.9	1.0	17.6	3.0	18.5
		Two	3.1	9.8	0.6	10.8	5.7	12.0
	Four spans	One	0.5	16.8	1.0	17.5	3.0	18.4
		Two	3.1	9.8	2.8	10.8	5.1	12.0
	Five spans	One	0.5	17.3	1.0	18.0	3.0	18.9
		Two	3.1	10.7	2.8	11.6	5.1	14.7
Single Interior	Simple	One	0.0	19.8	0.0	19.7	1.7	21.2
		Two	3.2	10.5	3.6	10.8	6.0	16.2
	Two Equal	One	0.0	22.3	0.0	22.7	3.0	24.1
		Two	3.3	10.7	3.7	11.2	6.6	12.8
	Three Spans	One	0.0	33.1	0.0	31.6	3.4	29.8
		Two	3.9	21.6	4.4	20.9	7.6	20.4
	Four spans	One	0.0	33.0	0.0	31.5	3.3	29.5
		Two	3.9	21.6	4.4	20.8	6.3	20.6
	Five spans	One	0.0	37.7	0.0	35.5	3.3	32.4

		Two	3.9	26.6	4.4	25.3	6.3	23.8
Multi Interior	Simple	One	10.8	24.0	13.5	25.3	19.6	30.2
		Two	4.8	11.6	6.0	12.2	8.8	14.1
	Two Equal	One	12.5	26.0	15.3	27.9	21.8	32.9
		Two	5.2	12.0	6.5	12.5	9.5	14.4
	Three Spans	One	15.2	33.5	18.9	34.0	26.7	38.3
		Two	6.4	21.1	7.9	21.0	11.2	21.3
	Four spans	One	15.1	33.4	18.8	34.0	26.5	38.1
		Two	6.3	21.1	7.9	21.0	11.2	21.3
	Five spans	One	13.4	38.2	16.1	37.6	21.1	42.8
		Two	6.3	25.7	7.9	25.5	11.0	24.6

Conclusions

This works produces live load distribution factor (LLDF) equations suitable for the truck loads specified by the Euro Code[6] and the Egyptian Code[5] for roadway bridges. Twenty-four equations are given for interior and exterior girders of slab and beam and box-girder concrete bridges, single and multiple loadings. Effects of skew angles and the existence of cross-girders are also investigated. Correction multipliers to the LLDF are given for several skew angles. The effects of one cross-girder at mid-span or two cross-girders at third-points on the LLDF for sagging moments are also given in the form of corrections to the proposed equations.

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References

1. AASHTO. (1991): Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials,.
2. AASHTO LRFD (1991): Bridge design Specifications Published by the American Association of State Highway and Transportation Officials, third edition, 2004.
3. Hambly E.C., (1991)"Bridge Deck behavior," second edition,.
4. SAP (2000): V14.2 Manual, http://www.csiberkeley.com/sap2000
5. Egyptian Code for Loads and Forces on Buildings and structures, EC 205(2008). September.
6. BS EN (1991): Actions on Structures - Part 2: Traffic loads on bridges - July 2002.
7. AASHTO. (1998): Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials,.
8. Zokaie, T., Osterkamp, T. A., and Imbsen, R. A. (1991): Distribution of wheel load on highway bridges," National Cooperative Highway Research Program Report 12-26/1, Transportation Research Board, Washington, D.C.
9. IPM SPSS Statistics Software http://www-01.ibm.com/software/analytics/spss/products/statistics/whats-new.html.
10. Ezz Eldin Kamel (2012): Live Load Distribution Factors for Concrete Bridges suitable for ECP 201 and Euro Code Loading," M.Sc. Thesis, Structural Engineering Department, Cairo University.

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