Offering a Mathematical Model for reducing Road Traffic in the Highways and solving it by Genetic Algorithm

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Abstract: Traffic systems and specially rail transportation systems have a sophisticated nature and the current and future situations of these systems are considerably dependent upon many factors that influence each other. Usually these effective factors include physical and human factors. So, it is difficult to present a proper model for all effective mechanisms on a transportation system. To release from this sophistication, a particular relation may be extended by different analyses. By offering a mathematical model and solving it by a genetic algorithm, this paper is aimed at distributing the traffic load in the urban transportation network properly. This model is a non-linear ideal programming that is among optimal traffic allocation matters. This model controls limitations specified in the decision making space and it modifies generation members which will improve rapid problem solving and evolutionary process. Solving the proposed model by the genetic algorithm showed that the time of solving the model by genetic algorithm with a population size 30 and generation number 300 is less than discrete algorithms. Increasing the number of generations and size of population for solving the model is a more efficient solution.
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Key words: network traffic allocation model, mathematical model, traffic modeling, genetic algorithm

1. Introduction

One of the problems of the present era is deficiency of urban transportation traffic in the big cities and also large networks of urban transportation. Lack of proper distribution of traffic load in the network level leads to increase in the environmental pollution, noise pollution, and waste of time of the network users. So, offering efficient managerial tools plays a determinant role in distributing network traffic load and can reduce the existing unfavorable factors in the urban transportation network to some extent.

Various researchers have studied traffic allocation problems from two perspectives. In the first perspective, problems are modeled based on user-optimized flows. This perspective studies the behavior of network users so that no user may reach shorter travel time by changing the path. The behavior of the users was firstly presented by Wardrop (Wardrop, 1952). He suggested that the travel time of all paths that are passed really by a vehicle equals or is less than travel time of those paths that have not passed. Ashtiani and Magnanti (Ashtiani and Magnanti, 1981) used non linear complementarity problem for modeling flow equilibrium problems in the network. Florian (1986) reported that important advantage of modeling the network flow problems in using non linear complementarity is that since the network is modeled based on arcs flow, solving the network models with larger dimensions by using non linear complementarity may be easier than solving models that have been modeled based on path flow. Chen and colleagues (Chen and Husheh, 1998) offered a model that selected path and time of user optimized movement. This type of modeling is not efficient for problems with larger dimensions in terms of time due to large number of limitations, and non-convex solution space and zero-one variables add to the model computational problems.

In the second perspective, problems are modeled based on system optimized flows. The objective of this perspective is to allocate paths to the vehicles in different periods so that the average travel time is minimized in the arcs (Wardrop second principle). Kaufman and colleagues (Kaufman et al., 1998) offered a model that was a linear programming model integrated with integers. This model is minimized by using Branch and bound method. Merchant and Nemhauser (1978) presented a traffic allocation model that was non linear and non-convex and had multiple origins and single destination. To solve the model, they used the linear version made based on zigzag line segments that was exerted on the objective function and non linear limitations. Ho (1980) developed a nested analysis algorithm and found the absolute optimal solution by testing the set of optimal solutions of linear program. Carey (1987) analyzed Merchant and Nemhauser model and converted it into a convex non-linear programming by adding flow control limitations to the Merchant and Nemhauser model limitations in order to reach the absolute optimal solution by using property of convex programming and testing Kuhen and Taker conditions. Also besides modifying structure of limitations and controlling the flow in the arcs, Kari referred to multiple destinations;

while in the Merchant and Nemhauser model, the problem had been modeled by only one destination.

2. Modeling Method

The present paper has offered a mathematical non linear, non-convex and multi objective model. In this model, an arc exit function is offered which is dependent upon traffic load at the beginning of the current period and a ratio of inflows of the arc in the current period. Also when the rates of outflows are increased, the proposed arc exit function shows it more precisely than the arc exit function offered in the Kari model.

Since the arc exit function depends upon the average travel time of vehicles in the arc, the offered mathematical model acts in a two level manner. Moreover, in the model offered by Merchant and Nemhauser and the modified model of Kari, timing of traffic lights in the intersections has not been stated in the arc exit function. This factor has been exerted in the proposed model in a parametrical manner and based on the ratio of green time to the cycle time in the arc exit function. This ratio may be exerted as the decision making variable in the model so that optimized timing of traffic lights in the intersections is determined based on flows interaction in the whole network. Another feature of the proposed model is that entrance (supply) and exit (demand) of flow are possible through arcs besides nodes.

Entrance and exit of flow through arcs are due to the fact that all demands and supplies cannot be exerted through nodes due to geometrical form of the urban transportation networks; because confining exertion of demands through nodes makes estimation of average travel time in the arcs inaccurate. In the proposed mathematical model offered based on Li research (1991), zero- one variables in the interval [0, 1] were considered continuous and solving time was reduced remarkably by exerting limitations related to zero-one variables. Then a particular genetic algorithm was designed and the proposed model was solved by that. The value of the proposed model objective function was estimated 138.6 by the genetic algorithm and solving time was obtained 2.2 minutes. The value of Kari model objective function was estimated 131.3 and solving time was obtained 1 minute. The reasons of 5.5% difference between the value of the objective function of the proposed model and Kari model were stated before. Prashker and Bekhor (2000) compared certain user equilibrium and probable user equilibrium models with the system optimal flow equilibrium model. They concluded that when flows have little congestion, flow allocation for probable user equilibrium is worse than certain user equilibrium. When flows have a moderate congestion, flow allocation for certain users is worse than probable users relative to system optimal flows. Also when flows have a high congestion, flows of certain user equilibrium is worse than certain users relative to the system optimal flows. This research calculated flows deviation of certain and probable user equilibrium relative to the system optimal flows. Friesz and colleagues (1993) offered a static and multi objective model for optimal design of transportation network and the model structure was non linear and non-convex. They solved their model by SA algorithm and reported that by weighing method and SA (Simulated Annealing) algorithm, 16.46 hours are required for each set of weights (in mini-computer 9370 IBM) in order to produce Pareto optimal set.

Comparison of genetic algorithm process in (Goldberg, 1989) and (Holland, 1992) with the innovative SA algorithm in (Friesz et al., 1993) shows that in SA algorithm, dependency between initial solution and final solution is not removed totally due to beginning from an initial point; while this problem is removed in the genetic algorithm by wider movements in the set of solutions. So by exerting operators, it is tried to search a large part of set of solutions as far as possible. Also by selecting a set of solution instead of one solution in the genetic algorithm, it is tried to reduce dependency of the final solution upon initial solution as far as possible (Feng-Tse et al., 1993). Also comparison of SA algorithm with TS (Tabu- Search) algorithm showed that for problems with large dimensions, logical agreement between quality and time of solving in the SA algorithm is more efficient than TS algorithm (*Ishibuchi and Murata, 1998)*.

Researchers (Prashker and Bekhor, 2000) showed that for solving mathematical programming problems by feed-forward neural networks, back propagation learning algorithm is a proper algorithm, but solving non linear programming problems by this method is slow due to needing small step length. Another algorithm for training neural networks is genetic algorithm. Efficiency of genetic algorithm relative to back propagation learning algorithm is not less, and while back propagation learning algorithm falls into the trap of local optimums, genetic algorithm removes this problem. By unifying formulation of evolutionary algorithms, *Fonseca and Fleming* (1998) used them for optimizing multi objective problems and controlling limitations. *Ishibuchi and Murata (*1998) offered a hybrid genetic algorithm for finding a set of non dominated solutions in order to optimize the multi objective problem.

2.1 traffic model for urban transportation network

In this paper, a transportation network with multiple origins and destinations is defined in a one-sided manner that has been composed of a set of nodes and arcs. Also the problem analysis periods are defined in the interval [0, 1]. Supply of origin points and demand of destination points are different in different periods and are dependent upon each other. With regard to the explanations, this paper is aimed at providing a mathematical model to distribute flows optimally between origins and destinations and optimize the related objectives ideally and satisfy limitations existing in the network. Our minor objectives in the model are namely, minimizing the average travel time for passing from arcs and average number of vehicles in the intersections, vacant capacity of arcs based on the related servicing level, and finally average costs of flows in the arcs. So, to reach these objectives, below hypotheses must be taken into consideration:

1) With regard to the hierarchical structure of urban road network, streets are divided into three feed lines (subsidiary streets), intermediary lines (main streets), and flows (highways).

2) Nodes existing in the geometrical form of the network are considered as terminal, square or intersection.

3) Streets have been considered one-sided.

4) Capacity of arcs in the related network is limited.

5) Only one vehicle (Passenger Car Equivalent) is considered.

6) The effect of accidents and wreck vehicles can be exerted in the model.

7) Daily travels are divided into working and shopping trips.

8) Travel production has been considered as flows entering to the network and travel attraction as flows exiting from the network.

8.1) In home based trips, regions with more travel production are residential regions.

8.2) In home based trips, regions with more travel attraction are administrative and commercial regions.

9) Traffic load distribution in arcs is considered uniform.

10) The vehicles move along the path with a moderate speed.

3. Mathematical model of the problem

In the proposed model, the final objective has not been considered due to lack of information related to cost of each unit of sent flows. The second objective refers indirectly to the costs of vehicles stop in the intersections. So, the rate of consumed fuel and waiting time caused by stop for the vehicles that have reached the red light earlier are more than the vehicles that have reached it later. As a result, the function related to the number of vehicles in the queue is calculated as per figure 1:

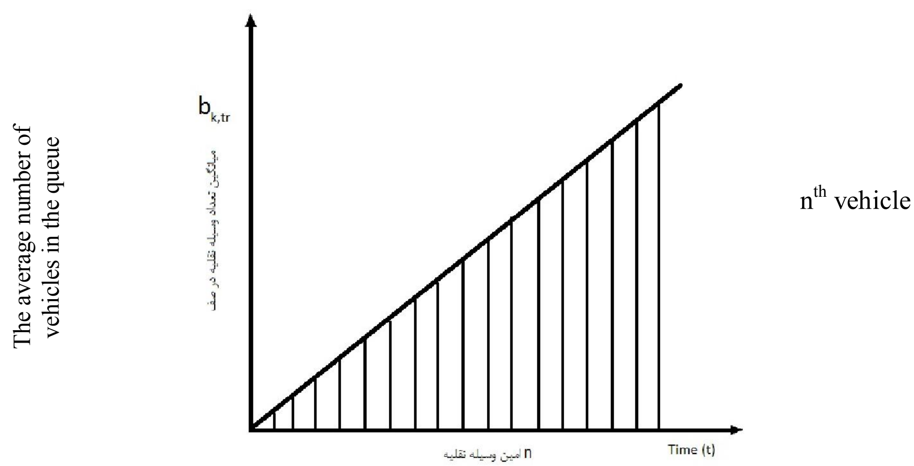

Figure 1- the average number of vehicles in the queue in the intersections

t_r : the stop time of the first vehicle

$t_r - \alpha$: the stop time of the second vehicle

$t_r - ۲\alpha$: the stop time of the third vehicle

$t_r - n\alpha$: the stop time of the n^{th} vehicle

$\propto$: difference of time of reaching the red light between two consecutive vehicles

$$t_r - n\alpha = 0 \quad \rightarrow \alpha = \frac{t_r}{b_{kt_r}}$$

$$n = b_{kt_r}$$

The total time wasted behind the red light in the t^{th} period in the k^{th} node is:

$$t_r + t_r - \propto + t_r - 2\propto + \cdots + t_r - n\propto - (n+1)t_r - \propto(1 + 2 + \cdots + b_{kt_r})$$
$$(n+1)t_r - \frac{t_r}{b_{kt_r}} * b_{kt_r}\left(\frac{(b_{kt_r}+1)}{2}\right) = t_r\left[\frac{(b_{kt_r}+1)}{2}\right]$$

Since b_{kt_r} denotes the vehicles accumulation size in the period t and also since the number the traffic light that will become red equals $\frac{\gamma}{t_c}$, we can write the third objective as below:

$$b_{kt} = \frac{\gamma}{t_c} b_{kt_r} \quad \Rightarrow b_{kt_r} = (b_{kt} * t_c)/\gamma$$
$$Min \sum_{k \in N} \sum_{t=1}^{T} b * t_r(b_{kt} * \frac{t_c}{\gamma} + 1)/2$$

The vehicles accumulation size is obtained by relations (1) and (2).

$$b_{kt} = \frac{t_r}{t_c}\left(x_{jt} + S_{jt} - S'_{jt} + \frac{\gamma - t_{jt}}{\gamma} f_{jt}\right) \text{ if } \quad t_{jt} \le \gamma \qquad \textbf{(1)}$$

$$b_{kt} = \frac{t_r}{t_c} * \frac{t_{jt}}{\gamma}(x_{jt} + S_{jt} - S'_{jt}) \text{ if } \quad t_{jt} > \gamma \qquad \textbf{(2)}$$

$$Min \sum_{t=1}^{T} \sum_{j \in A} t_{jt} \qquad (3)$$

$$Min \sum_{j \in A} \sum_{t=1}^{T} \left(1 - \frac{f_{jt}}{u_j} \Phi_j\right) \qquad (4)$$

$$Min \sum \sum^{T} \frac{b(t_r(b_{kt}+1))}{2} \qquad (5)$$

$$Min \sum_{t=1}^{T} \sum_{j \in A} a * l_j * f_{jt} \qquad (6)$$

Now for each objective, a certain value is determined as an ideal and then each objective is exerted in the model in the form of ideal limitations.

$$\sum_{t=1}^{T} \sum_{j \in A} t_{jt} + n_1 - p_1 = g_1$$
$$\sum_{k \in A} \sum_{t=1}^{T} \frac{b(t_r(b_{kt}+1))}{2} + n_2 - p_2 = g_2$$
$$\sum_{j \in A} \sum_{t=1}^{T} \left(1 - \frac{f_{jt}}{u_j} \Phi_j\right) + n_3 - p_3 = g_3$$

Since g_1 and g_2 and g_3 are the highest limits of ideals, the ideal programming will be as following considering weight coefficients for deviation from ideals.

$$Min z = \sum_{i=1}^{3} W_i P_i$$

3.1 limitations of the model allocated to urban transportation traffic network

In this regard, some limitations must be taken into account:

a. limitation of average travel time in the arcs

These limitations control limitations (4) and (5) by using travel time function that is dependent upon arc length and traffic volume. So one can write:

$$t_{jt} = \frac{l_j}{\bar{v}_j} + a x_{jt}^{\beta}$$

In this function, l_j denotes the length of the j[th] arc, $\bar{v}_j$ is the average speed in the j[th] arc, x_{jt}, traffic volume in the beginning of period t in the j[th] arc, and α & β are calibration parameters of the function. It must be noted that if traffic volume equals zero, the average free flow travel time will equal $\frac{l_j}{\bar{v}_j}$. But if traffic volume is larger than zero, the average travel time will increase. Now if each time period is considered 15 minutes, then we have:

$$t_{jt} \le 15 + M * \gamma_{jt}$$
$$t_{jt} > 15 - M * y_{jt}$$
$$\gamma_{jt} + y_{jt} - 1$$

So, if the average travel time is less than 15 minutes, then $\gamma_{jt} = 0$ and $\gamma_{jt} = 0$ and vice versa.

b. Limitation of the arc exit function

The j[th] arc exit function in period t is calculated by relations (7) and (8). So one can write:

$$t_{jt} \le \gamma \qquad (7)$$

If:

$$g_{jt} = \begin{cases} Min\left(q_{max}, \frac{t_g}{t_c}\left(x_{jt} + S_{jt} - S'_{jt} + \frac{\gamma - t_{jt}}{\gamma} f_{jt}\right)\right) \\ \text{if } t_{jt} > \gamma \\ Min\left(q_{max}, \frac{t_g}{t_c}(x_{jt} + S_{jt} - S'_{jt})\right) \end{cases} \qquad (8)$$

In relations (7) and (8), q_{max} denotes the maximum flow volume in the related intersection. The value of maximum flow volume depends upon geometrical form of the intersection, number of lines existing in the arc, and scheduling of traffic lights. Sometimes, the number of vehicles that reach the end of an arc in a certain period may not equal q_{max}. So, the arc exit function is calculated by the second part of relations (7) and (8). When the average travel time is less than γ time units, the second part of exit function depends upon traffic volume in the arc, the rate of entry and exit, and a fraction of inflows that may reach the end of arc inside the related period. The relation (7) defines the exit function for short arcs or arcs with a low traffic; while relation (8) defines it for long arcs or arcs with a high traffic. In this paper, q_{max} has been defined as a parameter and its value may be entered in the model based on the network specifications. So, in a certain arc and a certain period, the exit function will be written as relation (9).

$$g_{jt} = \left\{Min\left(q_{max}, \frac{t_g}{t_c}\left(x_{jt} + S_{jt} - S'_{jt} + \frac{\gamma - t_{jt}}{\gamma} f_{jt}\right)\right)\right\}(1 - r_{jt}) + \left\{Min\left(q_{max}, \frac{t_g}{t_c}(x_{jt} + S_{jt} - S'_{jt})\right)\right\}(1 - y_{jt}) \quad (9)$$

With regard to relation (9), if the average travel time in the j^th arc is less than γ time units, the first part of relation (9) specifies the exit function and if the average travel time in the j^th arc is more than γ time units, the second part of relation (9) will determine the exit function. So according to the first part of relation (9), the value of the exit function will equal q_{max} or $\frac{t_g}{t_c}\left(x_{jt} + S_{jt} - S'_{jt} + \frac{\gamma - t_{jt}}{\gamma} f_{jt}\right)$. Now, if $\frac{t_g}{t_c} = \frac{1}{2}$, limitations of the exit function may be rewritten as relations (10) and (11).

$$\frac{1}{2}\left(x_{jt} + S_{jt} - S'_{jt} + \frac{\gamma - t_{jt}}{\gamma} f_{jt}\right) \le q_{max} + MP_{jt} \quad (10)$$

$$\frac{1}{2}\left(x_{jt} + S_{jt} - S'_{jt} + \frac{\gamma - t_{jt}}{\gamma} f_{jt}\right) > q_{max} - \mathrm{M}(1 - P_{jt}) \quad (11)$$

Since the variable P_{jt} is zero and one, one of the above limitations will become active and the exit function will be calculated through relation (12).

$$z_{jt} = q_{max} * P_{jt} + \frac{1}{2}\left(x_{jt} + S_{jt} - S'_{jt} + \frac{\gamma - t_{jt}}{\gamma} f_{jt}\right) * (1 - P_{jt}) \quad (12)$$

Now if the average travel time in the j^th arc is more than γ time units, flows entering to the beginning of j^th arc in the related period will not reach the end of j^th arc in the same period and so will be converted into x_{jt}. So, limitations of the exit function are:

$$\frac{1}{2}(x_{jt} + S_{jt} - S'_{jt}) \le q_{max} + Me_{jt} \quad (13)$$

$$\frac{1}{2}(x_{jt} + S_{jt} - S'_{jt}) > q_{max} - \mathrm{M}(1 - e_{jt}) \quad (14)$$

Since the variable e_{jt} is zero and one, one of the above limitations will become active.

$$l_{jt} = q_{max} * e_{jt} + \frac{1}{2}(x_{jt} + S_{jt} - S'_{jt}) * (1 - e_{jt}) \quad (15)$$

And the value of the exit function will be calculated by relation (15).

c. Limitation of traffic situation control in the arcs

To control traffic volume in the beginning of period (t+1), the number of vehicles exiting from the arc during period t must be subtracted from total traffic volume in the beginning of period t and the flow entered during period t. and this relation must be rewritten as a chain for all periods.

$$x_{j,t+1} = x_{jt} + f_{jt} + S_{jt} - S'_{jt} - g_{jt} \quad (16)$$

It must be noted that S_{jt} and S'_{jt} are inflows and outflows of subsidiary arcs that will be exerted as the input information in the model. Now by placing values of the exit function from relations (12) and (15) in the relation (16), the latter relation is rewritten as relation (17).

$$x_{j,t+1} = x_{jt} + f_{jt} - (1 - r_{jt}) * z_{jt} - (1 - y_{jt}) * l_{jt} + S_{jt} - S'_{jt} \quad (17)$$

Since the sum of variables r_{jt} and y_{jt} equal one, one of the exit functions z_{jt} and l_{jt} will affect relation (17).

d. Limitation of keeping flow in the nodes (intersections)

In this group of limitations, it is tried to show that if there is no flow demand in the nodes, the total flows exiting from the end of arcs before the node k equals the total flows entering to the arcs after the node k. so one can write:

$$\sum_{j\in B(k)} g_{jt} = \sum_{j\in A(k)} f_{jt} \qquad ; \qquad t = 1,2,\dots,T \tag{18}$$

e. Limitation of flow propagation and demands supply

In this group of limitations, it is tried to transmit the flow from the beginning arcs to the final arcs, and the arcs demand are supplied through S'_{jt} during flow transmission. In these limitations, the input flow to the j[th] arc during period t may not transmit to the next arcs due to length of average travel time in the j[th] arc; rather it may transmit to the next arcs during period t to $\left\lceil t+\frac{t_{jt}}{\gamma}\right\rceil$. So, traffic volume and input flows during period t and total difference of inputs and outputs during periods t to $\left\lceil t+\frac{t_{jt}}{\gamma}\right\rceil$ will be converted into arcs before the node k, and flows of arcs after the node k during period t to $\left\lceil t+\frac{t_{jt}}{\gamma}\right\rceil$. So one can write:

$$\sum_{j\in B(k)} (f_{jt}+x_{jt}) + \sum_{j\in B(k)} \sum_{\tau=1}^{\left\lceil t+\frac{t_{jt}}{\gamma}\right\rceil} (S_{jt}-S'_{jt}) = \sum_{j\in A(k)} \sum_{\tau=1}^{\left\lceil t+\frac{\tau_{j_1t}}{\gamma}-1\right\rceil} f_{jt} + \sum_{j\in A(k)} f_{j,\left\lceil t+\frac{\tau_{j_1t}}{\gamma}\right\rceil} \tag{19}$$

$$t = 1,2,\dots,\left\lceil \frac{\tau_{j_1t}}{\gamma}+t\right\rceil$$

$$,\tau_{j_1t} = Max\{t_{jt}, j\in B(k)\}, \forall k\in N, j\in A \tag{20}$$

Relation (20) is an ideal relation that does not consider limitation of traffic light in the intersections. By considering this limitation, the left side of relation (20) will be always equal or bigger than its right side. In the other words, due to length of the average travel time in some arcs, some flows reach the end of arcs before the node k in the period $\left\lceil t+\frac{t_{jt}}{\gamma}\right\rceil$ and face with the red light at the same moment. So, these flows cannot pass from the node k. therefore, the first part of relation (20) will be always bigger than its second part.

f. Limitation of arcs capacity

$$0 \le f_{jt} \le u_j \qquad \forall j\in A, t\in T$$

g. Limitation of variables type

$$x_{j_0} = I_j \qquad \forall j\in A$$

$$f_{jt}, x_{jt} \ge 0 \quad , \quad r_{jt}, y_{jt}, P_{jt}, e_{jt} \quad \in \{0,1\}$$

3.2 Features of the research model

In the mathematical model of this paper, two groups of variables are continuous and three groups of variables are zero and one. The model has $11*A*T*+2*N*T$ limitations and $5*T*A$ decision making variables. T denotes number of periods, A is the number of arcs and N is the number of nodes. The presented model has 4 nodes and 9 arcs that by considering limitations and ideal variables, the number of limitations and variables will be 428 and 222, respectively. Among them, there are 144 zero and one variables. When flows are considered two sided, origin point and destination point change and the mode dimensions double in terms of number of limitations and variables. So solving the model with this dimension needs more time.

3.3 Application of the mathematical model presented in Hamedan case study

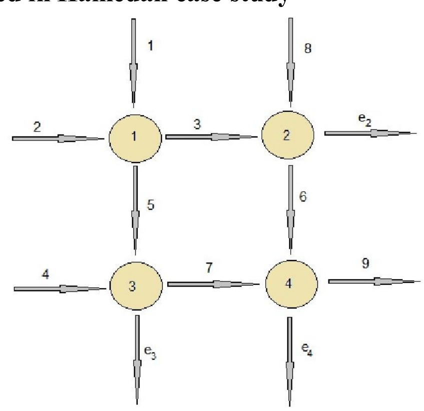

Figure 2- Central business district of Hamedan City

To apply the mathematical model, a case study was selected for the traffic allocation model. The location of implementing the model presented in this paper is the central business district (C.B.D) of Hamedan City. In the network of the central business district of Hamedan City, streets and intersections have been numbered according to figure (2).

Statistical results showed that there is a linear relation between traffic volume and travel time function. It must be noted that when the network is locked, validity of the linear relation is not confirmed. So, the travel time function is defined as relation (21) (Morlok, 1978):

$$t_{jt} = \frac{l_i}{\bar{v}_j} + \alpha x_{jt} \qquad (21)$$

Since in the relation (21), the statistics related to the average travel time, average speed and traffic volume in the arcs have been gathered, value of the coefficient α can be calculated. The statistical test related to the linearity of travel time function has been conducted in table (1) based on t statistic.

Also travel demand was for 7 to 8 am. The average travel supply in the arcs 1 and 2 and travel demand for the nodes 2, 3 and 4 were obtained according to table (2).

Table 1- travel time function in the arc

Street No.	*Travel time function based on minute*
1	$t_{11} = 1.263 + 0.03x_{11}$
2	$t_{21} = 1.44 + 0.053x_{21}$
3	$t_{31} = 1.371 + 0.0161x_{31}$
4	$t_{41} = 3.36 + 0.178x_{41}$
5	$t_{51} = 1.178 + 0.011x_{51}$
6	$t_{61} = 1.2 + 0.007x_{61}$
7	$t_{71} = 1.015 + 0.014x_{71}$
8	$t_{81} = 3 + 0.03x_{81}$
9	$t_{91} = 2.7 + 0.026x_{91}$

3.4 Solving the model by the genetic algorithm

In this section, it is necessary to introduce the general form of chromosomes used in solving the proposed model.

$x_{11} \ldots x_{94}$	$f_{11} \ldots f_{94}$	$r_{11} \ldots r_{94}$	$p_{11} \ldots p_{94}$	$e_{11} \ldots e_{94}$
X	*F*	*R*	*P*	*E*

Due to chain-like form (dependency) of situation (X) variables and also zero-one variables, these decision making variables are removed from the primary structure, and during solving process, their values calculated implicitly and are put at their place. So, the modified form of chromosomes that play a direct role in solving the model will be as per below.

$$f_{11} \quad f_{12} \quad f_{13} \quad f_{14} \quad f_{15} \quad \ldots \quad f_{91} \quad f_{92} \quad f_{93} \quad f_{94} \quad f_{95} \qquad (22)$$

Figure (4) shows genetic algorithm flow chart. According to figure 3, crossover, mutation, and reproduction operators are used. Among the above factors, coding system structure and form of fitness function are more critical than other factors (Grefenstette, 1986). Design of genetic algorithm is elaborated in five stages:

First stage: primary population production

Since the structure of the presented mathematical model uses two continuous and integer (zero-one) variables, design of coding system has a particular importance in this regard.

Table 2- information of travels supply and demand in the time periods

No.	Supply in the time period				Demand in the time period			
	1	2	3	4	1	2	3	4
1	256	250	273	253	-	-	-	-
2	256	250	272	253	70	95	130	130
3	-	-	-	-	250	330	420	313
4	-	-	-	-	70	130	225	180

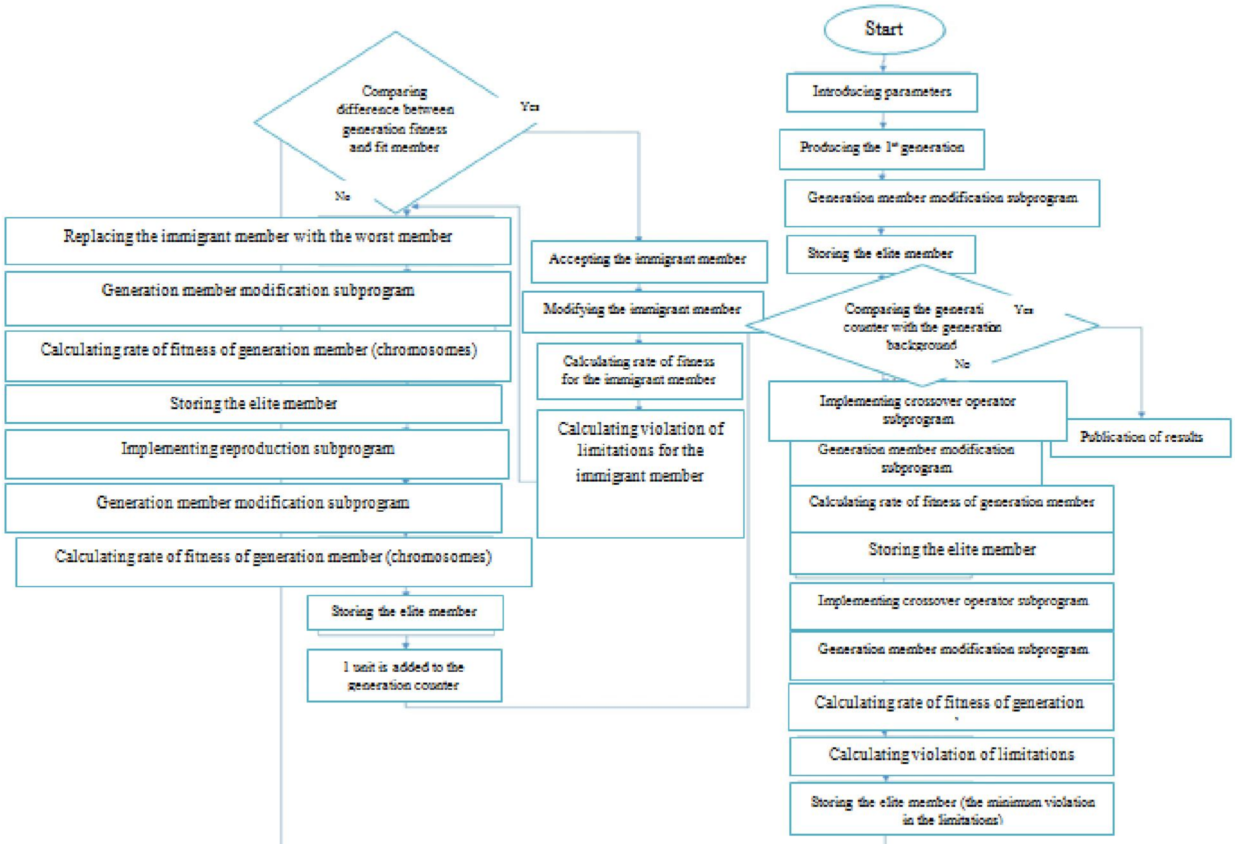

Figure 3- Algorithm design flow chart

So, if combined arrays (chromosomes) are defined as continuous and integer variables, it is very unlikely that such arrays can be applied in all limitations of solution space as components of these arrays are produced randomly and tested in the limitations. So, zero-one components leave arrays aside and undertake operation related to operators only by continuous variables. Then the primary population is produced while zero-one variables have no value. Then by placing random value of continuous components of each array, the value of zero-one components of each member of population is specified. This operation is done under a program titled as "generation member modification".

Second stage: testing the population produced in the problem limitations and measuring their rate of violation

In designing the genetic algorithm, an important point is to control limitations in the justified space. For unjustified chromosomes, repairing strategy is used for some limitations and penalty strategy is used for other limitations (Gen and Gheng, 1997). So, for each member of the generation, absolute value of deviations relative to the justified space is calculated.

$$\begin{matrix} X^1 & X^2 & \dots & X^m \\ \varepsilon_1 & \varepsilon_2 & \dots & \varepsilon_m \end{matrix}$$

ε_1 is absolute value of limitations deviation relative to solution X^1 and ε_2 is absolute value of limitations deviation relative to solution X^2 and ε_m is absolute value of limitations deviation relative to solution X^m.

$$\varepsilon = \varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_m$$

So, penalty function of limitations violation is defined as relation (22) (Gen and Gheng, 1997):

$$eval(\varepsilon) = (C * Gen)^a \sum_{j=1}^{k} \sum_{i=1}^{m} |\varepsilon_j(x^i)| \qquad (23)$$

(Gen: counter of the generation number)

Third stage: replacing the worst solution with the best solution

By using probable simplex method in (Yen et al., 1998), we replace the worst solution of each generation with the best solution of that generation.

Relations (23) and (24) state the method of new point production.

$$X_P = \bar{X} - \beta(\bar{X} - X_W) \quad (24)$$

β is a random variable that is produced by using triangular distribution function in [0,1] (Yen et al., 1998). If each member of the solution vector is defined as below:

$$\underset{\sim}{x} = \left[\underset{\sim}{x}_{\bullet\bullet}, \underset{\sim}{f}^R_{\bullet\bullet}, \underset{\sim}{r}^R_{\bullet\bullet}, \underset{\sim}{P}^R_{\bullet\bullet}, \underset{\sim}{e}^R_{\bullet\bullet}\right]^T$$

$$\underset{\sim}{x}_{\bullet\bullet} = [x_{11}, x_{12}, x_{13}, x_{14}, \dots, x_{91}, x_{92}, x_{93}, x_{94}]^T \qquad \in R^+$$
$$\underset{\sim}{f}^R_{\bullet\bullet} = [f_{11}, f_{12}, f_{13}, f_{14}, \dots, f_{91}, f_{92}, f_{93}, f_{94}]^T \qquad \in R^+$$
$$\underset{\sim}{r}^R_{\bullet\bullet} = [r_{11}, r_{12}, r_{13}, r_{14}, \dots, r_{91}, r_{92}, r_{93}, r_{94}]^T \qquad \in \{0,1\}$$
$$\underset{\sim}{P}^R_{\bullet\bullet} = [P_{11}, P_{12}, P_{13}, P_{14}, \dots, P_{91}, P_{92}, P_{93}, P_{94}]^T \qquad \in \{0,1\}$$
$$\underset{\sim}{e}^R_{\bullet\bullet} = [e_{11}, e_{12}, e_{13}, e_{14}, \dots, e_{91}, e_{92}, e_{93}, e_{94}]^T \qquad \in \{0,1\}$$

Among the above five subsidiary vectors, the value of subsidiary vectors components $\underset{\sim}{f}^R_{\bullet\bullet}$, $\underset{\sim}{r}^R_{\bullet\bullet}$, $\underset{\sim}{P}^R_{\bullet\bullet}$ and $\underset{\sim}{e}^R_{\bullet\bullet}$ are produced randomly in the determined spans and the value of each of them is copied in vector $\underset{\sim}{x}$. With regard to the primary conditions of the traffic volume and dependency of the variables of situation x_{jt} upon equations (17), their value is produced implicitly based on the size of decision making variables. Having produced the present population, the subsidiary vectors $\underset{\sim}{x}_{\bullet\bullet}$ and $\underset{\sim}{f}^R_{\bullet\bullet}$ are separated as per relation (25).

$$\underset{\sim}{x}' = \left[\underset{\sim}{x}_{\bullet\bullet}, \underset{\sim}{f}^R_{\bullet\bullet}\right]^T \quad (25)$$

Based on probable simplex method, first the solutions mean vector is calculated. So, corresponding components of all members of the population in each generation are added and divided into the number of population members. For example, if $\underset{\sim}{x}^m, \dots, x^2, x^1$ are solutions related to one generation, the mean vector will be:

$$\bar{\underset{\sim}{x}}' = \frac{\underset{\sim}{x}^1_{\bullet\bullet} + \underset{\sim}{x}^2_{\bullet\bullet} + \dots + \underset{\sim}{x}^m_{\bullet\bullet} + \underset{\sim}{f}^{1,R}_{\bullet\bullet} + \underset{\sim}{f}^{2,R}_{\bullet\bullet} + \dots + \underset{\sim}{f}^{m,R}_{\bullet\bullet}}{m} \quad (26)$$

So, components of the mean vector are:

$$\bar{\underset{\sim}{x}} = \left[\bar{x}_{11}, \dots, \bar{x}_{11}, \bar{f}^R_{11}, \dots, \bar{f}^R_{94}\right]^T$$

Now with regard to the value of solution vector components in the relation (26), travel time function vector is calculated in the relation (27).

$$\bar{t}_{jt} = \frac{l_j}{\bar{v}_j} + \alpha \bar{x}_{jt} \quad (27)$$

Then the value of time vector components is placing in the limitations. And dominant value of components of vectors $\bar{e}, \bar{P}, \bar{r}$ is modified by generation member modification subprogram and copied instead of prior random values. By implementing this program, the rate of violation of limitations related to zero and one variables and equations of situation (17) becomes zero. And violation of other limitations is calculated by using relation (22). For example, if travel time function is defined in the first arc and first period as below:

$$t = 3 + 0.04\,\bar{x}_{11},$$
$$\bar{t}_{11} \le 15 + M * \bar{r}_{11},$$
$$\bar{t}_{11} > 15 - M * \bar{y}_{11},$$
$$\bar{r}_{11} + \bar{y}_{11} = 1,$$

Now based on the value of $\bar{t}_{11}$, the value of variables $\bar{r}_{11}$ and $\bar{y}_{11}$ is modified. So, continuous and discrete components of the new point are produced and added to the current population.

Fourth stage: use of reproduction and crossover operators during the evolutionary process

In this stage, fitness of each member of the present generation is calculated based on proper scaling of fitness function. Also by using arithmetic operator, the operation related to crossover operator is done (Gen and Gheng, 1997). Assuming that X_1 and X_2 are two vectors of chromosomes, two new vectors of chromosomes may be produced by using uniform random numbers generator and two current vectors. If λ is a random number produced in [0, 1], two new points are:

$$X'_1 = \lambda X_1 + (1-\lambda)X_2$$
$$X'_2 = (1-\lambda)X_1 + \lambda X_2$$

Fifth stage: mutation subprogram implementation

Mutation operator used in this paper is Direction-Based Mutation operator. This operator was first offered by Gen and colleagues (Gen and Gheng, 1997). In the mutation subprogram, the member that has been received is modified by using generation member modification subprogram. Then by selecting randomly for each component of subsidiary vector $\tilde{f}$, a random value between zero and 500 (0-500) is produced and replaced. Then by generation member modification subprogram, the obtained member is modified. If violation of limitations for the obtained member is less than the member before changing the component value of vector $\tilde{f}$, the last member is replaced with the obtained member; otherwise, we disregard replacement and this operation is repeated for all components of vector $\tilde{f}$.

Some solutions existing in each generation may be unjustified and so this subprogram first modifies the member it has received by generation member modification subprogram. Then this operation is done for each component of vector f to ensure that all obtained solutions are justified. The best member has zero violation and the highest fitness. It must be noted by crossover and mutation operators, more points of solution space are searched and possibility of finding better solutions is more. Also mutation operator prevents from pre-mature convergence. Besides mutation operator, the immigrant member is also used in the evolutionary process in order to prevent from pre-mature convergence (Grenfenstette, 1986).

4. Conclusions and results analysis

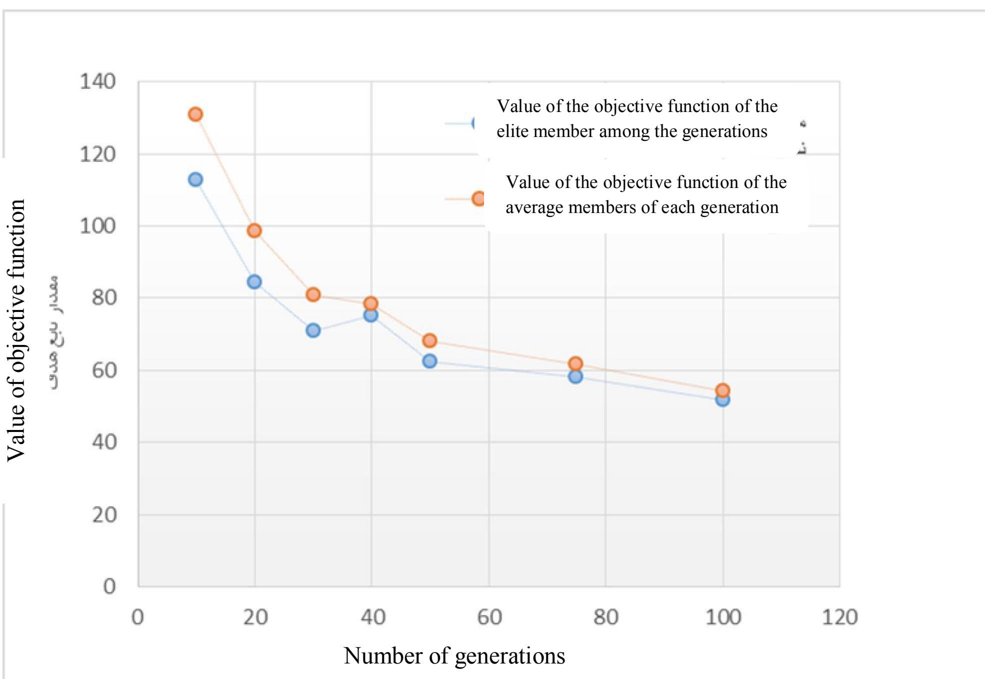

Diagram 1- number of generations based on value of objective function of elite member

To study function of the designed genetic algorithm, different tests were conducted. The diagram shows that by increase in the number of generations, the value of objective function of the elite member is reduced.

As mentioned earlier, since the problem objective function is minimal, fitness function is increased by reducing the objective function. The reduction of objective function value is due to the fact that based on evolutionary process of the genetic algorithm, by increase in the number of generations and exertion of genetic operators on the generation members, the value of objective function and also dependency of new generation members upon the primary generation members are reduced as per figure 3. It is inferred from the above diagram that the order of objective function value is not descending totally which is due to random structure of genetic algorithm and lack of consistency of chromosomes length with Goldberg theory (Goldberg, 1989). Because based on Goldberg theory, length of chromosomes must be short and also if there is a minor interaction between the components (genes), a proper coding method may strengthen the form of constituting elements. Since the length of chromosomes was not short in the structure of the presented mathematical model and the interaction of components is inevitable, the value of objective function is influenced.

According to figure 3, the results of the proposed genetic algorithm in terms of time of implementation and efficiency are presented in table 3.

Table 3- parameters of solving the proposed model by the genetic algorithm

Solving the model by the genetic algorithm		
Problem No.	*Implementation time (minute)*	*Value of the ideal objective function*
1	*1.25*	*118.5*
2	*1.25*	*112.7*
3	*1.3*	*104.2*
4	*1.7*	*97.8*
5	*1.8*	*88.1*
6	*2.2*	*84.4*
7	*2.6*	*71.06*

The model proposed in this paper receives the input data based on the information related to the average travel time in the arcs and estimation of the average travel time function in the arcs and also information of travels supply and demand between origins and destinations and saturated traffic in the intersections, and distributes flows optimally between origins and destinations based on the optimization process in order to fulfill limitations existing in the network and optimize the related goals. As mentioned earlier, when the zero-one variables are discrete, time of solving the model will be very long. By continuous zero-one variables in [0, 1] and solving the model in two stages, the time of solving the model was reduced considerably. Solving multiple problems showed that time of solving the proposed model in these conditions was reduced to ten to twelve minutes. Under these conditions, time of solving the model in the problems with larger dimensions is reduced less. That's why a particular genetic algorithm was designed and the proposed model was solved by that. Solving problems with different weights showed that time of solving the model in the genetic algorithm is shorter due to finite number of generations and also simultaneous search from several points instead of one point in the decision making space. Therefore, since the genetic algorithm is an innovative algorithm and acts based on a natural selection and evolutionary process, multiplicity and interaction of parameters of the genetic algorithm and finite number of generations are effective on the value of the objective function. Also the difference between the values of the objective function obtained from genetic algorithm solutions is due to different generation numbers. By increase in the number of generation, better local optimal solutions are obtained.

4.1 Suggestions based on the research findings

The future research may be divided into two major groups. First, some parameters of the proposed model may be designed in a fuzzy form and the results of solving the model optimization with fuzzy parameters and certain parameters can be compared. Also analysis of the model parameters sensitivity in the certain and fuzzy states may be compared and measured. Second, more efficient algorithms may be provided for solving the proposed model when decision making variables and situation variables are bound to integers.

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