Computing stiffness of co-central restrains with a circular yielding element

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Abstract: Restrained frames with yielding damper is one of useful usable systems in earthquake regions and areas in order to decrease the structure's respond in efforts resulted by an earthquake. We have studied steel frames with circular yielding element behavior in this paper and examined dimension, size and bending stiffness at rate of absorbing energy rate under cycle loading in them. We also studied circular element stiffness inside the frame and the amount of side stiffness was computed considering radius and inetria moment and it is shown that how we can increase this stiffness three times more by adding two members inside the circular yielding element. Finally we studied these systems' behavior by push over analysis and determined the effect of using circular damper at increasing structures behavioral coefficient.

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1. Introduction

This frame includes an external frame with co-centered restrain which the damping element is placed at the center of this restrains. This system is classified among inactive energy absorbers which is able to act as a damping system when earthquake happens without any energy receiving and absorbs the energy. This damping element can be applied in different ways; the rectangular one includes a central frame which is connected to surrounding frame by means of axle members. We could also connect a steel shit inside the central frame to increase this damping mechanism. This kind of elements stiffness is evaluated by means of patience and stability. Fewer research have been executed about circular elements, in a research by Mklek using a circular element with a box surface was suggested and its ability to waste more energy compared to a damping mechanism called TADAS under cycle loading was examined. Mofid and Tajamolian have shown that using a circular or rectangular element in center of the frame would lead to similar behavior when an earthquake emerges and to choose among one of these two could be done based on architectural or executive requirements.

2. Circular element stiffness

To obtain the stiffness of retrain frame with circular damping mechanism under side loads first we should determine each of their effect in stiffness of a frame.

To do so first we present stiffness of circular element under loading based on its axial. Based on relations in materials resistance shown on figure (1) a slim circular bar whit radius of R is under load in manner of an axial, decrease in length of AB and increase in CD is computed based on figure (1) and (2).

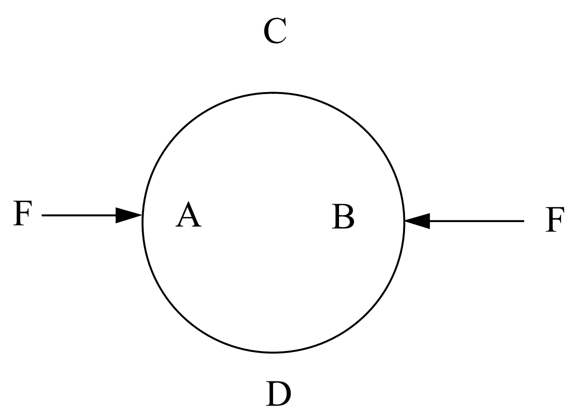


Figure 1

$$|\Delta AB| = \frac{FR^3}{4EI}\left(\square - \frac{8}{\pi}\right) \quad (1)$$

$$|\Delta CD| = \frac{FR^3}{2EI}\left(\frac{4}{\pi} - 1\right) \quad (2)$$

I and E are creating elastic and inertia at the axial level of circular element.

We could determine the amount of change in axial in axial members by means of simulation of the element placement situation in a frame with similar height and inset.

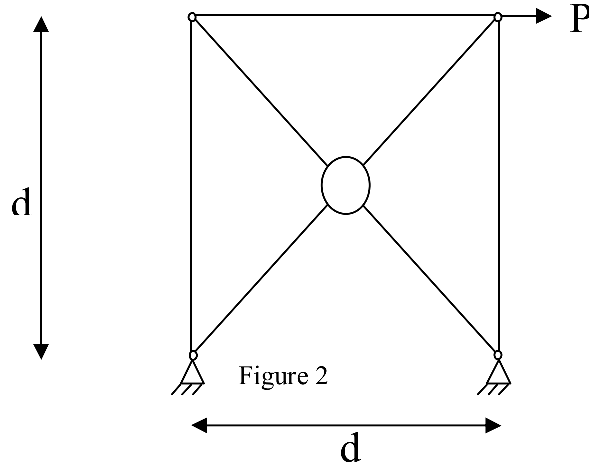


Figure 2

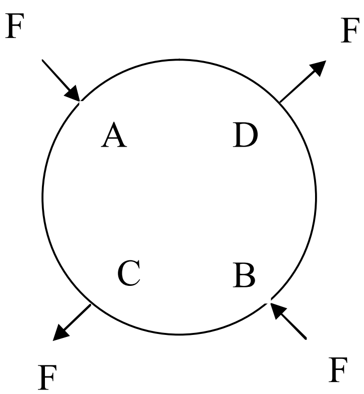


Figure 3. Determination of the stiffness of the circular element in the diagonal direction

$$|\Delta_{AB}| = |\Delta_{CD}| = \frac{FR^3}{2EI}\left(\frac{4}{\pi} - 1\right) + \frac{FR^3}{4EI}\left(\pi - \frac{8}{\pi}\right) \quad (3)$$

In fact in mentioned condition the amount of transition is equal to transitions resulted by changing the element shape along inserted powers of axial member.

Stiffness of circular element based on axial members is possible by means of relation number (4).

$$K_{CI} = \frac{F}{\Delta} = \frac{2EI}{R^3\left[\left(\frac{4}{\pi} - 1\right) + \frac{\pi}{2} - \frac{4}{\pi}\right]} = \frac{2EI}{R^3\left[\frac{\pi}{2} - 1\right]} = \frac{3.5039EI}{R^3} \quad (4)$$

Determining stiffness of damping element and axial members based on frame's axial by means of equivalent spring method:

Relative transition based on frames axial under side weight is equal to bending form change of circular element plus change of axial members' length. Therefore we can compute stiffness resulted from members by simulating these members as equivalent springs.

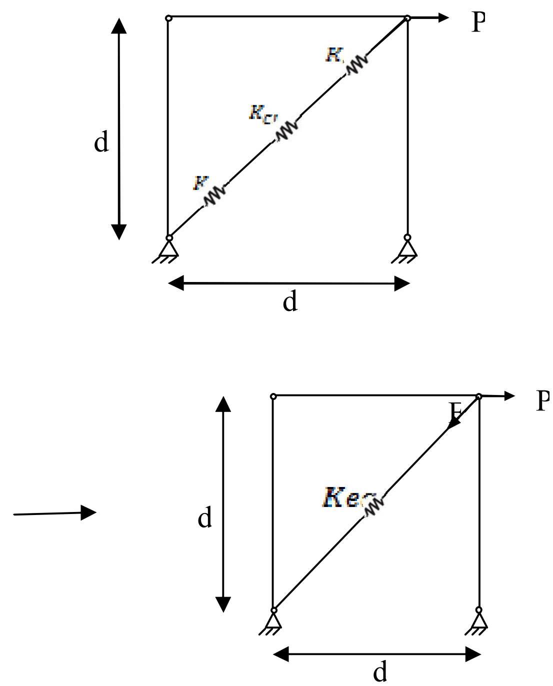


Figure 4

$$U_{eq} = U_b + U_{CI} + U_b \rightarrow \frac{F}{K_{eq}} = \frac{2F}{K_b} + \frac{F}{K_{CI}} \quad (5)$$

$$K_{eq} = \frac{K_b K_{CI}}{K_b + 2K_{CI}} \quad (6)$$

In mentioned relations F is in put reaction based on frame's axial, Ub amount of length change of each axial member based on the F and U_{CI} the amount of shape shift of circular element based on F. K_b and K_{CI} express stiffness of axial members and circular element.

Kb is evaluated by following relation:

$$K_b = \frac{2EA_b d}{(1-n)a} \quad (7)$$

A_b of every axial member's segment area and n ratio of length of circular element to frame's axial length and a is equal to frame's axial length which is obtained by number (8) relation.

$$a = d\sqrt{2} \quad (8)$$

d is equal to frame's inset.

By replacing circular element stiffness and axial members; equivalent stiffness based on axial members is obtained as relation in figure (9).

$$K_{eq} = \frac{7.0078EA_b I}{2A_b R^3 + 7.0078(1-n)aI} \quad (9)$$

We should note that to evaluated relation (9) we have used relation (4). Therefore equivalent stiffness amount (Keq) in mentioned relation is evaluated by powers interact based on AB and Ac as figure (3). This condition is based on border condition of circular element in restrained frame with a cross sectional restrain.

Evaluating total side stiffness:

In order to evaluate system's total side stiffness we should illustrate the stiffness resulted from axial members and circular element based on each of frame's axials.

Since circular element stiffness is equal based on two axials we could compute these two by relation (9).

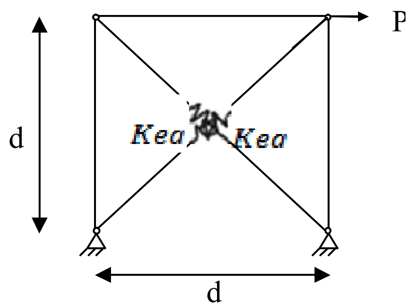


Figure 5

By means of equivalent stiffness and considering rigid tiroston with joint connections whole system's is determined by means of fallowing relation.

$$K = Keq\cos^2\frac{\Pi}{4} + Keq\cos^2\frac{\Pi}{4} = 2Keq\cos^2\frac{\Pi}{4} \qquad (10)$$

In case frame's size is changed we could compute circular element stiffness by means of relation (12). This relation shows axial figure change in each point of element affected by axial power.

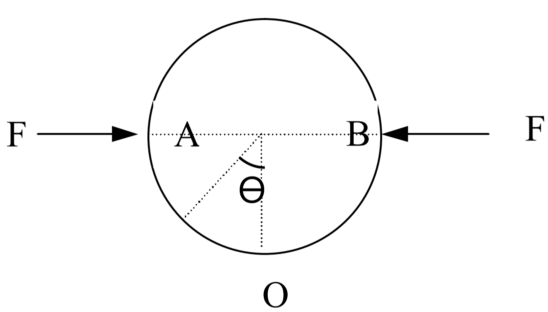


Figure 6

$$\Delta = \frac{FR^3}{4EI}\left(\cos\theta + \sin\theta - \frac{4}{\pi}\right) \qquad (12)$$

If in mentioned relation θ is rewritten based on axial member angle, circular element stiffness is evaluated by relation (13).

$$K_{CI} = \frac{4EI}{R^3[\Pi - 2\sin 2\beta - (\Pi - 4\beta)\cos 2\beta]} \qquad (13)$$

In mentioned relation βof the angle between axial member and horizon is based on figure (7).

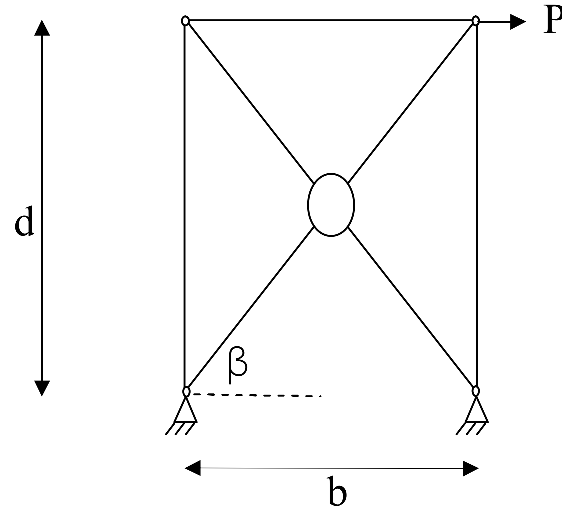


Figure 7

Total stiffness of the system is computed by considering stiffness of axial members by means of relations (5)to (11) by just this difference that instead of angle π/4 we apply amount of β in our evaluations and amount of A is also obtained based on following relation.

$$(14)\quad a = \sqrt{d^2 + b^2}$$

d and b are respectively equal to length of inset and frame's height.

Effect of bars' and pillars' stiffness in total stiffness

By means of springe idea which was mentioned in previous section we could also apply effect of pillars stiffness at the whole frame. We should note that in structures usually only porch of the roof has a high stiffness and since bars deal with these porches totally the same we could claim that pillars with rigid elements are joined to each other in floors level. Therefore to evaluate stiffness of single inset frame we consider a rigid bar but role of pillar at decreasing stiffness is computable as fallow.

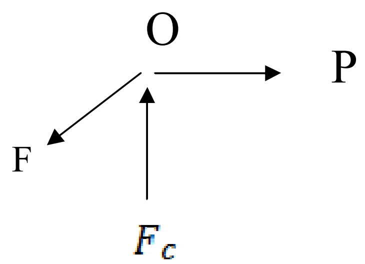


Figure 8

$\longrightarrow \quad \sum F_Y = 0 \qquad F_c = F\sin□$ (15)

$\longrightarrow \quad \sum F_X = 0 \qquad P = F\cos\beta$ (16)

Fc and F are respectively pillar's axial energy and axial members'.

Since β angle is clear based on frame's size transition amount is computed based on following relation.

$\frac{P}{k_{eq}} * \frac{a}{d} = \Delta_{eq}$ (17)

$\frac{P}{k_c} * \frac{d}{b} = \Delta_c$ (18)

In mentioned relation Kc and Keq are respectively pillars stiffness and equivalent stiffness of axial member and circular element.

Δ c and Δ eq are respectively amount of transition in these members affected by side energy of P.

In mentioned relation considering the small amount of transition energy's path is considered stable after side transition of frames.

In triangles shaped from frame's members and considering the transition we could write the fallowing relations to determine total stiffness.

The mentioned relations A is frame's size after transition and B is equal to its picture based on horizon.

Though stiffness is a subsidiary of F in this relation and if amount of F changes amount of Δ would also change based on that so the amount of K is stable. But considering primarily assumption no changes based on energies exact amount of K is obtained when side transition is at its minimum amount which means:

k=Lim (24)

Δ_{eq} F/((a+)sinα-b)

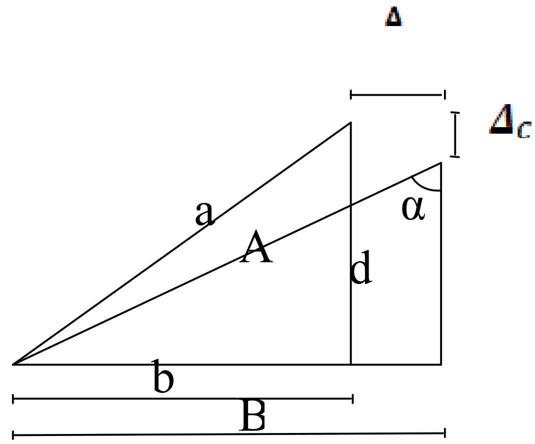


Figure 9

$\alpha = Arccos\left(\frac{d - \Delta_c}{a + \Delta_b}\right)$ (19)

$a + \Delta_{eq} = \frac{B}{\sin\alpha}$ (20)

$\Delta = B - b$ (21)

$k = \frac{F}{\Delta} = \frac{F}{(a + \Delta_{eq})\sin\alpha - b}$ (22)

$k = \lim_{F\to 0} \frac{F}{(a + \Delta_{eq})\sin\alpha - b}$ (23)

Increasing stiffness in circular element: We could increase side stiffness of the frame without any changes in frame's size and circular element's cross section by adding to axial elements inside the ring. These elements can be placed on each other horizontally and vertically and should have bending rigidity and be connected by joining rigid to the circular element because if joint connections are used shape shift of element is limited and the possibility of turning of circular element is still operative at the connection point.

Figure 9 shows the location of joined elements placement. Limited members analysis indicate that maximum turning in circular element happens at a, b, c and since Δab and Δ cd are close to zero. Therefore using joined elements with rigid connections could lead to stiffness in circular element.

To study this issue finite element analysis were used.

To do so a ring with inerty of 7854 mm4 and radius of 500mm were modeled in abaqus software and loaded in illustrated points like figure (10). Point C was connected to point a and b was connected to point d by means of a rigid element and rigid connection. Analyses results indicate that stiffness of this element id equal to 289N/mm. compared to joint connection we can observe that stiffness is more than six times. This stiffness was obtained by creating a joint connection equal to 46.2 N/mm.

Therefore type of connection is very effective at ring's stiffness.

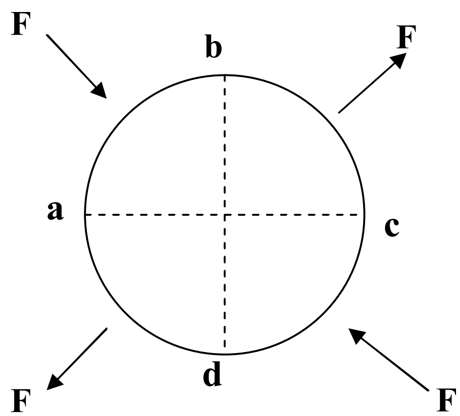


Figure 10

Parametric study of yielding elements under cycle loading

In this section twenty single inset yielding circular elements with an inset and height equal to 3 meters were studied. Bar, pillar, axial members and circular element are simulated by rigid connections. Cross section of pillars is IPB220 and bars are equal to IPE200 and axial members' cross section is equal to 2UNP140. Middle ring size and cross section is chosen based on table 1 and numbered.(Table1)

Table 1. Middle ring size and cross section

Model No	ratio Of circular element's Diameter to frame's Diameter	Profile
1	0.178	IPE14
2	0.252	IPE14
3	0.309	IPE14
4	0.357	IPE14
5	0.399	IPE14
6	0.178	IPE14
7	0.252	IPE14
8	0.309	IPE14
9	0.357	IPE14
10	0.399	IPE14
11	0.178	IPE16
12	0.252	IPE16
13	0.309	IPE16
14	0.357	IPE16
15	0.399	IPE16
16	0.178	IPE18
17	0.252	IPE18
18	0.309	IPE18
19	0.357	IPE18
20	0.399	IPE18

In first five models circular element profile is bent compare to weak axial and in other models to strong axials. We have used two line cycloid with a surrounding tension of 240 MPa and elastic of E1= 210000MPa and E2=600MPa for steel. These samples were first side loaded in analyzed elastic region and their stiffness was indicated. In figure (11) stiffness differentiation of mentioned models in table and restrained frame is shown for models in ratio of $\frac{I}{R3}$.

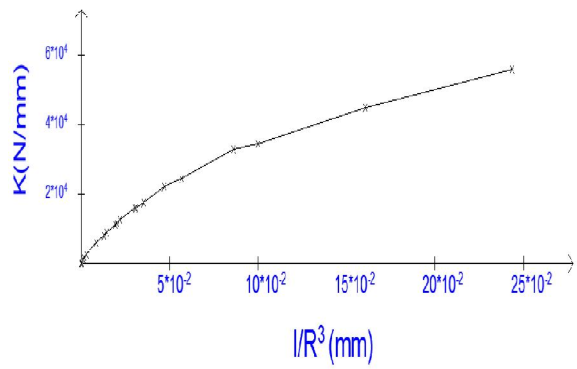


Figure 11

$K = K_f - K_{bf}$

Kb fis the stiffness of frame without restrain and Kf is the stiffness of frame with surrounding restrain.

We can observe based on figure (11) that by increase in amount of $\frac{I}{R3}$ stiffness of the frame is also increased and this increase amount is more in models with less $\frac{I}{R3}$. These issues are adjusted with formula number (9) and indicate that more the circular element is flexible ration changes in $\frac{I}{R3}$ has a greater effect in its total stiffness changes.

At the next part indicated models in figure (13) were cycle loaded. So nineteen side loading cycle entered above frame bar by increasing liner domain and maximum domain of 19 mm and bending energy of old plastic were evaluated by ring element by the program. Figure (12) illustrates loading diagram throughout the time.

In figure (13) Noting high axial stiffness of axial elements compared to side stiffness of ring elements we could assume that transition amount in all rings is similar.

As you can see models with higher stiffness start to surrounded in lower transitions and lower cycles, and also when comparing two models with equal stiffness with higher inerty and smaller radius they surrender sooner but total wasted energy amount is depending on transition amount after the stiffness as by increase of growth rate in cycles loading wasted energy in models with more inerty increases.

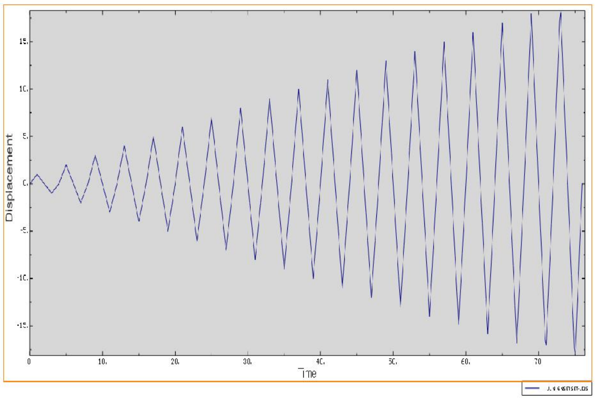


Figure 12

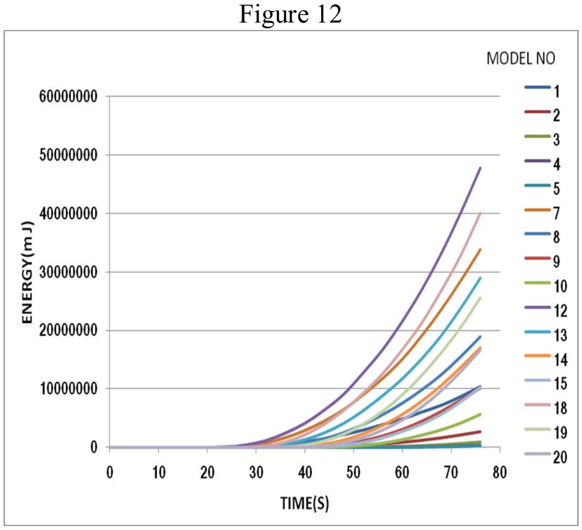


Figure 13

Figure (14) shows amount of stiffness increase and wasted energy in these models compared to the time that added elements were not applied based on percentage.

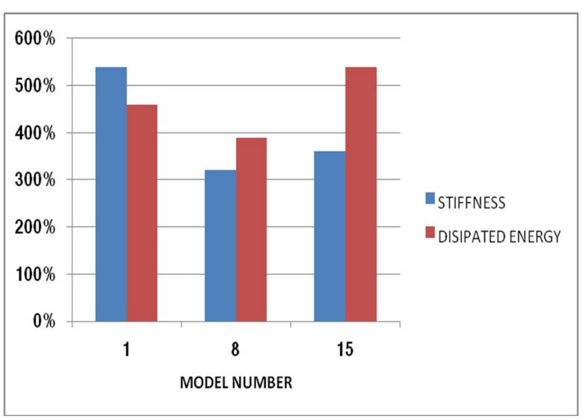


Figure 14

In these samples the location of plastic joints in circular element like models without added element in joint parts is presented. In other worlds by means of added elements eight plastic joints are created at the ring and without using them only four plastic joints are placed at the connection point of axial members.

Figure (15) illustrates the point and location of creating plastic joints.

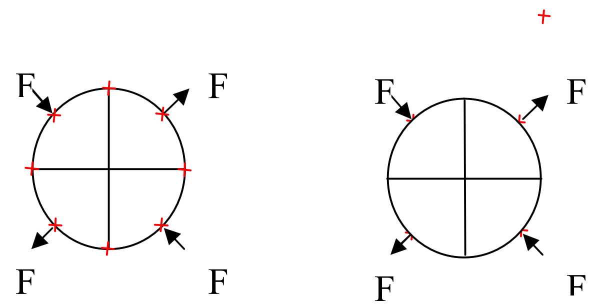


Figure 15

Studding the correctness of stiffness relation computed by results of limited members

Considering that stiffness relation for ring element is evaluated by means of ruling relations on bending slim bars foe examining the difference between stiffness computed by means of formula and stiffness obtained from analysis of limited members' results first a bar shaped model with a circle cross section equal to 7854mm2 was models by means of abaqus software and analyzed based on side load and models characteristics are as fallow:

Bar, pillar and axial members are modeled in a rigid form and their connections are modeled as joints. Frame's inset and height is 3 meters. Cross section of axial elements is equal to 4080mm2 and young is 210000MPa. Ring element radius is equal to 500mm and cross section inerty is equal to 4908739 mm4. Obtained side stiffness from analysis is equal to 12631 N/mm.

if we obtain stiffness amount by formula 10 the fallowing amount would be determined.

$$K = 13024\frac{N}{m}$$

It is observed that difference in stiffness amount which is evaluated by software and by formula is 3%.

More than mentioned model 4 other models are also modeled and analyzed. Cross sections of circular element used in these models with a circle form cross section, box form cross section, are I and H form shaped. Circular element radius in these models is equal to 500 mm and inerty around the bending axial is almost equal. These models characteristics and the stiffness difference between limited members' analysis and stiffness computed by formula are presented in table 2. (Table 2)

Table 2. The models characteristics and the stiffness difference between limited members' analysis and stiffness computed by formula

Precent Error	moment of inertia around the In-Planeaxis mm^4	moment ofinertia around the bending axis mm^4	Cross section Area mm^2	profile	Model
3%	4908739	4908739	7854	50ϕ	1
10%	83600000	6040000	5380	IPE300	2
25%	449000	5410000	1640	IPE140	3
%10	6030000	6030000	2820	120BOX	4
%8	5890000	5890000	2640	PIPE139.7	5

In order to study wasting energy ability of mentioned models under cycle loading models are also analyzed. Figure (16) presents results obtained by analysis beside stiffness and cross section of them.

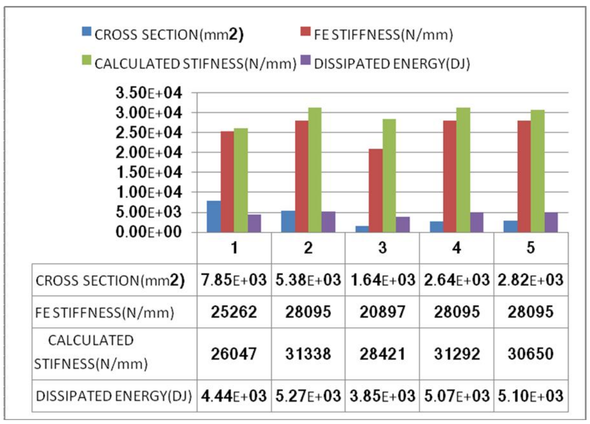
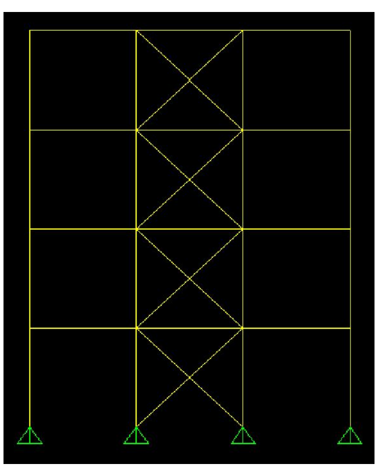
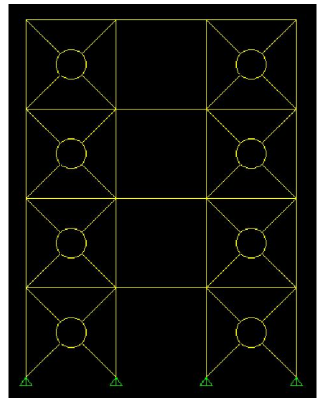
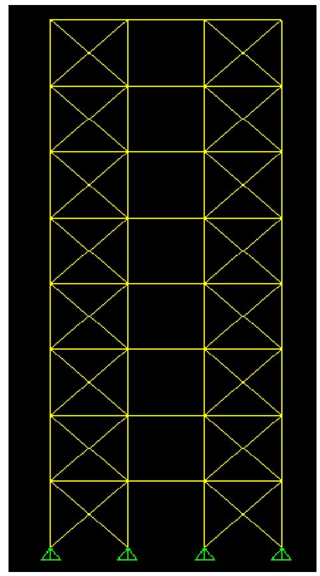


Figure 16


CON-1


CI-2


CON-2

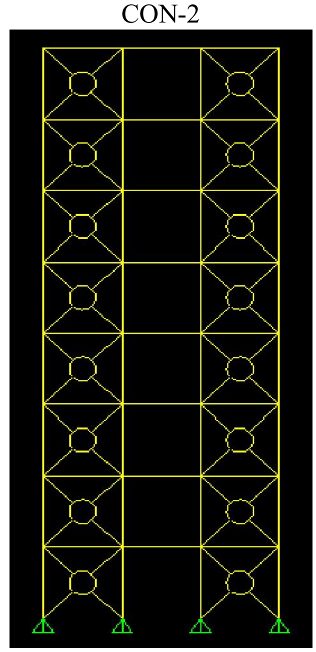

CI-1

Figure 17

Considering above figure it is observed that by means of I cross section more stiffness is obtained and wasted energy amount is also noticeable. Therefore we use I shape cross section as circular restrain at push over analysis.

Push over analysis

In this part we have used push over analysis to study circular restrain behavior facing side loads. To do so four two dimensions frames which are named in figure (17) were studied.

System model CON-1 and CON-2 are frames with a convergence restrain which are designed by SAP2000 software based on ASD89 regulation. Dead and alive loads on these models' bars are respectively 2400 Kg/m and 800 Kg/m. used steel kind is st37. Cross sections used for IPE bars, for IPB pillars and restrains are double tube profiles. Models number CI-1 and CI-2 system is a frame type with simple connections with circular restrains. Cross section and circular restrain size is designed by presented relations in previous sections and these frames elastic stiffness is almost equal to CON-1 and CON2 models.

Dedicated plastic joints to circular restrains are bending kind which is dedicated to them at the joining point of ring to axial members. This location is the part where these members bear maximum bending and results of the executed analysis with ABAQUS software also indicate creation of plastic joints at these parts.

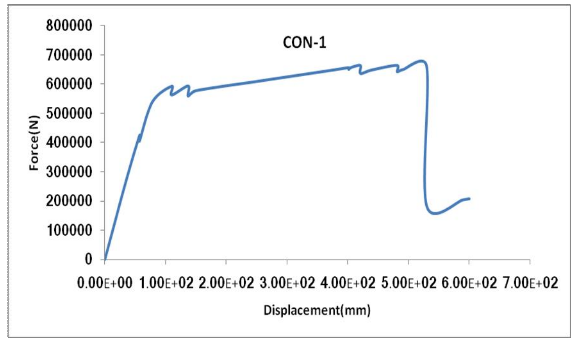
Side load pattern for push over analysis in models CON-1 and CI-1 are adjusted to first mood of vibration and is executed in CON-2 and CI-2 models based on spectrum analyze on the models based on spectrum regulation of UBC.

In figure (18) structure's push over cycloid for models CON-1 and CI-1 are presented.

In studding push over cycloid of model CON-1 it is observed that the point where the frame has a high resistance drop or in other world structure's destruction point, happens when resistant plastic joints get to a resistant drop point in a level. This frame shows a noticeable shape changing before getting to this level and frame's figure making before this level is prepared by surrounding pressure restrains.

Comparing between two above systems we observe that in CI-1 model at the moment model has a high resistance drop the transmission is much less. This means that dedicated joints to circular element get to plastic level and resistant drop from elastic level much faster than CON-1 by much less transmission and the structure shows a mealy behavior. Resistant drop happens in this structure when in all four joint points of circular elements we observe a special level of resistant drop in plastic joints.

Therefore using circular elements as a part of the side loading system could not solely guarantee structure's resistant under big side loads. This is indicated by evaluating coefficient of structure's behavior.



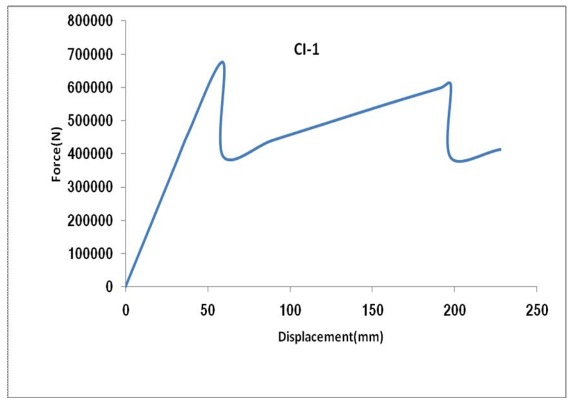


Figure18

To do so we have used push over diagram to evaluate behavioral coefficient of models. We used New mark and Hall's relations to compute coefficient decrease resulted by shape changing. Design decrease coefficient by legal tension method was considered equal to 1.5. Table 3 shows the behavioral coefficient of studied models and their obtaining parameters. (Table 3).

Table 3. The behavioral coefficient of studied models and their obtaining parameters.

Response Modification Factor **R**	Ductility Reduction Factor R_μ	Over strength Factor R_S	**Model**
7.86	3.36	1.56	CON-1
7.13	1.59	2.99	CON-2
2.32	1.06	1.46	CI-1
2.30	1.45	1	CI-2

By applying side loading due to that connection amount in defined joints in circular element is increased very fast and models CI-1 and CI-2 experience low amount of transitions before reaching to drop point therefore behavioral coefficient amount determined for these two frame is less than frame with central restrain. But in case using circular restrain at the bending frame structure this behavior of circular restrain would lead to energy waste at the structure while side resistant of the structure after creating the joints could be beard by the bending frame.

To study behavioral coefficient of structural system having a circular restrain combined by bending frame tow models with two dimensions as frames with four floors and eight floors with rigid connections were designed. These two frames respectively were named models MF-1 and MF-2. These models underwent push over analysis and their behavior coefficient was computed. Then circular restrain was added to them and they went through the same process again. Models with circular restrain were named MFCI-1 and MFCI-2.(Table 4)

Table 4 shows these models behavior coefficients and parameters to indicate these coefficients. Obtained results indicate that using a circular restrain in mentioned frames would lead to 50% increase at their behavioral coefficient.

There for to use these systems as a phase and to waste energy combining with bending frames is advised.

Table 4. Models with circular restrain named MFCI-1 and MFCI-2

Response Modification Factor ***R***	Ductility Reduction Factor ***Rs***	Over strength Factor ***Rμ***	**Model**
5.65	2.04	1.46	MF-1
5.54	2.31	1.60	MF-2
8.52	3.55	1.60	MFCI-1
8.61	4.07	1.41	MFCI-2

Discussion and Conclusion

1. Considering presented relations we could compute side stiffness of frames with circular element that obtained stiffness if circular element with inerty connection has direct relation and divers' relation with third exponent of radius.
2. Wasted energy at cycle loading at circular element is connected with its stiffness as its amount increases by increase in I or decrease in R3 but these changes growth rate is dependent to transition amount and increase in transition rate of wasted energy amount is higher in circular element with higher inerty connection.
3. By adding axial elements to the ring we could increase stiffness and ability to absorb energy and this increase amount is different based on elements size as in studied models increase amount was almost equal to 300% and absorption of energy was 400%.
4. Due to creating high stiffness and also suitable energy absorption compared to other cross sections using I cross section is suggested as circular slow making.
5. Considering structures behavior by circular restrain using this system is only suggested with a combination of bending frames.

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