APPLICATION OF ELZAKI TRANSFORMATION WITH LOW CRITICAL BUCKLING LOAD

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Abstract: These are considerably long in comparison with their lateral dimensions and hence buckle when the axial load approaches a certain critical value known as critical buckling load. In this paper, we present Elzaki transformation means for discussing the Euler's theory of very long columns with low critical buckling loads to obtain the Euler's formula for critical or buckling load. It is a powerful mathematical means which is generally applied in different areas of science, engineering and technology for solving ordinary or partial differential equations without finding their general solutions.
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INTRODUCTION

Elzaki Transformation applied in solving boundary value problems in most of the science and engineering disciplines [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. It also comes out to be very effective tool to analyze differential equations with delta function [12,13, 14,15,16,17]. The differential equations are generally solved by adopting Laplace transform method or convolution method of residue theorem method [18,,19, 20,21,22,23, 24]. In this paper, we present a new technique called Elzaki transform to analyze differential equations with delta function.

BASIC DEFINITIONS

2.1 Elzaki Transform

If the function $\hbar(y)$, $y \geq 0$ is having an exponential order and is a piecewise continuous function on any interval, then the Elzaki transform of $\hbar(y)$ is given by

$$E\{\hbar(y)\} = \bar{\hbar}(p) = p\int_0^\infty e^{-\frac{y}{p}}\hbar(y)dy.$$

The Elzaki Transform [1, 2, 3] of some of the functions are given by

- $E\{y^n\} = n!\,p^{n+2}$, where $n = 0,1,2,..$
- $E\{e^{ay}\} = \frac{p^2}{1-ap}$,
- $E\{sinay\} = \frac{ap^3}{1+a^2p^2}$,
- $E\{cosay\} = \frac{ap^2}{1+a^2p^2}$,
- $E\{sinhay\} = \frac{ap^3}{1-a^2p^2}$,
- $E\{coshay\} = \frac{ap^2}{1-a^2p^2}$.

2.2 Inverse Elzaki Transform

The Inverse Elzaki Transform of some of the functions are given by

- $E^{-1}\{p^n\} = \frac{y^{n-2}}{(n-2)!}$, $n = 2,3,4 ...$
- $E^{-1}\{\frac{p^2}{1-ap}\} = e^{ay}$
- $E^{-1}\{\frac{p^3}{1+a^2p^2}\} = \frac{1}{a}\sin ay$
- $E^{-1}\{\frac{p^2}{1+a^2p^2}\} = \frac{1}{a}\cos ay$
- $E^{-1}\{\frac{p^3}{1-a^2p^2}\} = \frac{1}{a}\sin hay$
- $E^{-1}\{\frac{p^2}{1-a^2p^2}\} = \frac{1}{a}\cos hay$

2.3 Elzaki Transform of Derivatives

The Elzaki Transform [1, 2, 3] of some of the Derivatives of h(y) are given by

- $E\{h'(y)\} = \frac{1}{p}E\{h(y)\} - p\,h(0)$
  $or\ E\{\hbar'(y)\} = \frac{1}{p}\bar{\hbar}(p) - p\,\hbar(0),$
- $E\{\hbar''(y)\} = \frac{1}{p^2}\bar{\hbar}(p) - \hbar(0) - p\hbar'(0),$
  and so on.

METHODOLOGY

Let a very long vertical column AB of length ‘a’ and having uniform area of cross-section, where A is the upper end of column and B is its lower end. Let ‘y’ be the lateral deflection of a section of the column at a distance ‘x’ from the lower end B, I be the moment of inertia of the section, ‘Y’ is the Young’s modulus of elasticity of the column and‘P’ be the critical buckling load. Now we will discuss four different cases:

Case-I: When both ends A and B of the column are pinned or hinged

In this case, the bending moment [1] at the section is given by

$\ddot{y}(x) + k^2 y(x) = 0$, where $k = \sqrt{\frac{P}{YI}}$.   (4)

Taking Elzaki Transform of equation (4), we get

$E[\ddot{y}(x)] + k^2 E[y(x)] = 0$

This equation gives

$\frac{1}{p^2}\bar{y}(p) - y(0) - p\dot{y}(0) + k^2\bar{y}(p) = 0$ (5)

Applying boundary condition: $y(0) = 0$, equation (5) becomes,

$\frac{1}{p^2}\bar{y}(p) - p\dot{y}(0) + k^2\bar{y}(p) = 0$ Or $\frac{1}{p^2}\bar{y}(p) + k^2\bar{y}(p) = p\dot{y}(0)$ (6)

In this equation, $\dot{y}(0)$is some constant.

Substitute$\dot{y}(0) = A$, equation (6) becomes

$\frac{1}{p^2}\bar{y}(p) + k^2\bar{y}(p) = Ap$ (7)   Or $\bar{y}(p) = \frac{Ap^3}{(1+p^2 k^2)}$

Taking inverse Elzaki transforms [3] of equation (7), we get

$y(x) = \frac{A}{k}\sin(kx)$ (8)

Applying boundary condition: $y(a) = 0$, equation (8) gives $\frac{A}{k}\sin(k a) = 0$

Since A cannot be equal to zero because for A = 0, y = 0. This means that column will not bend at all, which is not possible.

Therefore, sin (k a) = 0

Or ka = n π, where n is an integer greater than equal to zero.

Or$k = \frac{n\pi}{a}$ (9)

The least practical value of n is 1, therefore considering n = 1, we have

$k = \frac{\pi}{a}$

Or $\sqrt{\frac{P}{YI}} = \frac{\pi}{a}$

Or $P = \frac{\pi^2 YI}{a^2}$

This equation provides the Euler’s formula for critical buckling load for very long column which is pinned at its both the ends.

Case-II: When lower end B of the column is fixed and the other end A is free

In this case, the bending moment [1] at the section is given by

$\ddot{y}(x) + k^2[d - y(x)] = 0$, where‘d’ is the deflection at the free end A due to critical buckling load.

Taking Elzaki Transform of equation (4), we get

$E[\ddot{y}(x)] + k^2 E[y(x) - d] = 0$

This equation gives

$\frac{1}{p^2}\bar{y}(p) - y(0) - p\dot{y}(0) + k^2\bar{y}(p) = k^2 p^2 d$ ….. (5)

Applying boundary conditions: $y(0) = 0$ and $\dot{y}(0) = 0$ as the slope at x = 0 is zero, equation (5) becomes,

$\frac{1}{p^2}\bar{y}(p) + k^2\bar{y}(p) = k^2 p^2 d$

Or $\bar{y}(p) = \frac{k^2 p^4 d}{(1+p^2 k^2)}$ (6)

Or $\bar{y}(p) = \frac{k^2 p^4 d}{(1+p^2 k^2)}$ (7)

Or $\bar{y}(p) = \frac{d}{p} - \frac{dp}{(p^2 + k^2)}$

Taking inverse Laplace transforms [4] of equation (7), we get

$y(x) = \frac{d}{k^2} - \frac{d}{k^2}\cos(kx)$

(8)

Applying boundary condition: $y(a) = d$, equation (8) gives

d = d - d cos (k a)

Or cos (k a) = 0

$or\ ka = \frac{(2n-1)\pi}{2}$, where n is an integer greater than equal to zero.

Or $k = \frac{(2n-1)\pi}{2a}$

The least practical value of n is 1, therefore considering n = 1, we have

$k = \frac{\pi}{2a}$

Or $\sqrt{\frac{P}{YI}} = \frac{\pi}{2a}$

Or $P = \frac{\pi^2 YI}{4a^2}$

This equation provides the Euler’s formula for critical buckling load for very long column whose lower end is fixed and upper end is free.

Case-III: When both the ends A and B of the column are fixed

In this case, the bending moment [1] at the section is given by

$\ddot{y}(x) + k^2 y(x) = M$, Where $M = \frac{M_0}{EI}$, $M_0$is the restraint moment at each end.

Taking Elzaki Transform of equation (4), we get

$E[\ddot{y}(x)] + k^2 E[y(x)] = M$

This equation gives

$\frac{1}{p^2}\bar{y}$ (p) $- y$ (0) $- p\dot{y}(0) + k^2\bar{y}$ (p) $=$ M E{1} (5)

Applying boundary conditions: $y(0) = 0$ and $\dot{y}(0) = 0$ as the slope at x = 0 is zero, equation (5) becomes,

$\frac{1}{p^2}\bar{y}(p) + k^2\bar{y}(p) = Mp^2$

Or $\bar{y}(p) = \frac{Mp^4}{(1+p^2\ k^2)}$ (7)

Or $\bar{y}(p) = Mp^2 - \frac{Mp^2}{(1+p^2\,k^2)}$

Taking inverse Laplace transforms [5] of equation (7), we get

y (x) $= \frac{M}{K^2} - \frac{M}{K^2}$ cos(kx) (8)

Applying boundary condition: $y(a) = 0$, equation (8) gives $\frac{M}{K^2} - \frac{M}{K^2}$ cos (k x) = 0

Therefore, cos (k a) = 1

Or k a = 2n π, where n is an integer greater than equal to zero.

Or k = $\frac{2n\pi}{a}$ (9)

The least practical value of n is 1, therefore considering n = 1, we have

k = $\frac{2\pi}{a}$

Or $\sqrt{\frac{P}{YI}} = \frac{2\pi}{a}$

Or P = $\frac{4\pi^2 YI}{a^2}$ (10)

This equation provides the Euler's formula for critical buckling load for very long column whose both ends are fixed.

Case-IV: When lower end B of the column is fixed and the upper end A is hinged or pinned

In this case, the bending moment [1] at the section is given by

$\ddot{y}(x) + k^2 y(x) = H(a-x)$, where $H = \frac{H_0}{EI}$, H_0 is horizontal force at the fixed end B.

Taking Elzaki Transform of equation (4), we get

$E[\ddot{y}(x)] + k^2 E[y(x)] = HE(a-x)$

This equation gives

$\frac{1}{p^2}\bar{y}$ (p) $- y$ (0) $- p\dot{y}(0) + k^2\bar{y}$ (p) $= HE[(a-x)]$ (5)

Applying boundary conditions: $y(0) = 0$ and $\dot{y}(0) = 0$ as the slope at x = 0 is zero, equation (5) becomes,

$\frac{1}{p^2}\bar{y}(p) + k^2\bar{y}(p) = = H\left[\left(\frac{a}{p} - \frac{1}{p^2}\right)\right]$ (6)

Or $\bar{y}$ (p) $= H[\frac{ap}{(1+p^2\,k^2)}$ - $\frac{1}{(1+p^2\,k^2)}$] (7)

Or $\bar{y}(p) = H[\frac{ap}{k^2} - \frac{ap^2}{k^2(1+p^2\,k^2)}$- $\frac{p^3}{k^2} + \frac{ap^3}{k^3(1+p^2\,k^2)}$]

Taking inverse Laplace transforms [6] of equation (7), we get

$y(x)$ =H[$\frac{a}{k^2}$ - $\frac{a}{k^2}$ cos (k x) -$\frac{x}{k^2}$+$\frac{\sin kx}{k^3}$] (8)

Applying boundary condition: $y(a) = 0$, equation (8) gives

H [$\frac{a}{k^2}$ - $\frac{a}{k^2}$ cos (k a) -$\frac{a}{k^2}$ +$\frac{\sin ka}{k^3}$] = 0

Or [- $\frac{a}{k^2}$ cos (k a) +$\frac{\sin ka}{k^3}$] = 0

Or $\frac{a}{k^2}$ cos (k a) =$\frac{\sin ka}{k^3}$

Or tan (k a) = ka

On expanding $\tan ka$ upto 5[th] power of ka and solving we get

Or ka = 4.5 radians

Or $\sqrt{\frac{P}{YI}}a = 4.5\,radians$

Or P = $\frac{20.25YI}{a^2}$

Or P = $\frac{2\pi^2 YI}{a^2}$

This equation provides the Euler's formula for critical buckling load for very long column whose lower end is fixed and upper end is pinned.

CONCLUSION

This paper discussed the Euler's theory of very long columns with low buckling axial loads by means of Elzaki transformation tool. An attempt has made to exemplify the Elzaki transformation method for discussing the Euler's theory of very long columns with low buckling axial loads for obtaining the Euler's formula of critical buckling load. In all the cases discussed, we found that the critical buckling load for very long columns which are subjected to axial loads , is inversely proportional to the square of the length of the column.

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